Bayesian Adjustment For Calibration Uncertainty In Spectral Analysis

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OUTLINE

- Background
- Problem Description
- Methodology Research
- Results
- Future Work

BACKGROUND

- Systematic instrumental uncertainties in astronomical analysis have been generally ignored.
- Error bars on spectral model parameters could be underestimated since the ignorance.
- Despite this, calibration uncertainties are rarely incorporated because no robust principled method is available.
- Our goal is to incorporate the uncertainties by Bayesian Methods.

BACKGROUND

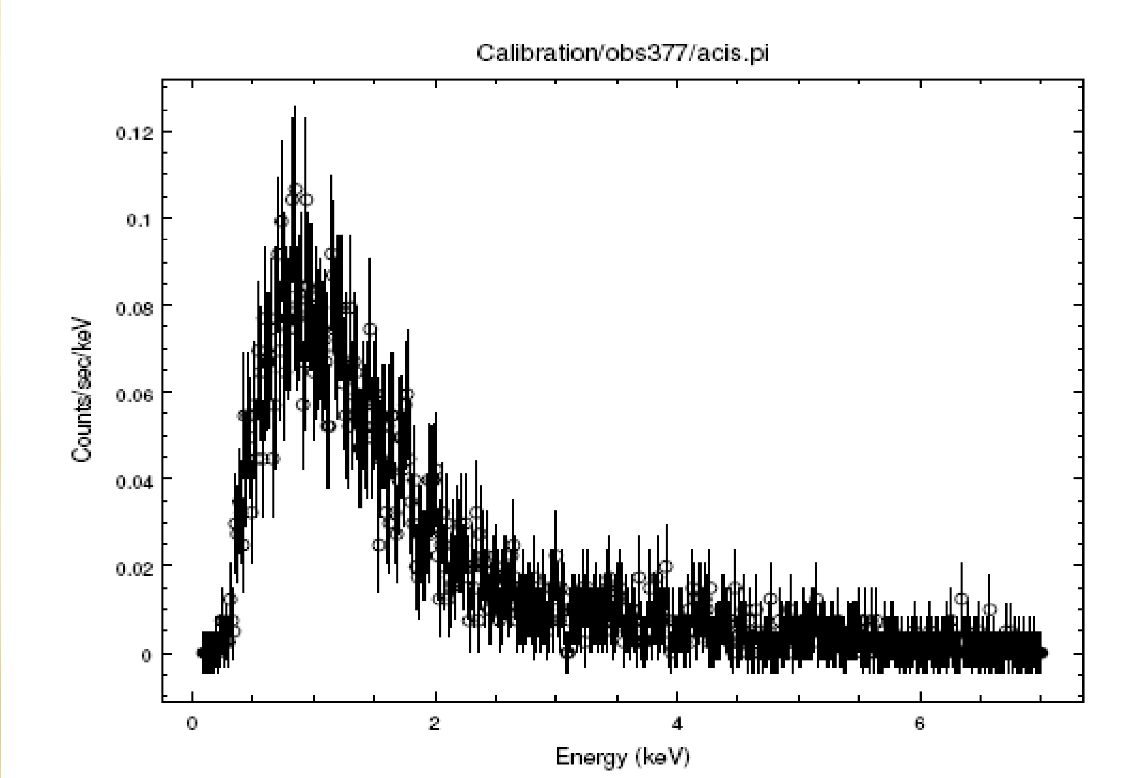
General idea of spectral analysis.

- For example, dataset ob377
- TITLE = EXTENDED EMISSION AROUND A GIGAHERTZ PEAKED RADIO SOURCE
- DATASUM = 4170859839
- DATE = 2006-12-29T16:10:48
- DATE-END = 2000-10-11T00:01:18
- EXPOSURE = 27601.21
- FILTER: 0.0949-7.0007 Energy (keV)

BACKGROUND

General idea of spectral analysis.

- Basically speaking, we are interested in pattern between the photon counts and energy, that's to parameterize the relationship.



BACKGROUND

- A model of telescope response that assumes position and time invariance:

$$\mathcal{M}(E^*;\theta) = \sum_E S(E;\theta)A(E)P(E)R(E^*;E),$$

- E^* : the detector channel at which a photon of energy E is recorded.
- θ : parameters of source model.
- A, P, R are effective area, point spread function, and energy redistribution matrix of the detector.

BACKGROUND

- Set source model as Poisson Distribution with expectation equals to

  xsphabs.abs1*powlaw1d.p1

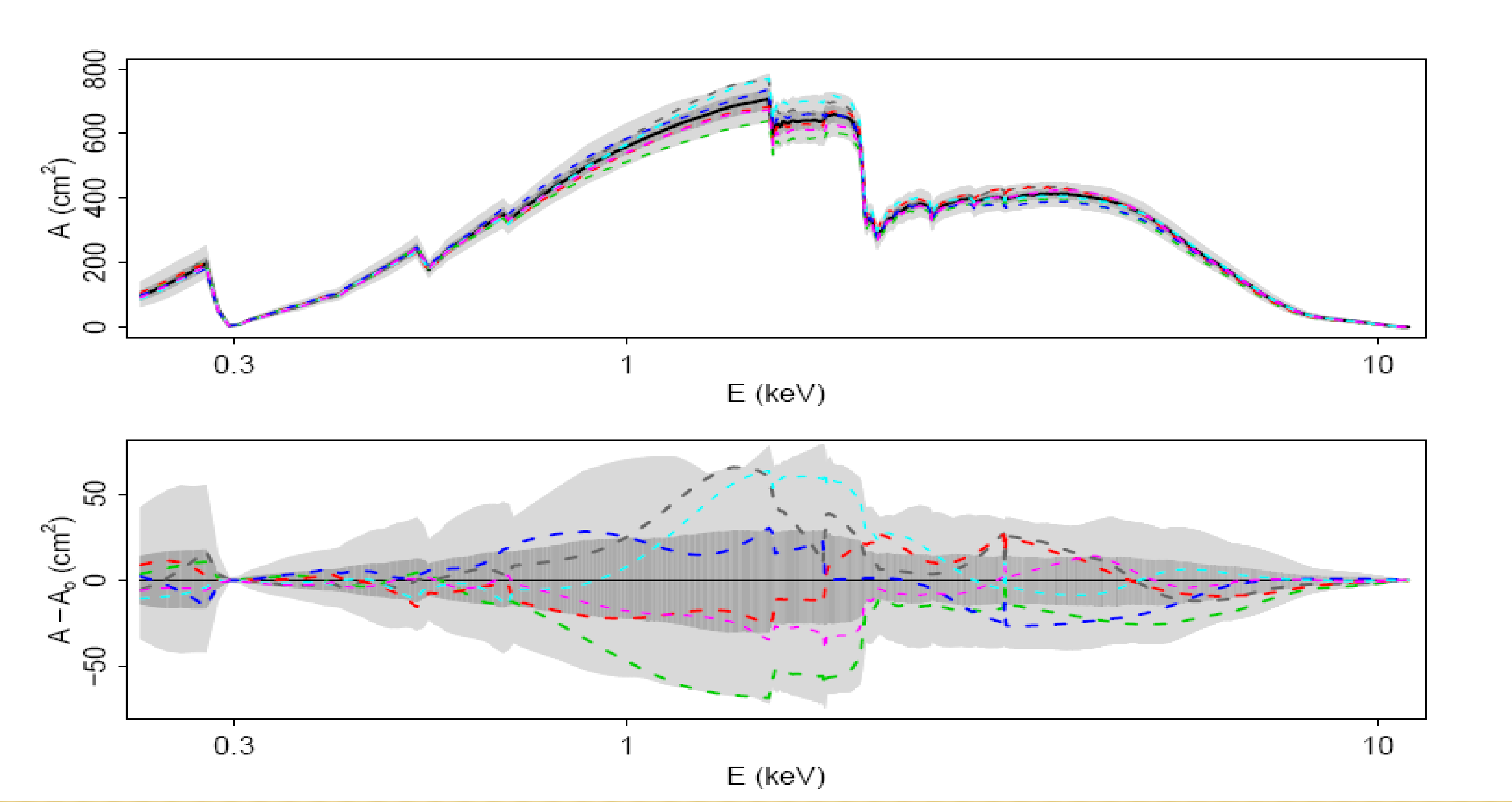
- xsphabs is photo-electric absorption model with one parameter, abs1.nH.
- powlaw1d is a 1-D power-law model with two parameters, p1. gamma and p1.amp1.
- There is another parameter for background, bkg_mdl_c1.factor.

BACKGROUND

- Here, we focus on uncertainty in the effective area for Chandra/ACIS-S in spectral analysis. The goal is to incorporate effective area uncertainties to source parameters' estimate.
- Drake et al. (2006), suggests to generate calibration samples of effective area curves to represent the uncertainty.
- $\{A_1, A_2, A_3 \cdots A_L\}$: calibration samples, where A_l is one of the effective area curve.

BACKGROUND

- Calibration Samples



PROBLEM DESCRIPTION

- The main target and interest is to estimate the source parameter θ , that is $p(\theta | Y, A)$, Y is observed photon counts, and A is the unobserved true affective area curve.
- The prior for $p(A)$ doesn't have parameterized form. Fortunately, we have the calibration samples.

METHODOLOGY RESEARCH

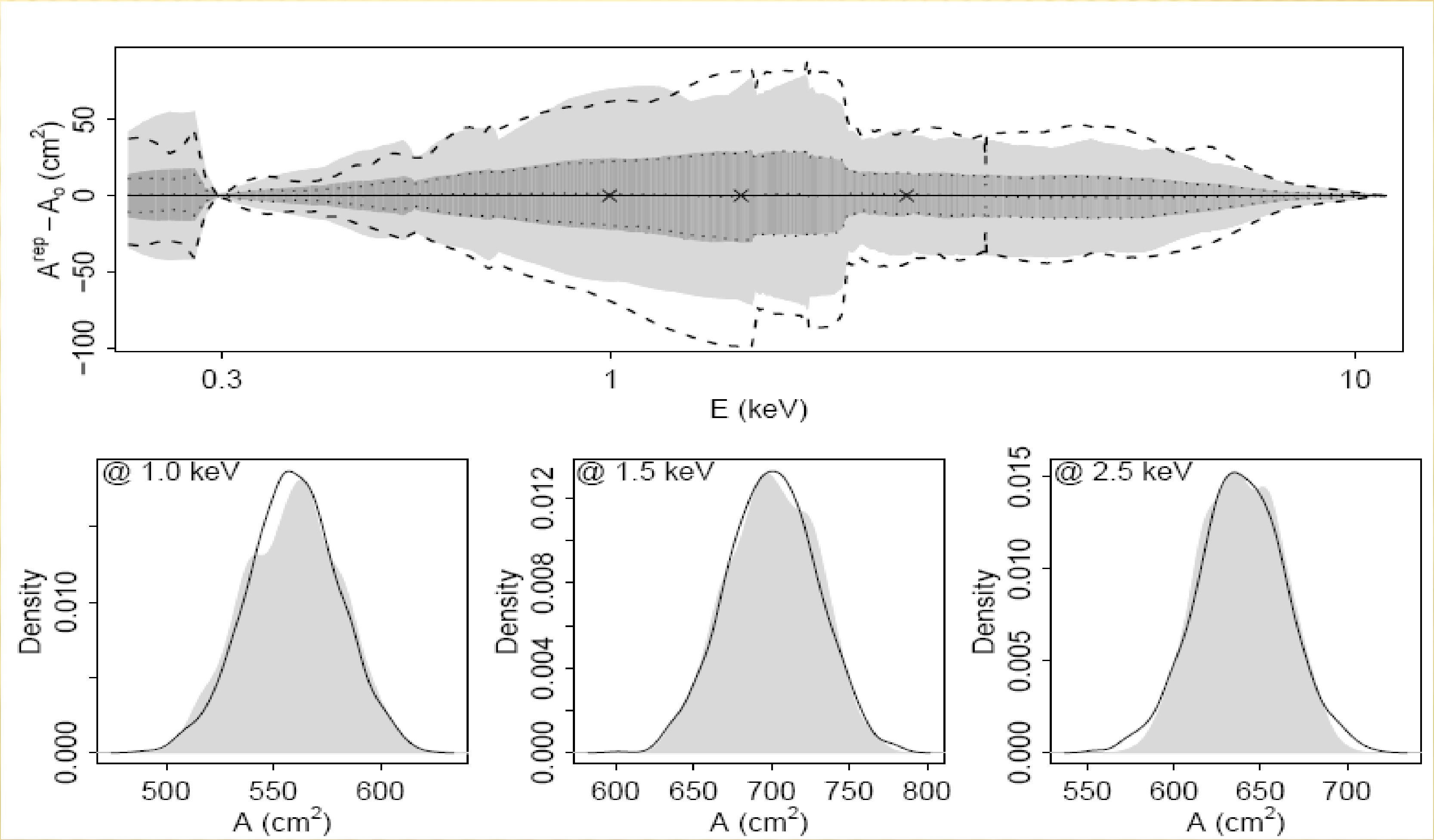
- First, we use PCA to parameterize $p(A)$,
- Suppose $\mathrm{A} = [A_1 - \overline{A}, \cdots, A_L - \overline{A}]_{n*L}$, use SVD, get $\mathrm{AA}^T = V\Lambda V^T$, where V is the eigenvector matrix, and $\Lambda = diag(\lambda_1, \lambda_2, \cdots)$ is the eigenvalue matrix. Here, n is the number of energy bins.
- Then we assume

$$X \sim MN(0, diag(\lambda_1, \lambda_2, \cdots \lambda_k, 0, \cdots, 0))$$

- And, $A = \overline{A} + VX$, k is the number of principle components we pick.

METHODOLOGY RESEARCH

- PCA method nicely parameterized $p(A)$



METHODOLOGY RESEARCH

- Three Different Sampling Schemes for source parameter θ .
- Fixed affective area curve scheme, that is $A = A_0$, where A_0 is the default affective area curve, may not be the true one.
- Sample θ from

$$p(\theta | Y, A_0) \propto L(Y | \theta, A_0) p(\theta)$$

METHODOLOGY RESEARCH

- PragBayes Scheme
- Step One:

  Sample A from $p(A)$
- Step Two:

  Sample θ from

  $$p(\theta \mid Y, A) \propto L(Y \mid \theta, A)\, p(\theta)$$

METHODOLOGY RESEARCH

- FullBayes Scheme
- Step One:

  Sample A from

  $$p(A|Y,\theta) \propto L(Y|\theta,A)\,p(A)$$

- Step Two:

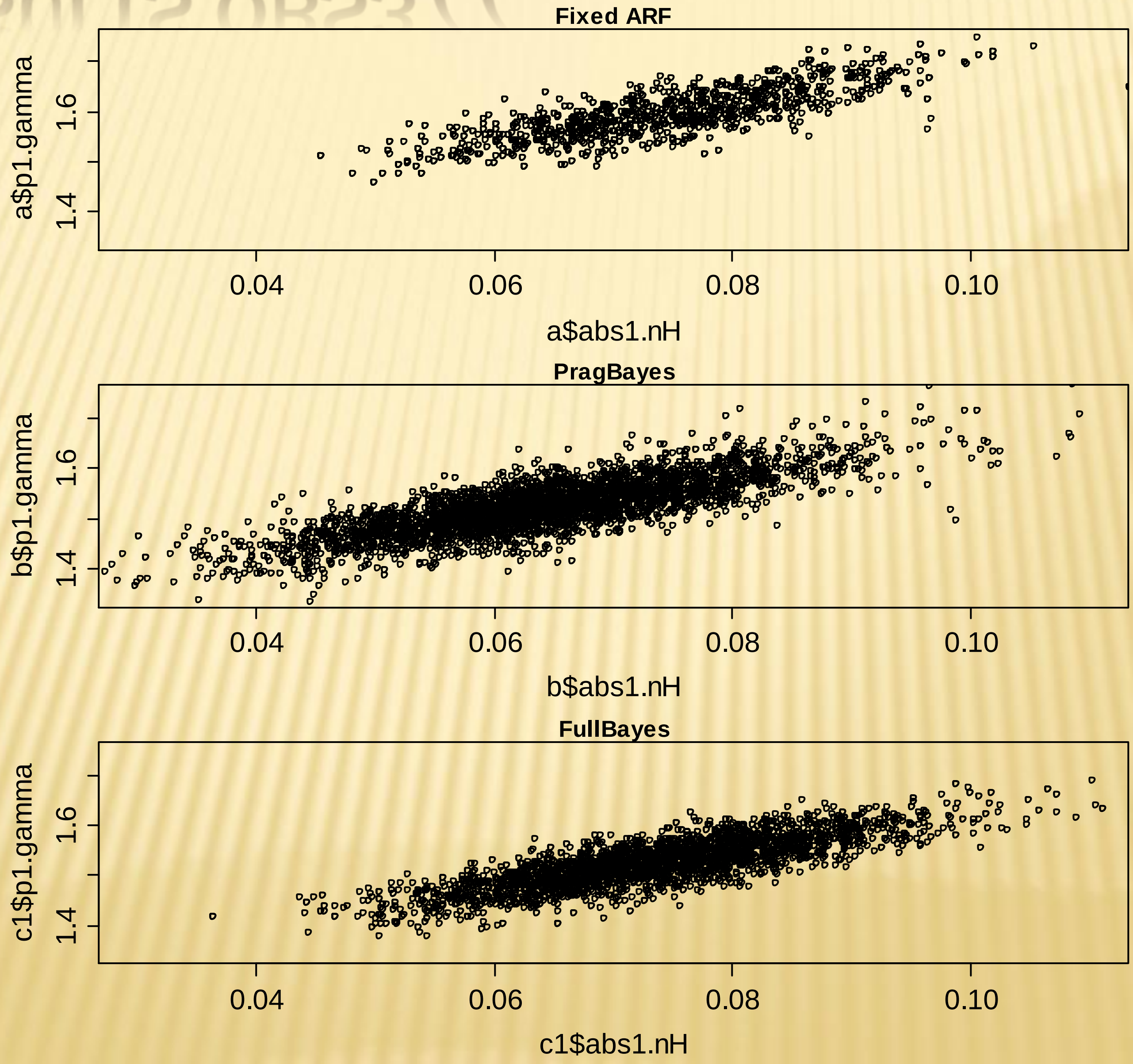
  Sample θ from

  $$p(\theta|Y,A) \propto L(Y|\theta,A)\,p(\theta)$$

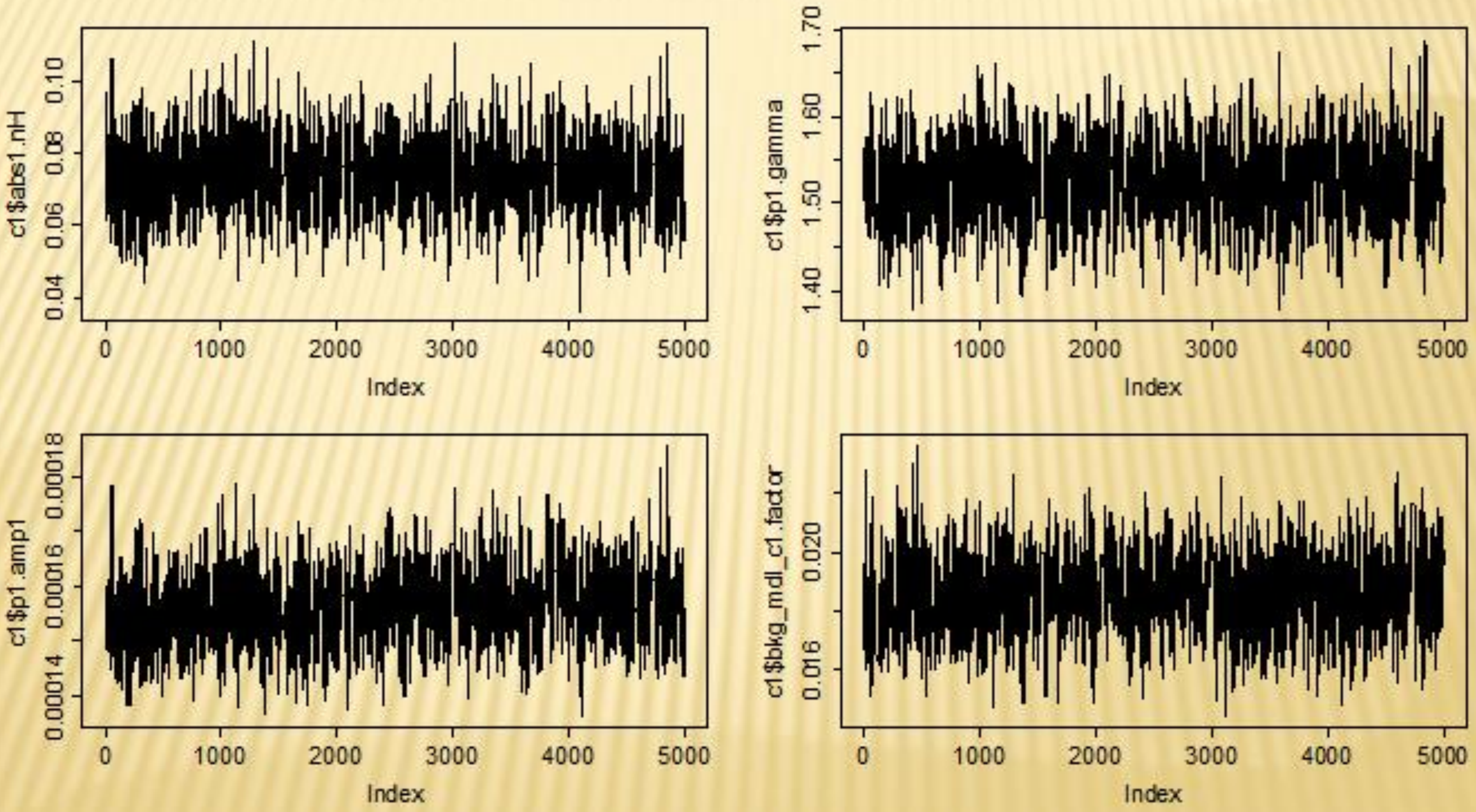
RESULTS, ACCEPTANCE RATE SUMMARY

	Fixed A	**Prag-Bayes**	**Full-Bayes**	
Dataset	Parameter Draw	Parameter Draw	Parameter Draw	A Draw
377	**0.17**	**0.64**	**0.49**	**0.56**
836	**0.19**	**0.62**	**0.38**	**0.51**
866	**0.16**	**0.64**	**0.47**	**0.54**
1602	**0.17**	**0.64**	**0.48**	**0.56**
3055	**0.25**	**0.57**	**0.21**	**0.43**
3056	**0.43**	**0.62**	**0.33**	**0.50**
3097	**0.18**	**0.65**	**0.47**	**0.56**
3098	**0.26**	**0.65**	**0.47**	**0.56**
3099	**0.25**	**0.54**	**0.17**	**0.44**
3100	**0.40**	**0.63**	**0.36**	**0.51**
3101	**0.47**	**0.62**	**0.35**	**0.57**
3102	**0.22**	**0.61**	**0.14**	**0.40**
3104	**0.22**	**0.64**	**0.51**	**0.57**
3105	**0.18**	**0.66**	**0.51**	**0.59**
3106	**0.45**	**0.60**	**0.34**	**0.59**
3107	**0.41**	**0.64**	**0.31**	**0.48**

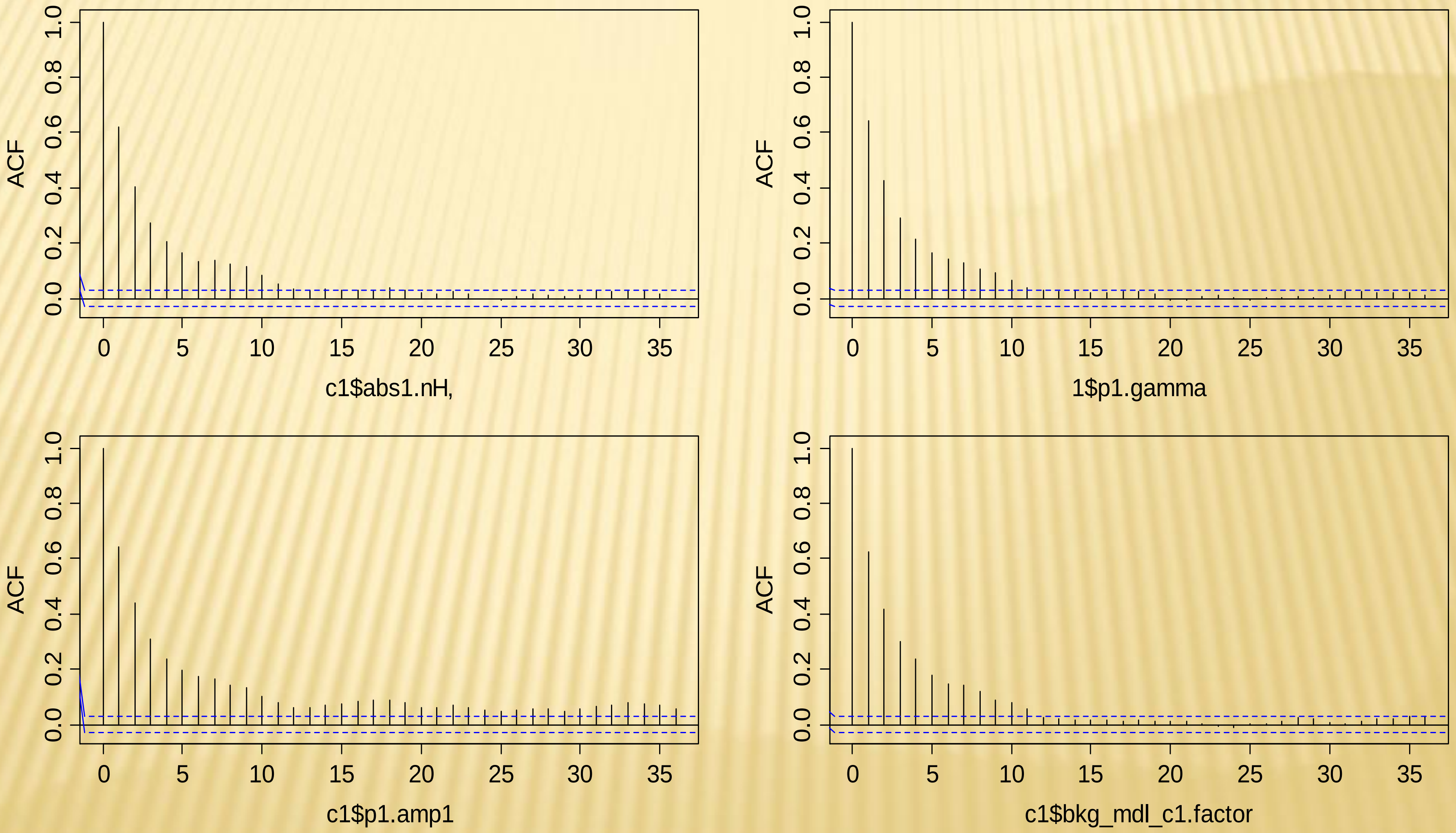
RESULTS,OBS377



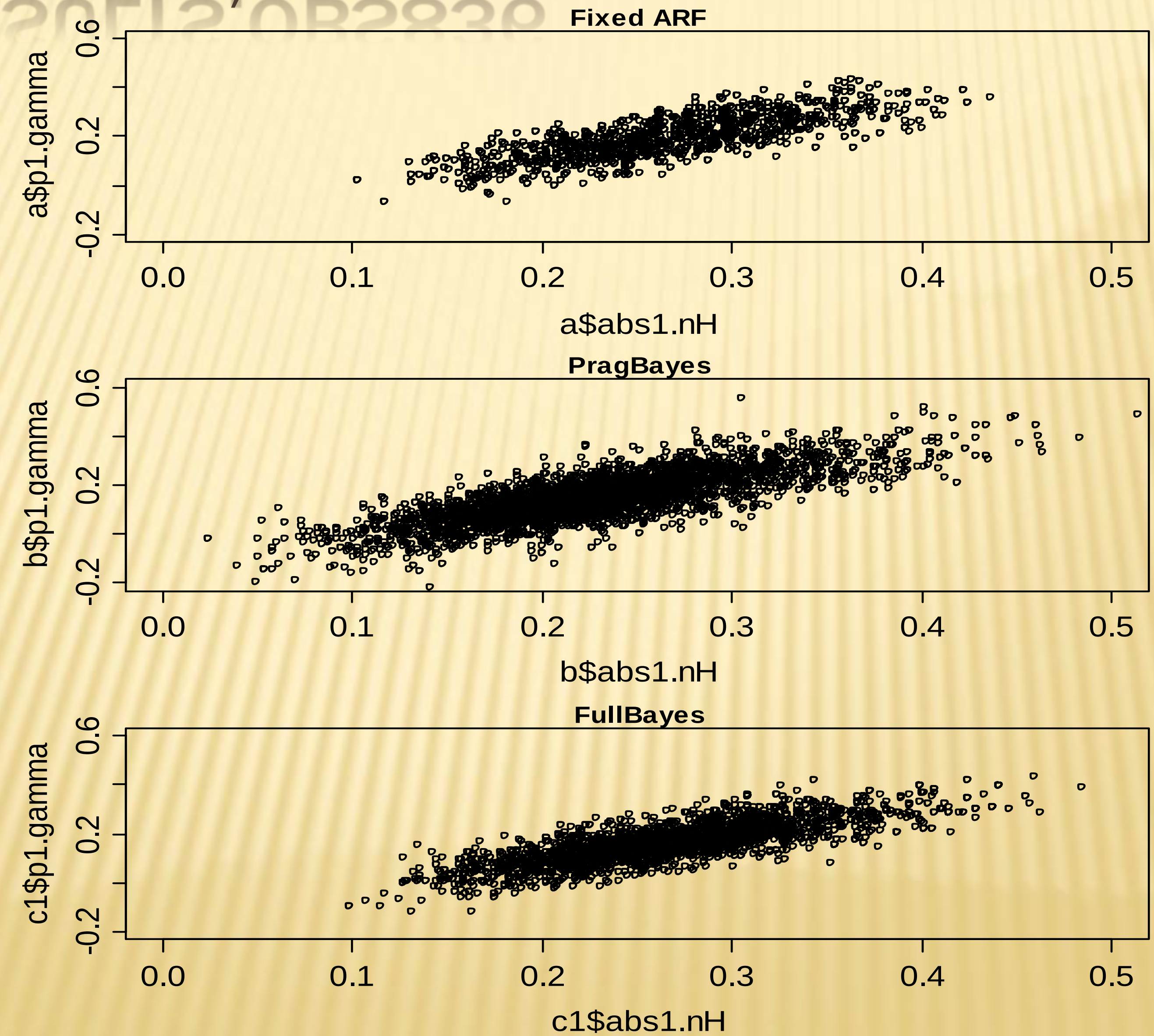
RESULTS,OBS377, FULLBAYES



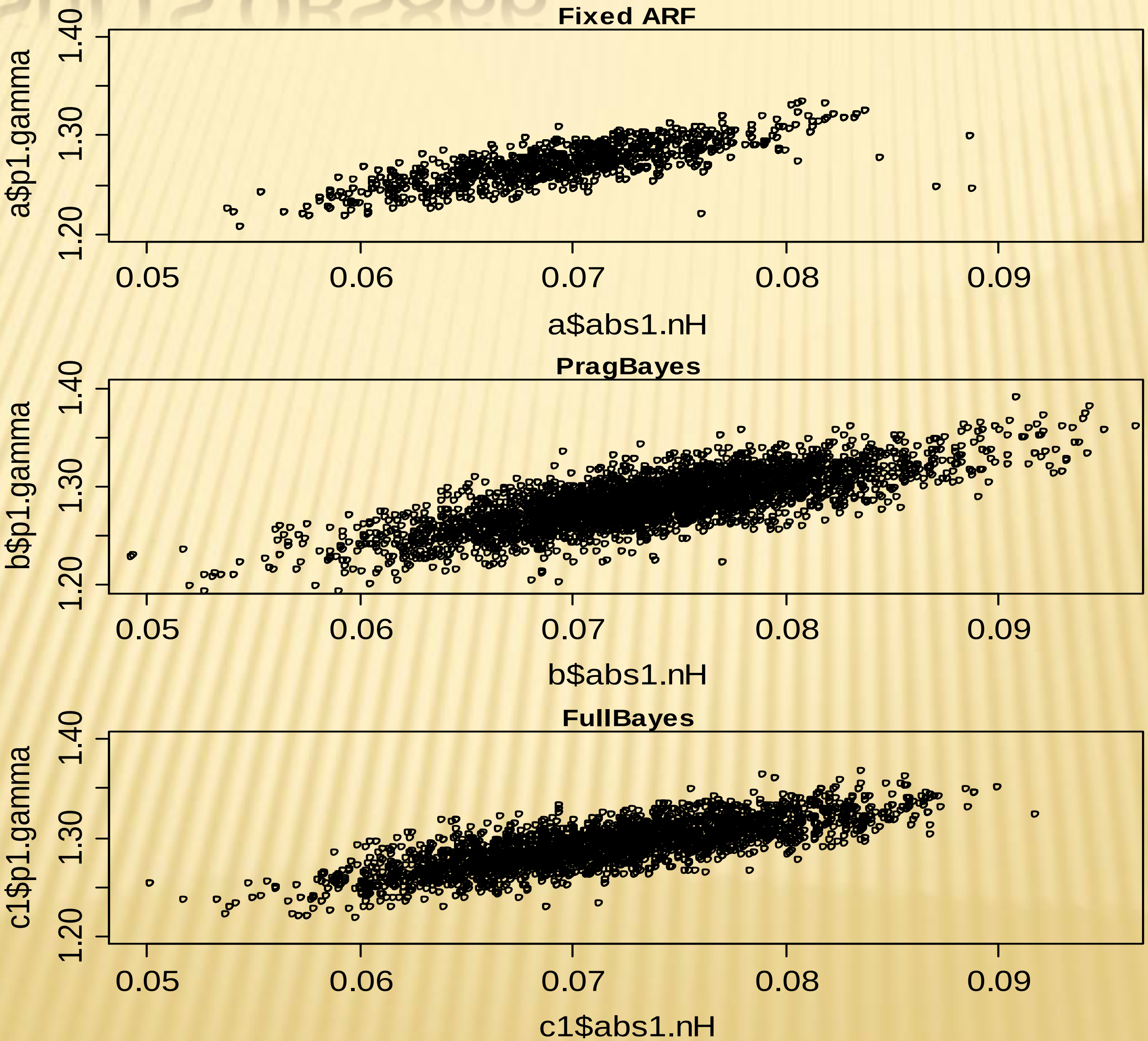
RESULTS,OBS377, FULLBAYES



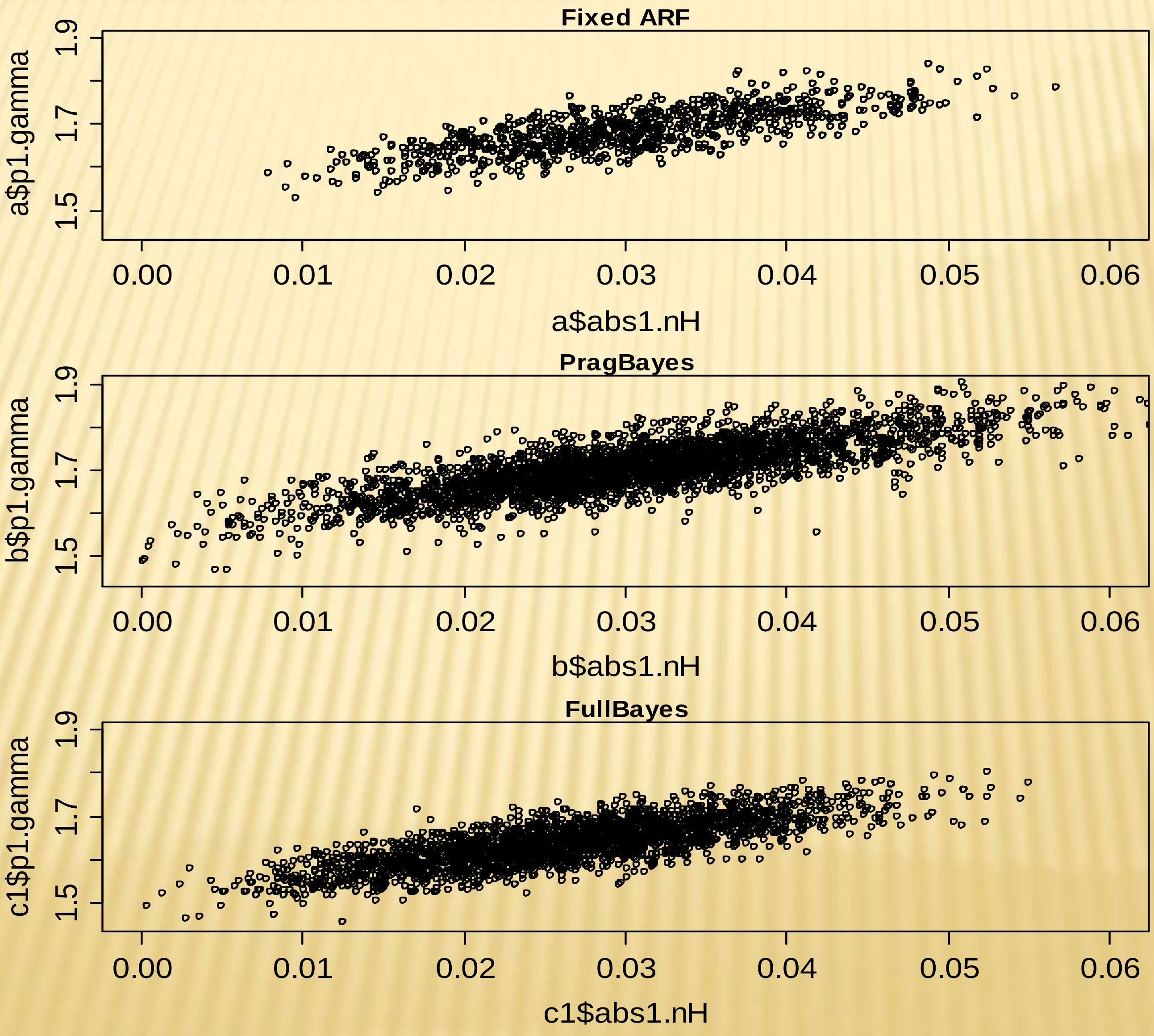
RESULTS,OBS836



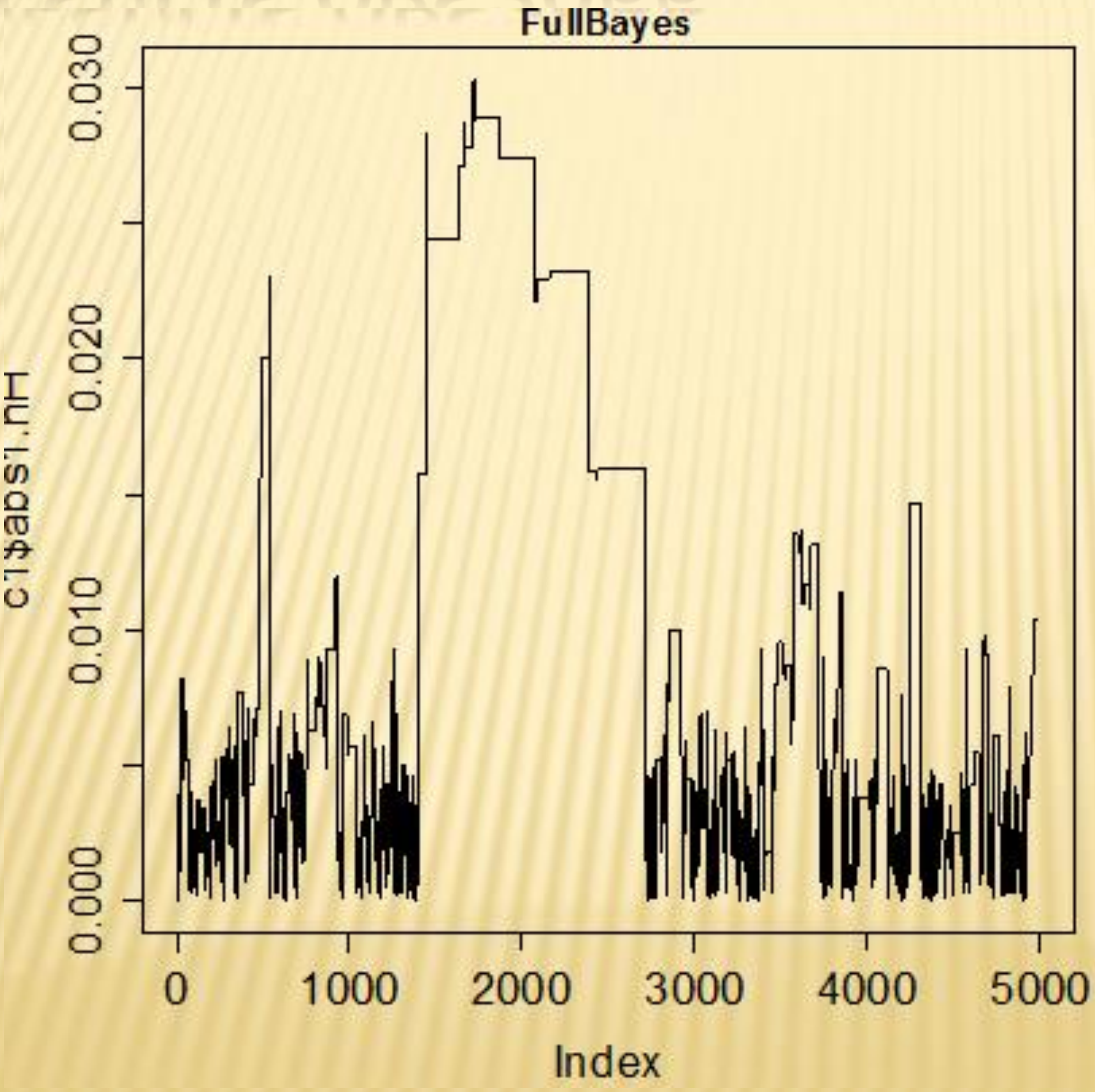
RESULTS,OBS866



RESULTS,OBS1602

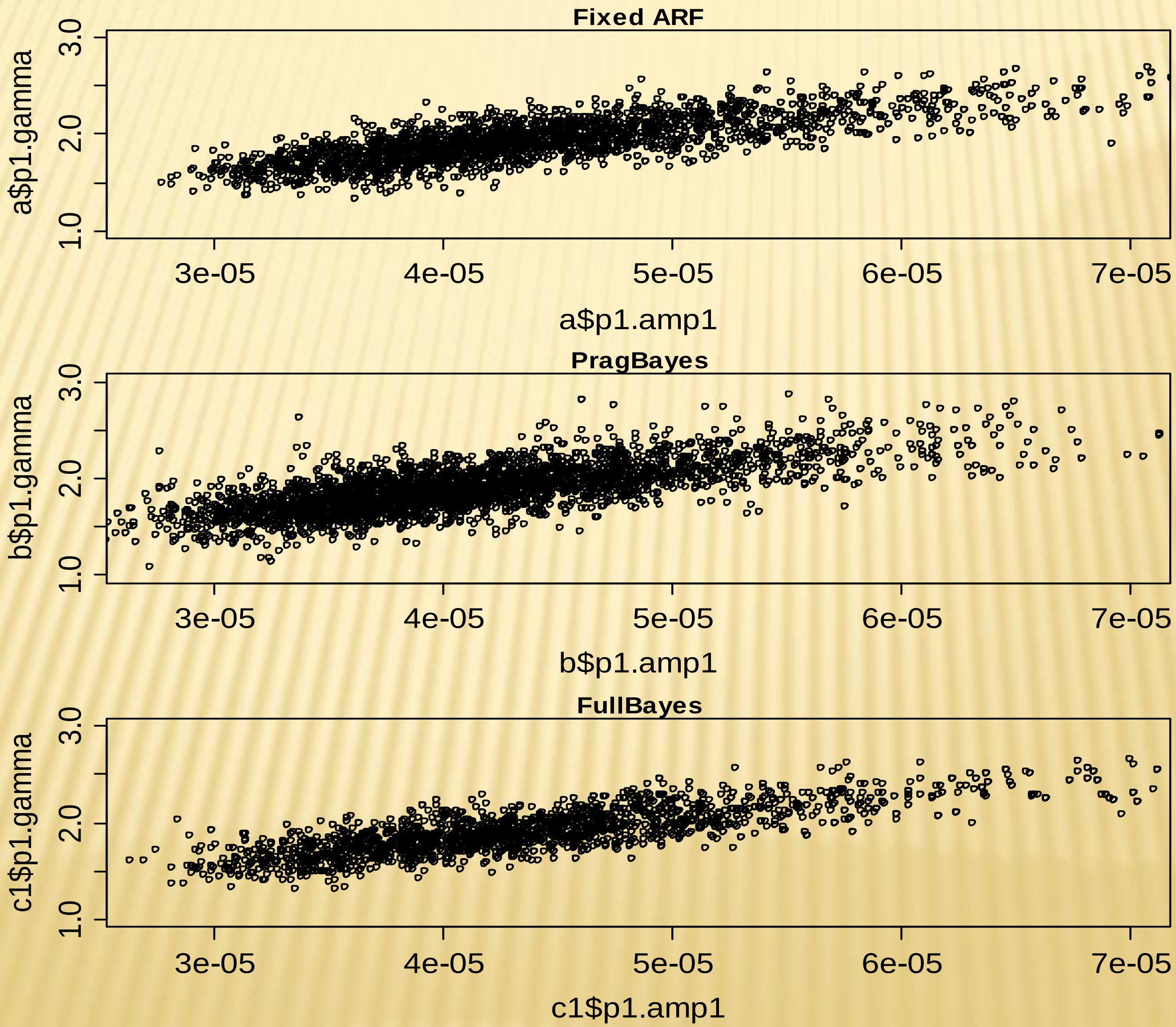


RESULTS,OBS3055

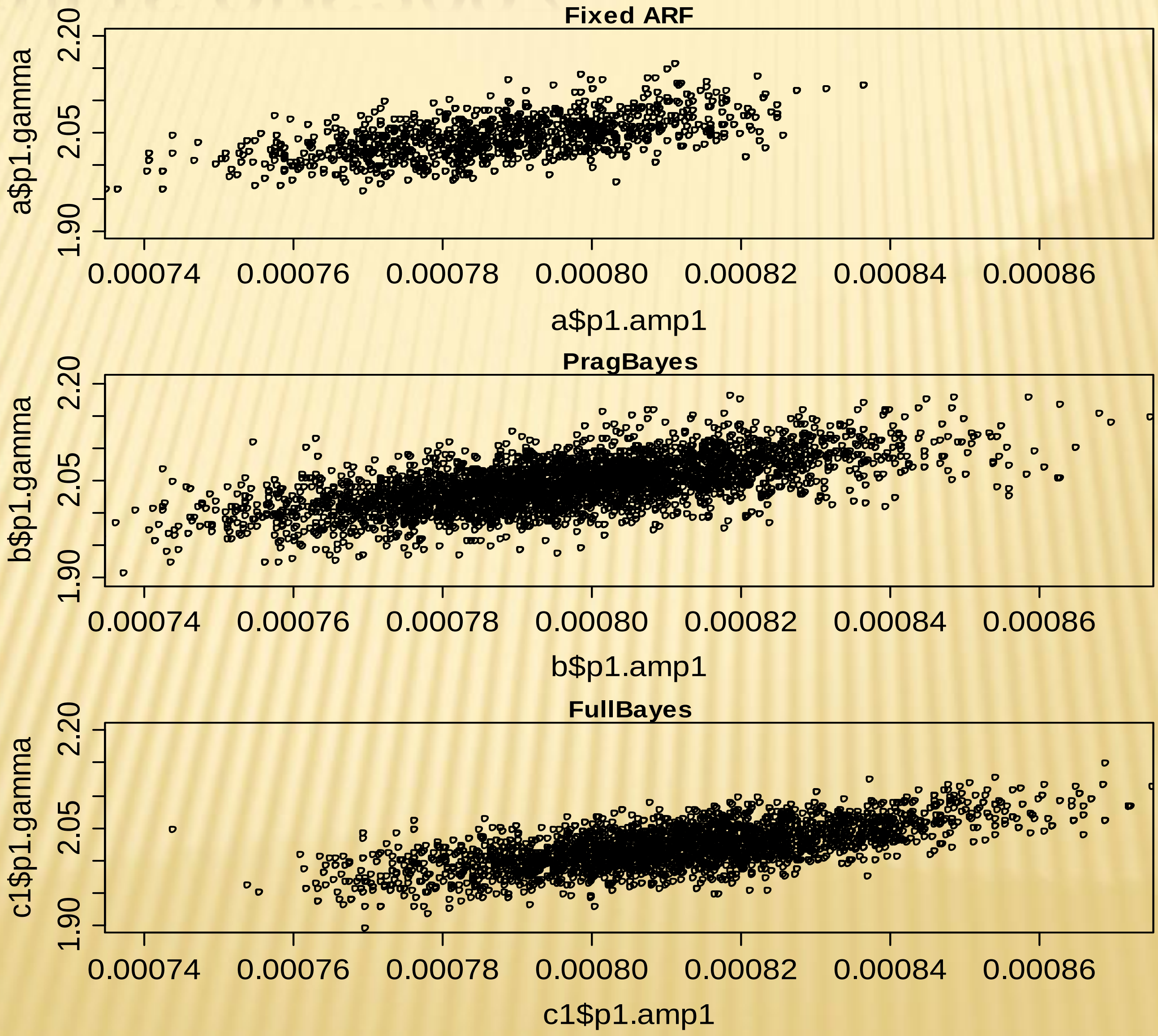


Full Bayes Draws are stuck somewhere for this dataset!!!

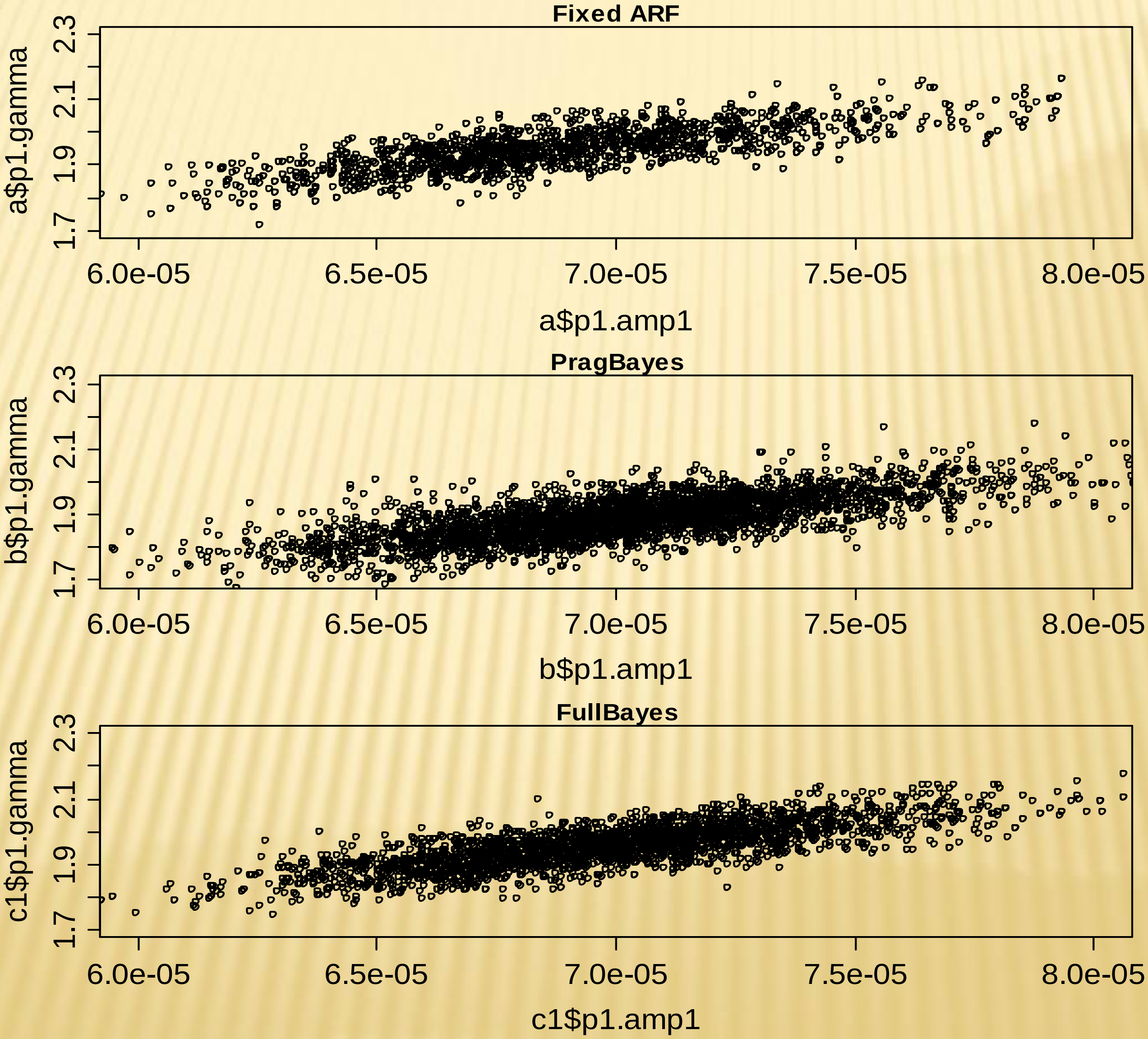
RESULTS,OBS3056



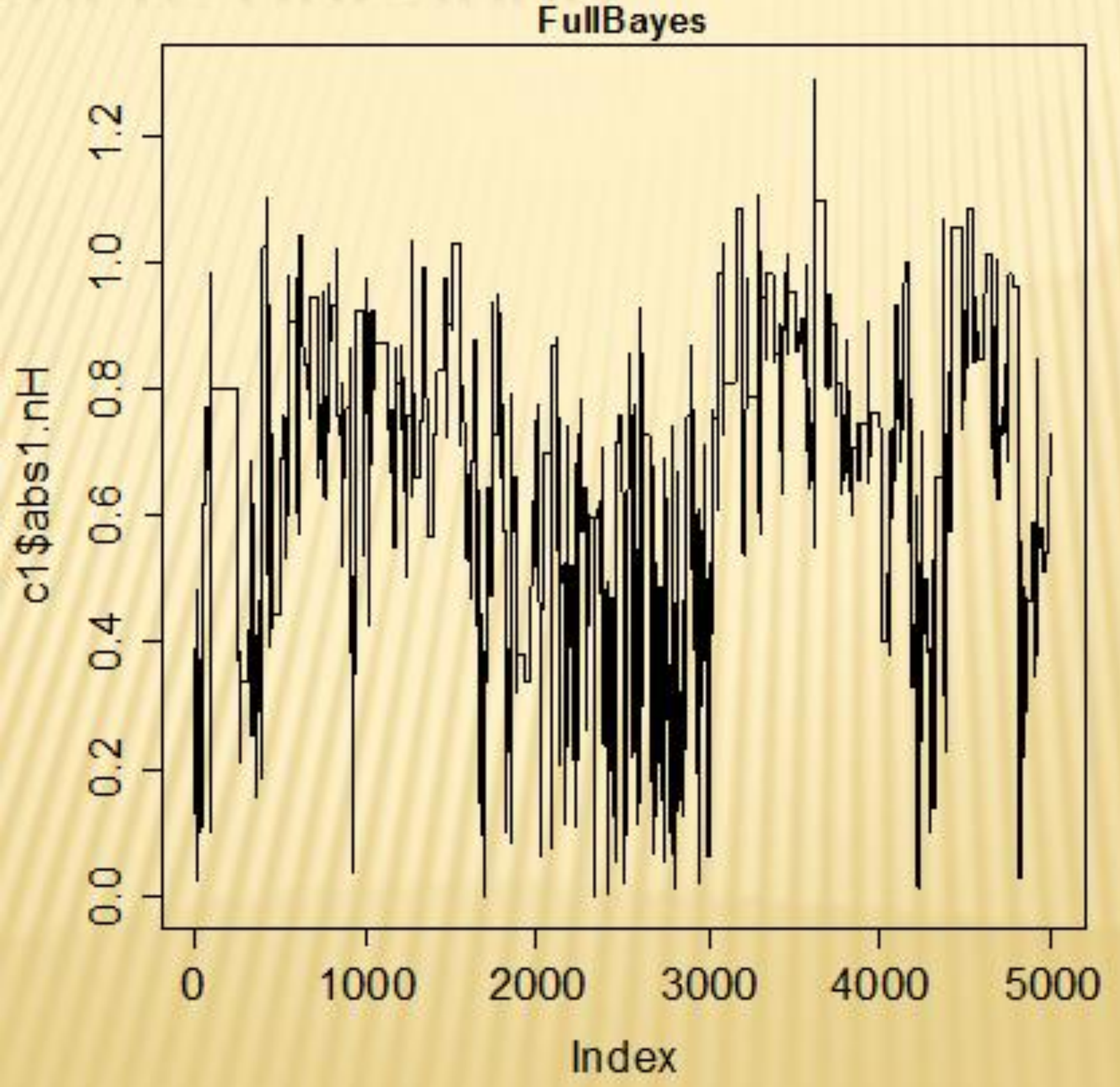
RESULTS,OBS3097



RESULTS,OBS3098

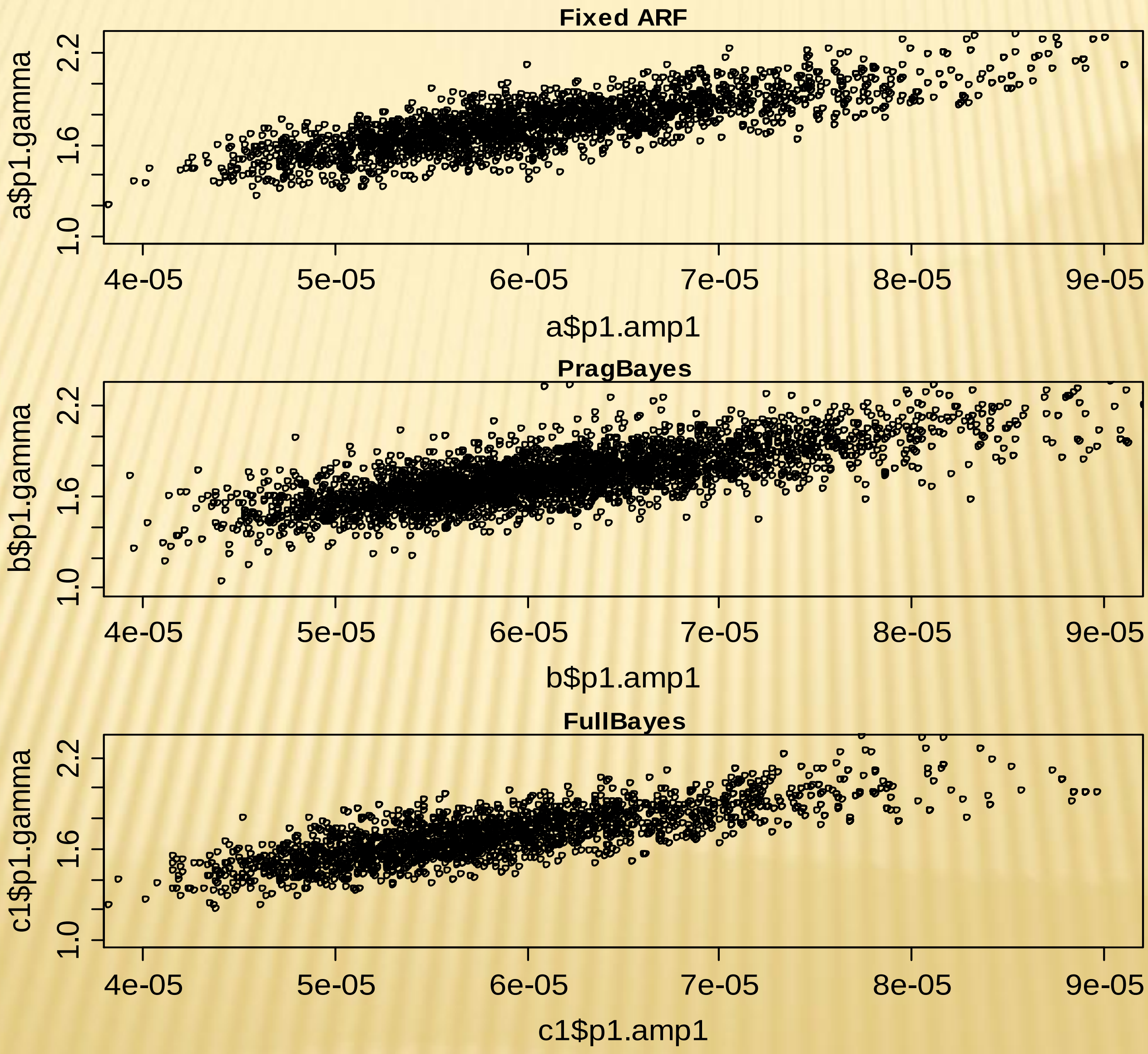


RESULTS,OBS3099

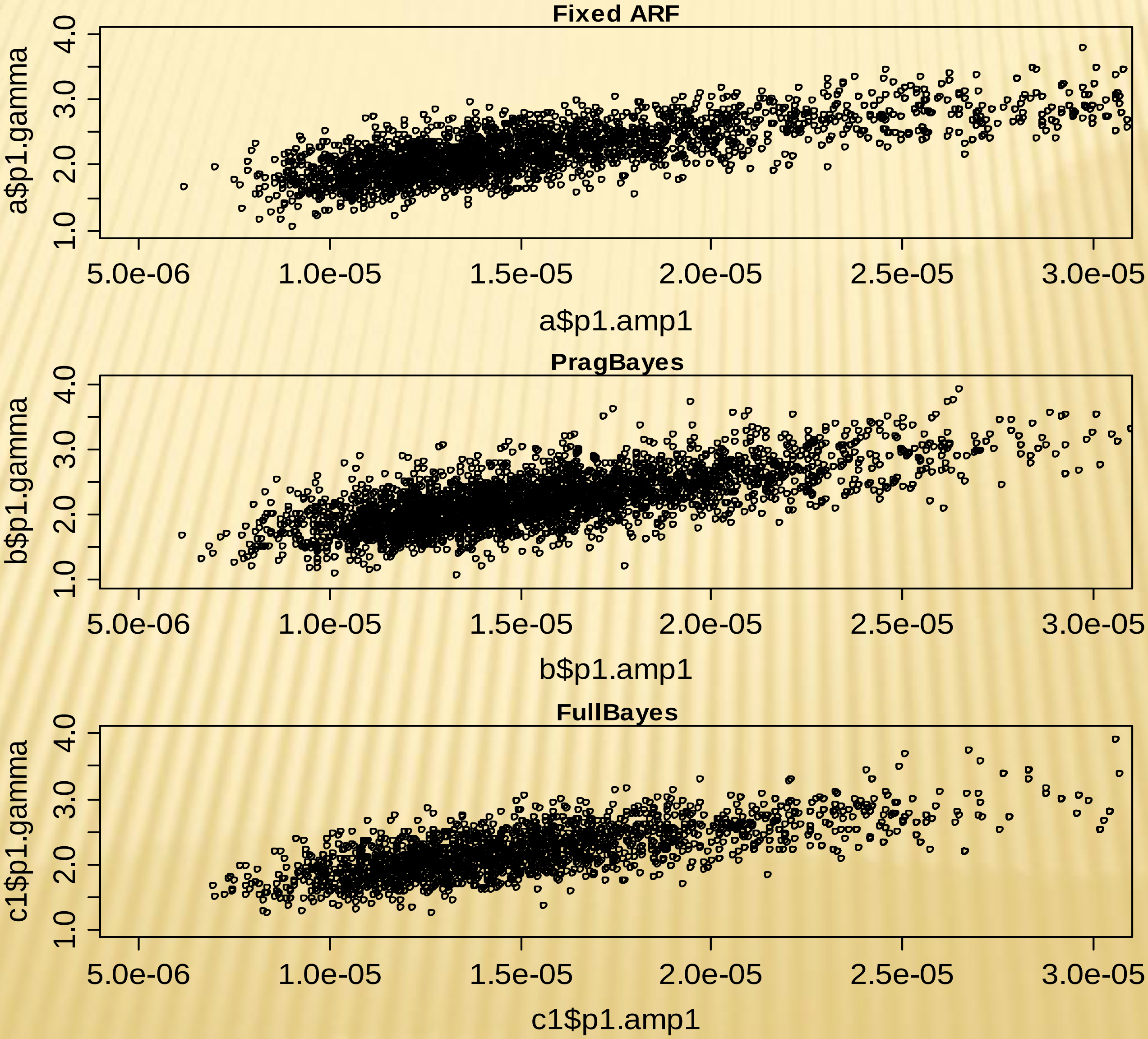


Need more iterations! Maybe need several weeks' run.

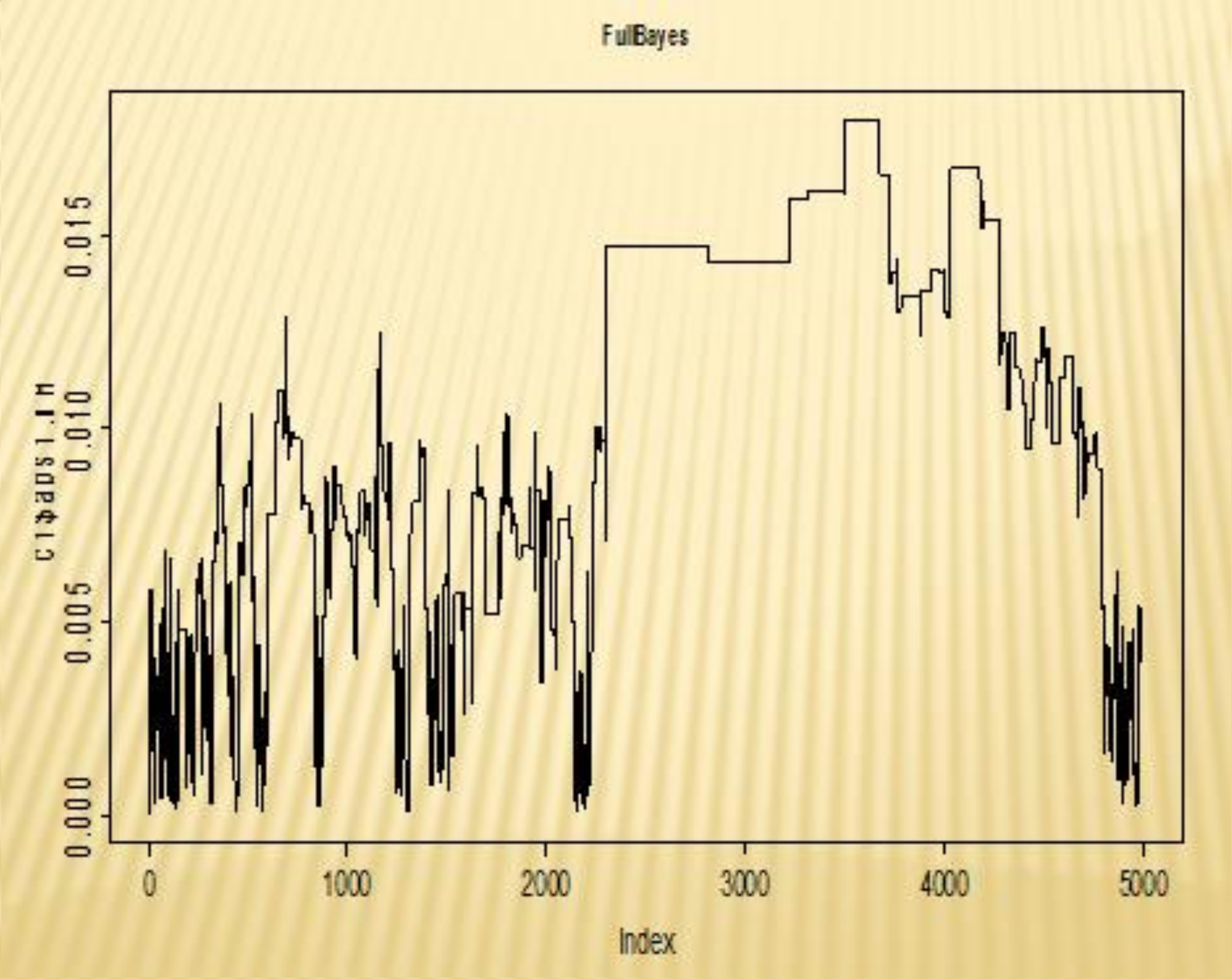
RESULTS,OBS3100



RESULTS,OBS3101

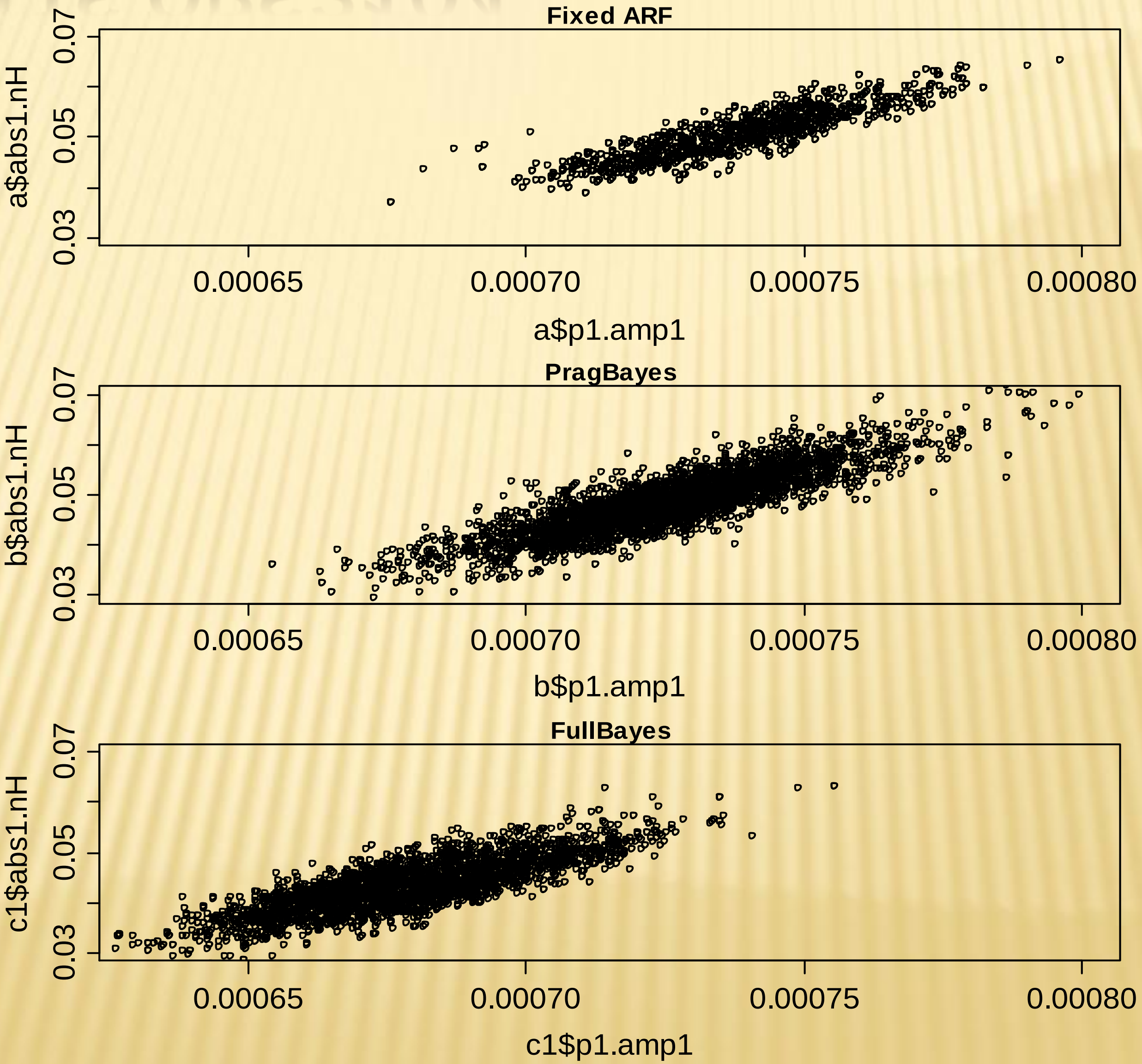


RESULTS,OBS3102

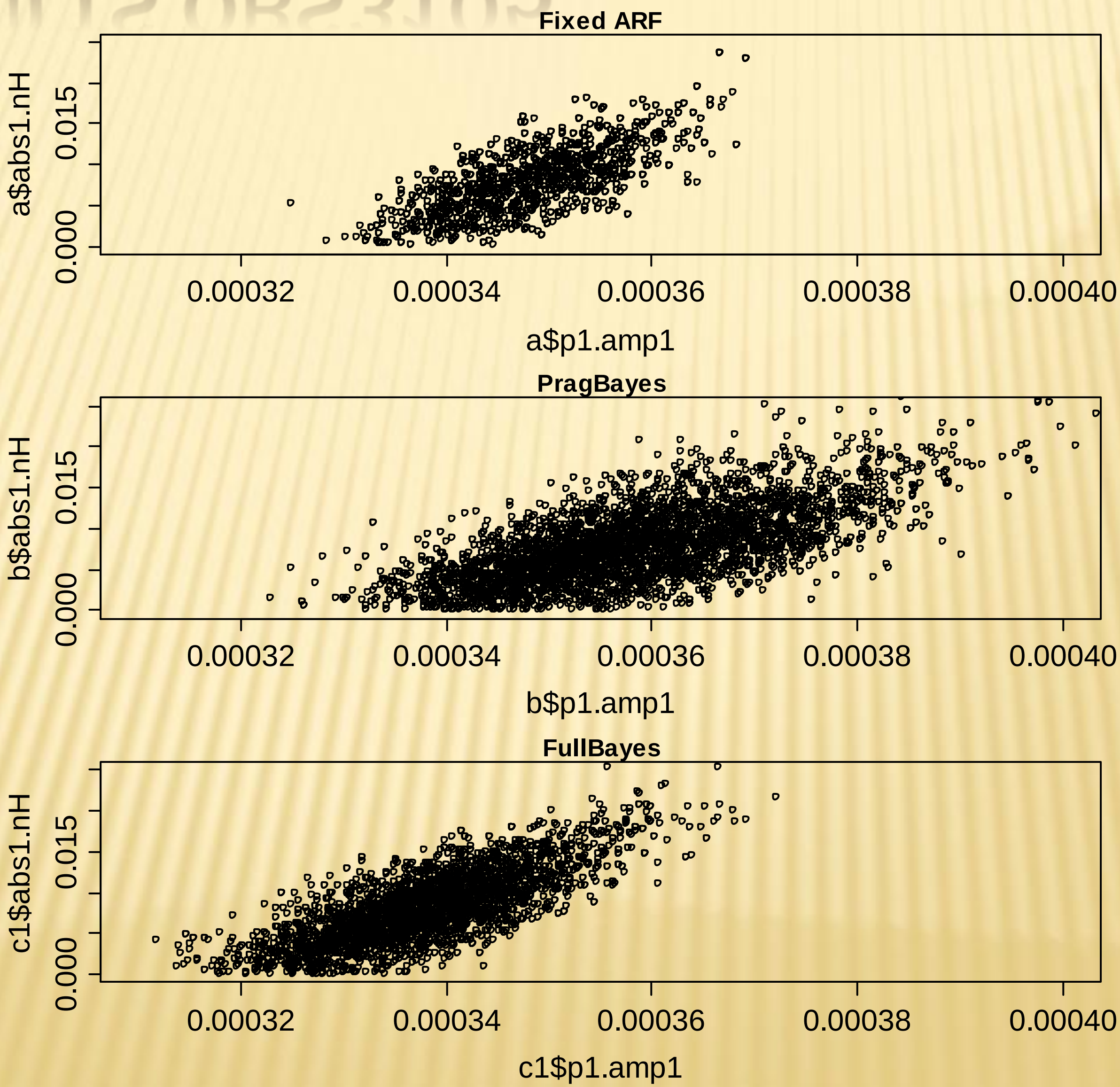


Similar as 3055 dataset result!

RESULTS,OBS3104

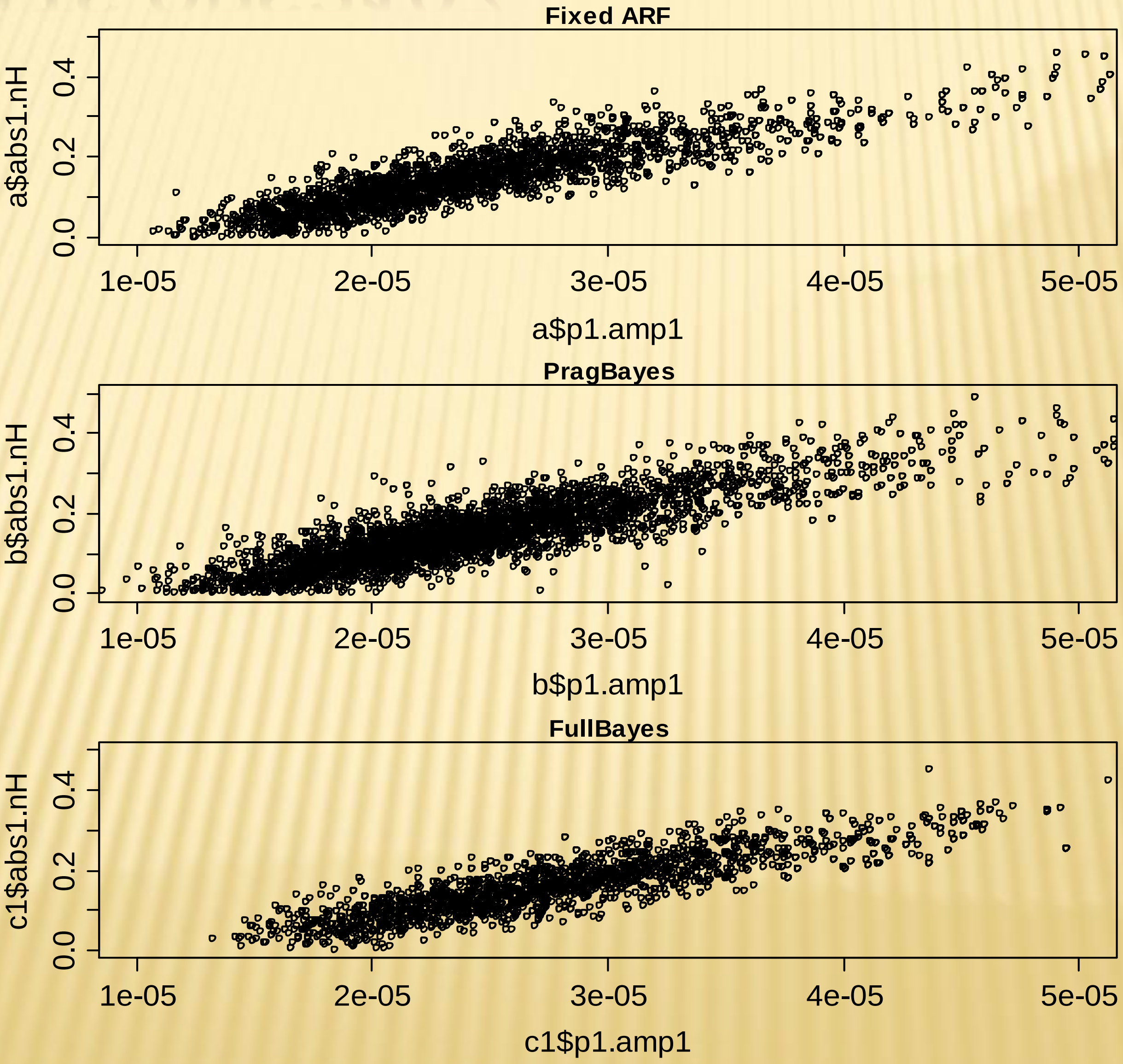


RESULTS,OBS3105



It seems hitting the boundary!

RESULTS,OBS3107



RESULTS SUMMARY

- Basically Speaking $Var_{prag} > Var_{full} > Var_{fixed}$
- Comparing PragBayes Model with FullBayes Model, results differ with the dataset.
- Datasets, 377,836,3097,3104,3105,and 3107, show that PragBayes differs a lot from FullBayes in both location and variance.
- Datasets, 866,1602,3056,3098,3100,and 3101, demonstrate that the results are similar.
- FullBayes Model fails for datasets 3055,3102 and 3099

RESULTS SUMMARY

- Full Bayes Model has the advantage that it automatically chooses the most possible affective area curves according to the dataset during iterations, which may give us the best estimates of parameters.
- The main disadvantage of Full Bayes Model is the low acceptance rate compared to Prag Bayes. Even worse, for some datasets, Full Bayes fails.

FUTURE WORK

- Try different source models in Pyblocx.
- And…

- Thank you!!