Type Ia SN Light Curve Inference: Hierarchical Models for Nearby SN in the Rest-Frame Near Infrared and Optical
Related Papers

Type Ia Supernova Light Curve Inference: Hierarchical Models in the Optical and Near Infrared.  

Type Ia Supernova Light Curve Inference: Hierarchical Bayesian Analysis in the Near Infrared.  

Do Spectra improve distance measurements of SN Ia?  
2011, A&A 526, A81

Wood-Vasey, et al. Type Ia Supernovae are Good Standard Candles in the Near Infrared: Evidence from PAIRITEL.  
Outline

• Type Ia SN and Cosmology

• Statistical Inference with SN Ia Light curves
  • Hierarchical Framework for Structured Probability Models for Observed Data
  • Describing Populations & Individuals, Multiple Random Effects, Covariance Structure

• Statistical Computation with Hierarchical Models
  • BayeSN (MCMC)

• Application & Results: Hierarchical Model for Nearby CfA NIR (PAIRITEL) and Optical (CfA3) SN Ia light curves
Standard Candle Principle

1. Know or Estimate Luminosity $L$ of a Class of Astronomical Objects

2. Measure the apparent brightness or flux $F$

3. Derive the distance $D$ to Object using Inverse Square Law: $F = \frac{L}{4\pi D^2}$

4. Optical Astronomer’s units: $m = M + \mu$
Type Ia Supernovae are Almost Standard Candles

- Progenitor: C/O White Dwarf Star accreting mass leads to instability
- Thermonuclear Explosion: Deflagration/Detonation
- Nickel to Cobalt to Iron Decay + radiative transfer powers the light curve
- SNe Ia progenitors have nearly same mass, therefore energy

Credit: FLASH Center
Supernova Cosmology: Constraining Cosmological Parameters using Luminosity Distance vs. Redshift

Credit: Gautham Narayan

- Nearby Hubble law is linear
- High-z depends on cosmology
- Host Galaxy Dust is a Major Confounding Factor
Cosmological Energy Content

Dark Energy Equation of state $P = w\rho$

Is $w + 1 = 0$? Cosmological Constant
Reading the Wattage of a SN Ia: Empirical Correlations

- **Width-Luminosity Relation**: an observed correlation (Phillips)
- Observe optical SN Ia Light Curve Shape to estimate the peak luminosity of SN Ia: ~0.2 mag
- **Color-Luminosity Relation**
- **Methods**:
  - $\Delta m_{15}(B)$
  - MLCS, Abs LC vs Dust
  - SALT, App. Color single factor

Intrinsically Brighter SN Ia have broader light curves and are slow decliners
Statistical inference with SN Ia

• SN Ia cosmology inference based on empirical relations
• Statistical models for SN Ia are learned from the data
• Several Sources of Randomness & Uncertainty
  1. Photometric errors
  2. “Intrinsic Variation” = Population Distribution of SN Ia
  3. Random Peculiar Velocities in Hubble Flow
• Apparent Distributions are convolutions of these effects
• How to incorporate this all into a coherent statistical model? (How to de-convolve?)
Advantages of Hierarchical Models

• Incorporate multiple sources of randomness & uncertainty
• Express structured probability models adapted to data
• Hierarchically Model (Physical) Populations and Individuals simultaneously: e.g. SN Ia and Dust
  • Intrinsic Covariance: Color/Luminosity/Light Curve Shape
  • Dust Reddening/Extinction
• Full (non-gaussian) probability distribution = Global, coherent quantification of uncertainties
• Completely Explore & Marginalize Posterior trade-offs/degeneracies/joint distributions
• Deals with incomplete/missing data problems
  • Make best inference/estimate for the observed data
• Modularity
Directed Acyclic Graph for SN Ia Inference with Hierarchical Modeling

- Intrinsic Randomness
- Dust Extinction & Reddening
- Peculiar Velocities
- Measurement Error

"Training" - Learn about Populations

Generative Model

Global Joint Posterior Probability Density Conditional on all SN Data
Representing SN Ia Light curves: Differential Decline rates

- Gaussian Process over Decline Rates at different Wavelengths / Phases and Peak Luminosities
- Goal: Infer the Intrinsic Covariance Structure of SN Ia light curves over multiple wavelengths and phases
- Use to make “best” distance predictions
Positive Dust only
Dims and Reddens

$A_V \sim \text{Expon}(\tau = 0.37 \pm 0.04)$
Directed Acyclic Graph for SN Ia Inference: Distance Prediction

\[ A^s_V, R^s_V \]

\[ \mu_s \rightarrow \mathcal{Z}_s \]

\[ \text{Dust Pop} \]

\[ \text{SN Ia AbsLC Pop} \]

\[ A^p_V, R^p_V \]

\[ \mu_p \rightarrow \mathcal{D}_p \]

\[ \text{Training} \]

\[ s = 1, \ldots, N_{SN} \]

\[ \text{Prediction} \]

\[ \text{AbsLC}_s \rightarrow \text{AppLC}_s \]

\[ \text{AbsLC}_p \rightarrow \text{AppLC}_p \]

\[ \mathcal{D}_s \]

\[ \mathcal{D}_p \]
Statistical Computation with Hierarchical SN Ia Models: The BayeSN Algorithm

- **Strategy:** Generate a Markov Chain to sample global parameter space (populations & all individuals) => seek a global solution
- Chain explores/samples trade-offs/degeneracies in global parameter space

Multiple chains globally converge from random initial values
BayeSN

• Metropolis-Hastings within Gibbs Sampling Structure to exploit conditional structure

• Requires (almost) no tuning of jump sizes

• Generalized Conditional Sampling to speed up exploring trade-off between dust and distance: $(A_v, \mu) \rightarrow (A_v, \mu) + \gamma(1, -x)$

• Run several (4-8) parallel chains and compute Gelman-Rubin ratio to diagnose convergence
BayeSN in Graphs

Dust Pop

AbsLC Pop

Av, Rv #1

AbsLC #1

AppLC #1

μ₁

Δ₁

μₙ

Δₙ

z₁

zₙ

AbsLC #N

Av, Rv #N

AppLC #N
BayeSN in Graphs

AbsLC
#1

AbsLC
Pop

AbsLC
#N
BayeSN in Graphs

![BayeSN Graph Diagram]

- AbsLC
- AppLC
- Av, Rv
- $\mu_1$
- $D_1$

Tuesday, January 25, 2011
BayeSN in Graphs

\[ \text{Av, Rv}\ #1 \rightarrow \text{AbsLC}\ #1 \rightarrow \text{AppLC}\ #1 \rightarrow \mu_1 \rightarrow z_1 \]

AbsLC Pop
BayeSN in Graphs

Dust Pop

Av, Rv #1

μ₁

AbsLC #1

AppLC #1

AbsLC Pop
BayeSN in Graphs

AbsLC Pop

Av, Rv #N

AppLC #N

AbsLC #N

\[ \mu_N \]

\[ D_N \]
BayeSN in Graphs

AbsLC Pop

Av, Rv #N

AbsLC #N

AppLC #N

\( \mu_N \)

\( z_N \)
BayeSN in Graphs

Dust Pop

AbsLC Pop

Av, Rv #N

AppLC #N

\( \mu_N \)

AbsLC #N
BayeSN in Graphs

Dust Pop

AbsLC Pop

Av, Rv #1

AbsLC #1

Av, Rv #N

AbsLC #N

μ₁

μₙ

z₁

D₁

zₙ

Dₙ

AppLC #1

AppLC #N
Practical Application of Hierarchical Model: NIR SN Ia

Why are SN Ia in NIR interesting?

- Host Galaxy Dust presents a major systematic uncertainty in supernova cosmology inference
- Dust extinction has significantly reduced effect in NIR bands
- Observe in NIR!: PAIRITEL / CfA
Nearby SN Ia in the NIR: PAIRITEL

Observed in NIR
J ($\lambda=1.2 \, \mu m$)
H ($\lambda=1.6 \, \mu m$)
Ks ($\lambda=2.2 \, \mu m$)

Credit: Michael Wood-Vasey, Andrew Friedman
Figure 1: 142 CfA Light curves from 2000-2004 (UBVRI) and 2004-2007 (UBVri)
Optical+NIR Hierarchical Model Inference
PTEL+CfA3 Light-curves    Marginal Posterior of Dust

SN2005eq (CfA3+PTEL)

Obs. Days Since B_{max}

Obs. Mag. − k_{c − mwx}

Dust Law Slope R_{V}

Extinction (mag)

A_{V}

A_{H}

SN2005eq
Optical+NIR Hierarchical Model Inference

PTEL+CfA3 Light-curves
Marginal Posterior of Dust

Obs. Mag. – kc – mwx

Obs. Days Since B_{max}

SN2006ax (CfA3+PTEL)

Extinction (mag)

Dust Law Slope R_V

A_V

A_H

SN2006ax

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SN NIR Population Inference: Peak Absolute Magnitudes

Marginal Distributions of SN Ia NIR Absolute Magnitude Variances

\[
\begin{align*}
\mu(M_J) &= -18.26 \\
\sigma(M_J) &= 0.17 \\
\mu(M_H) &= -18.00 \\
\sigma(M_H) &= 0.11 \\
\mu(M_{Ks}) &= -18.24 \\
\sigma(M_{Ks}) &= 0.18
\end{align*}
\]

Mandel et al. 2009

Fig. 15.—Dispersion in peak magnitude (measured at the first light curve maximum) as a function of wavelength band for the models of Fig. 10 with 56Ni masses between 0.4 and 0.9 \( M_\odot \). [See the electronic edition of the Journal for a]
Optical and Near Infrared Luminosity vs. Decline Rate
Population Analysis

H-band provides nearly uncorrelated information on luminosity distance
Host Galaxy Dust

• Previous Analyses assumed all SN host galaxy dust has same R_v

• Estimated R_v = 1-1.7 (Astier06, Conley07) if attribute all color variation to dust

• But Rayleigh Scattering: R_v = 1.2

• But Hicken09 found R_v = 1.7 with MLCS

• For individual high Av SN, R_v < 2

• But R_v may have a distribution, or depend on Av (e.g. grain growth)
(Av, Rv) for Host Galaxy Dust
Assuming Linear Correlation

- Apparent Correlation of High Av / Low Rv
- Low Av Rv ≈ 2.5 : High Av has Rv ≈ 1.7
- Circumstellar dust at High Av ?
- Multiple Scattering (Goobar 2008)
Bootstrap Cross-validation

- Test Sensitivity of Statistical Model to Finite Sample
- Avoid using data twice for training and distance prediction
- Prediction/Generalization Error

- Low-z SN data
- Training Set $\{D_s, z_s\}$
- Test Set $\{D_t\}$
- Test Set $\{z_t\}$
- SN Ia Model
- Predict $\{\mu_t\}$
- Hubble $\{\mu(z_t)\}$
- Compare

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Nearby Optical+NIR Hubble Diagram

(Opt Only) rms Distance Prediction Error = \(0.15\) mag

(Opt+NIR) rms Distance Prediction Error = \(0.11\) mag

Aggregate Precision \(\sim (0.15/0.11)^2 \approx 2\)

\[ h = 0.72 \]

127 BVRI(JH) SN Ia (CfA3+PTEL+CSP+lit)

CV Pred Err (All, cz > 3000 km/s) = 0.14 mag (0.134 \(\pm\) 0.010 intr.)

CV Pred Err (Opt only & cz > 3000 km/s) = 0.15 mag (0.142 \(\pm\) 0.013 intr.)

CV Pred Err (Opt+NIR & cz > 3000 km/s) = 0.11 mag (0.113 \(\pm\) 0.016 intr.)
Improved Distance Precision for Individual Opt+NIR LCs

- Precision = 1/Variance
- On avg, 2.2x better BVRI vs BV
- 3.6x better BVRIJH vs BV
- 60% better BVRIJH vs BVRI
Summary

• Hierarchical models are useful statistical methods for discerning multiple random effects

• BayeSN: an efficient MCMC Sampler for computing inferences with SN hierarchical models

• Apparent differential trend of Rv vs Av (local dust at high Av?)

• NIR Light Curves have low correlation with optical, provide independent information on distance

• SN Ia Optical with NIR: Better dust and distance estimates than with Optical alone
Future Work & Problems

- Application to Larger Sample of Opt+NIR SN Ia
- Application to high-z SN Ia & Cosmological Inference
- Accounting for Selection Effects
- Using Auxiliary Information
  - Host Galaxy Information (e.g. P. Kelly, et al. 2010)
  - Spectral? Blondin, Mandel, & Kirshner 2011
  - Foley & Kasen 2011 (Color / Ejecta Velocity)
Spectral Info correlate with SN Ia luminosity and light curves?

Blondin, Mandel, Kirshner 2011

Multiple Comparisons Problem
Correlating Spectral Ratios with Luminosity
Blondin, Mandel, Kirshner 2011

Multiple Comparisons  K-fold Cross-Validation
Open Problems

• Photometric Classification of SN Light Curves

• Classification of SN by Spectra