

Detecting stellar flares using conditional volatility

Giovanni Motta

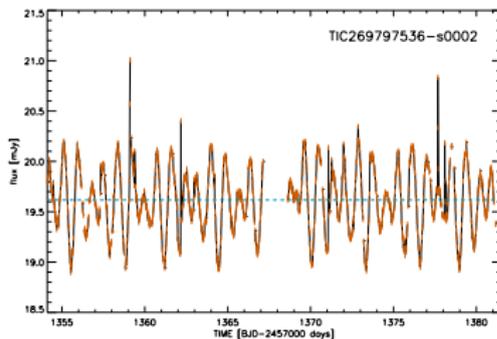
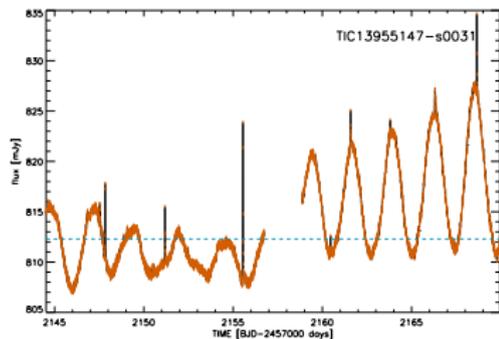
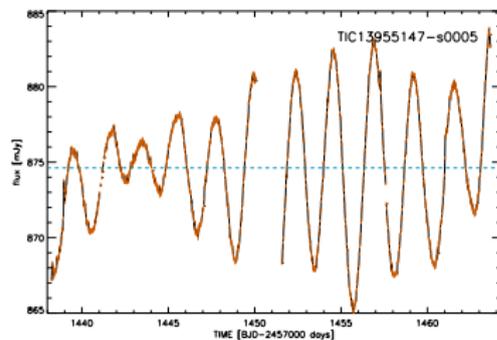
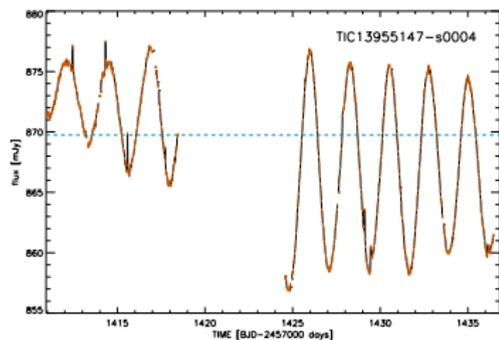
Data Science Institute, Columbia University

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JSM, Portland (Oregon)

8-7-2024

The 2-minute cadence flux light curves of TIC13955147 & TIC269797536



Flares

- Stellar flares
- Why Studying stars

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 - Pin down parameters and processes that control the nature of the corona;
 - Here we develop a method to detect flare occurrences in TESS light curves in order to gain insight into flare distributions, cadence, and onset.

Our contribution

We have studied several white light flares from the Transiting Exoplanet Survey Satellite (TESS), and modeled them via a GARCH process.

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 - define the modeling process

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 - Applied extensively in econometrics to understand stock price fluctuations, and we show that stellar flares can be described by the same mathematics.

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 - Applied extensively in econometrics to understand stock price fluctuations, and we show that stellar flares can be described by the same mathematics.
 - Model the so-called volatility index, and we find that it is a good guide to flare occurrence. This is a remarkable new result, and promises to open new avenues of exploration, to understand the precise nature of flare onset.

Our novel semi-parametric approach

Our approach combines a non-parametric method that removes the harmonic trend with a parametric method that detects the flares

1 Non-parametric step

We remove the trend using a time-varying harmonic fit combined with a NP-test for the local variance, so to capture changes in the deterministic amplitude of the light curve.

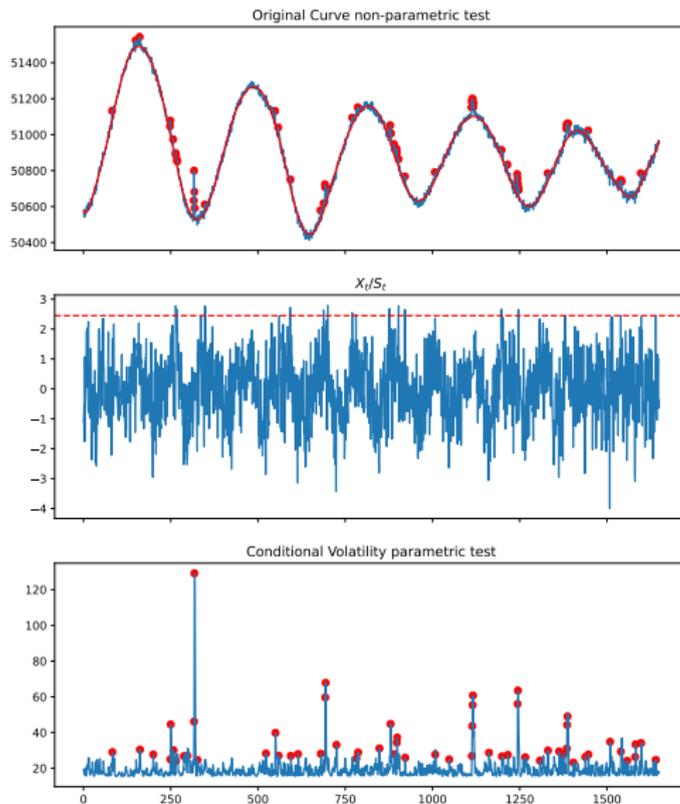
2 Parametric step

We enlighten the analogy between the stochastic part of the light curves and GARCH processes, and detect the flares by exploiting the parametric structure.

- We demonstrate that flares can be detected as significantly large deviations from the baseline.
- We apply the method on exemplar light curves from flaring stars **TIC13955147** (s4, s5 and s31) and **TIC269797536** (s2).

A snapshot of our novel method

TIC13955147 s-31



Outline

Flares

Our Statistical Model

Deterministic Trend: Time-varying Harmonic Oscillations

Stochastic Errors: Conditional Heteroskedastic Volatility

Our novel semi-parametric approach

Data: TESS Light Curves

Our Statistical Model

Our novel model is the sum of a non-random trend and a stochastic error:

$$Y_t = \mu(t) + X_t, \quad t = 1, \dots, n$$

- The function $\mu(t)$ is a deterministic, periodic time-trend that allows for time-variation in its coefficients, whereas
- the errors X_t follow a GARCH model

$$X_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, 1),$$

where the stochastic conditional variance $\sigma_t^2 = \text{Var}(X_t | \mathcal{I}_t)$ is an ARMA(p, q)

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i X_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

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Deterministic Trend: Time-varying Harmonics

Data:

- random observed vector $\mathbf{Y} = (Y_1, Y_2, \dots, Y_N)'$
- time vector $\mathbf{t} = (t_1, t_2, \dots, t_N)'$

Modulation model

$$Y_i = \sum_{k=1}^K \{g_{1k}(t_i) \cos(w_k t_i) + g_{2k}(t_i) \sin(w_k t_i)\} + X_i, \quad i = 1, \dots, N,$$

- smooth time-varying amplitudes
- K number of harmonics
- $\mathbf{X} = (X_1, \dots, X_N)'$ follow a GARCH(p, q) process

We estimate the parameters using penalized B-splines (Motta et al., 2002)

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Stochastic Errors: Heteroskedastic Volatility

A GARCH(p, q) model has a multiplicative form

$$X_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0, 1),$$

where

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i X_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, \quad t \geq \max(p, q),$$

with $\alpha_0 > 0$, $\alpha_1, \dots, \alpha_p \geq 0$, $\beta_1, \dots, \beta_q \geq 0$.

Stochastic Errors: Heteroskedastic Volatility

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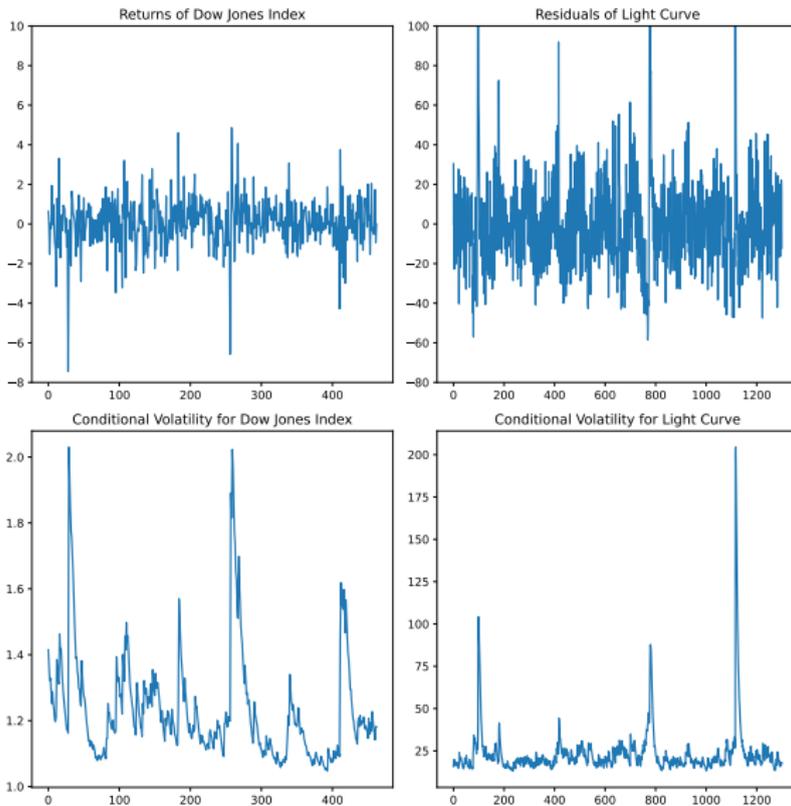
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Properties

- $\mathbb{E}(X_t | X_s, s < t) = \mathbb{E}(X_t) = 0$
- $\mathbb{E}(X_t X_s) = 0$, $s \neq t$
- $\mathbb{E}(X_t^2 | X_s, s < t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i X_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$
- $\mathbb{E}(X_t^2) = \alpha_0 / (1 - \sum_{i=1}^p \alpha_i - \sum_{j=1}^q \beta_j)$
- $X_t^2 \sim \text{ARMA}(p, q)$ and $\rho_{X^2}(h) \geq 0$ for all h

Stochastic Errors: Heteroskedastic Volatility



Heteroskedastic Volatility

Stylized facts of financial returns

- **heavy tailed**

$$\Pr(|X| > x) \sim Cx^{-\alpha}, \quad 0 < \alpha < 4$$

- **uncorrelated**

$\rho_x(h)$ near 0 for all lags $h > 0$

- $|X_t|$ and X_t^2 have **slowly decaying autocorrelations**

$\rho_{|x|}(h)$ and $\rho_{x^2}(h)$ converge to 0 slowly as h increases

- **stochastic and persistent** volatility

- large (small) fluctuations in the data tend to be followed by fluctuations of comparable magnitude
- this is reflected by GARCH models through the correlation in the sequence $\{\sigma_t^2\}$ of conditional variances

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Main idea: examine time-varying volatility within its local neighborhood.

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- Given the residual value $X_t = Y_t - \mu(t)$, we test

$$H_0 : X_t | \mathcal{I}_t \sim WN(0, \sigma_0^2) \quad \text{vs} \quad X_t | \mathcal{I}_t \sim WN(0, \sigma_t^2).$$

Under H_0 , $\text{Var}(X_t | \mathcal{I}_t) = \sigma_0^2 \forall t$. Under H_1 , $\text{Var}(X_t | \mathcal{I}_t) = \sigma_t^2 > \sigma_0^2$ for some t .

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Under H_0 , $X_t^2 / S_t^2 \sim F_{1, n-1} \xrightarrow{d} \chi_1^2(\alpha)$ as $n \rightarrow \infty$.

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- $(X_t / S_t)^2 > \chi_{1(\alpha)}^2$ is equivalent to $|X_t| / S_t > Q_{(\alpha)}^{\text{HN}}$, where $\alpha = \Pr(\mathcal{H} > Q_{(\alpha)}^{\text{HN}})$, \mathcal{H} being a Standard Half-Normal random variable.

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- Test-statistic: $|X_t| / S_t$, where S_t is a **robust, time-varying** estimator of σ_t .

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- The presence of flares makes $\hat{\mu}$ biased. For this reason, we (detect and) delete the flares from Y and then re-fit μ to the ‘flares-filtered’ data.

A semi-parametric approach

PARAMETRIC flares-detection

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \sigma_{t-i}^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, \quad t \geq \max\{p, q\}$$

Parametric test statistics based on the past conditional volatility

- Time-varying generalized χ^2 to model the conditional variance σ_t^2 :

$$G_t = \sum_{i=1}^p w_{t,i} \varepsilon_{t-i}^2 + C_t,$$

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- Test statistics: $\hat{G}_t = \sigma_t^2 | \hat{\sigma}_{\mathcal{I}_t}^2$. We decide that $\{t, X_t\}$ is an extreme residual if

$$P_t = \Pr(\hat{G}_t > \sigma_t^2) \leq \alpha. \quad (1)$$

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- The inequality in (1) is **our novel definition** of a flare.
- We only retain those with positive residual, i.e., we decide $\{t, Y_t\}$ is a flare if:

$$\mathbb{1}_{\{P_t < \alpha\}} \times \mathbb{1}_{\{X_t > 0\}} = 1$$

Our algorithm

Non-parametric detrending

Define $X^{(0)} = Y^{(0)} - \mu^{(0)}$ and $\alpha^{(0)} = \alpha_{\max}$

for i in $0 : (I - 1)$ **do**

for all t in $1:n$ **do** $\pi_t^{(i)} = \Pr(\mathcal{H} > \frac{|X_t^{(i)}|}{S_t^{(i)}})$

if $\pi_t^{(i)} < \alpha^{(i)}$ **then**

delete $\{t : t + 9, Y_t : Y_{t+9}\}$ (20 minutes forwards)

else

$Y_t^{(i+1)} = Y_t^{(i)}$

$X^{(i+1)} = Y^{(i+1)} - \mu^{(i+1)}$, and $\alpha^{(i+1)} = \max_{t: \pi_t^{(i)} < \alpha^{(i)}} \pi_t^{(i)}$

Parametric Flares-detection

Fit a GARCH(p, q) to $X^{(I)} = Y^{(0)} - \mu^{(I)}$

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Define $F_t = Y_t$

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α is guaranteed to decrease over i because the inequality $\pi_t^{(i)} < \alpha^{(i)}$ is strict

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Stochastic Errors: Conditional Heteroskedastic Volatility

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Data: TESS Light Curves

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- We demonstrate the application of our method to TESS (Transiting Exoplanet Survey Satellite)¹ datasets which have been observed multiple times.
- We choose two stars on which flares have been unambiguously observed (see Feinstein et al. 2022).
- Both are eruptive variables, at similar distances, and one is solar-like, and the other is a low-mass star.
- We apply our method to detect flares in the PDCSAP (Pre-Search Data Conditioning Simple Aperture Photometry) electron count rate light curves, which have backgrounds subtracted and systematics corrected for cotrending basis vectors².
- We show the light curves, converted from electron count rate to mJy³ for TIC 13955147 and for TIC 269797536.

¹<https://tess.mit.edu>

²https://spacetelescope.github.io/mast_notebooks/notebooks/TESS/beginner_tour_lc_tp/beginner_tour_lc_tp.html

³<https://tess.mit.edu/public/tesstransients/pages/readme.html> 

List of stars and datasets selected

We downloaded the processed light curve data from the TESS Science Processing Pipeline (TESS-SPOC)⁴ from the MAST Portal (Mikulski Archive for Space Telescopes)⁵ for the sectors listed in the Table below.

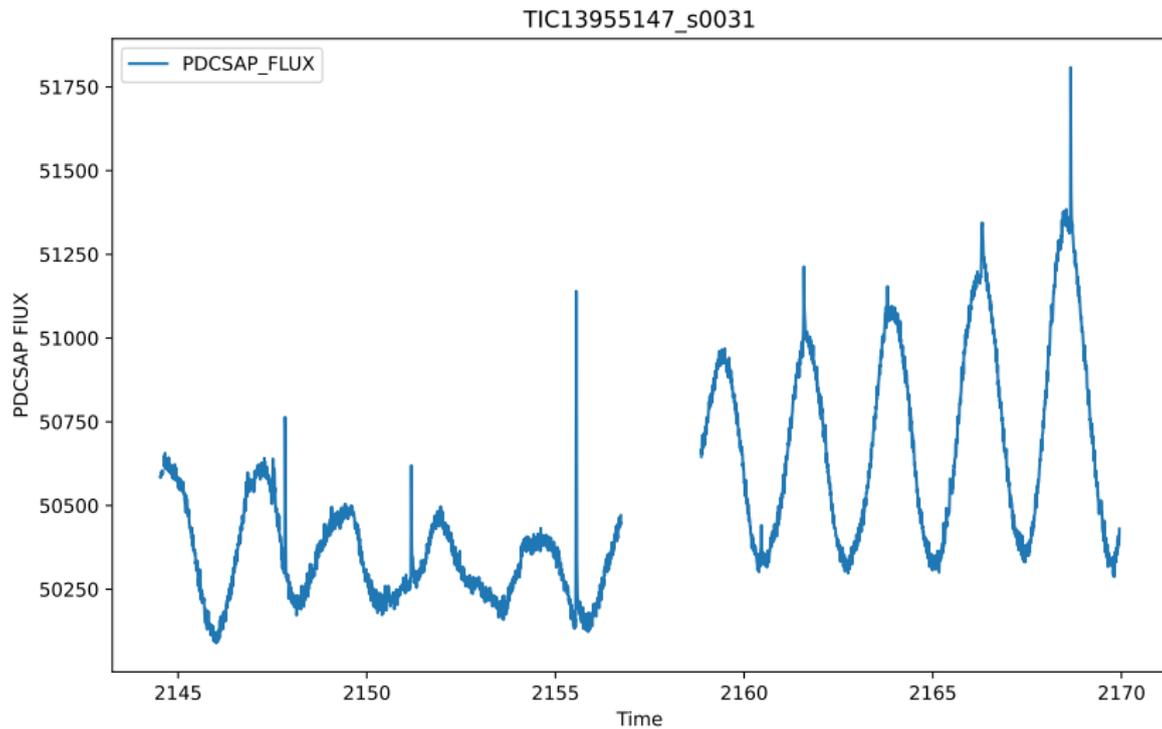
TIC	Other Names [†]	Spectral Type [†]	Distance [†] [pc]	Sectors
13955147	HD 32372 2MASS J05005186-4101065 Gaia DR3 4813691219557127808 1RXS J050051.7-410100	G5 V	78	4, 5, 31, 32
269797536	2MASS J04363294-7851021 Gaia DR3 4622912654918835200	M4 V	70	2, 5, 8, 11, 12, 13, 27, 28, 29, 32, 35, 38, 39

[†]: From the SIMBAD database (Wenger et al. 2000)

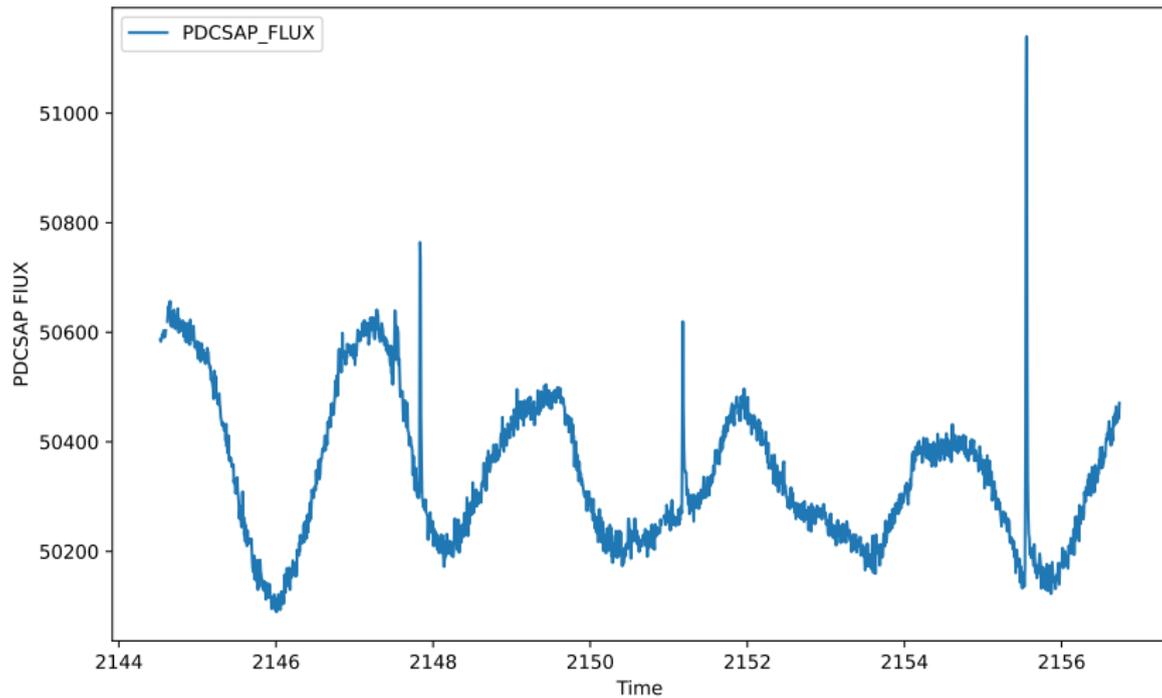
⁴<https://archive.stsci.edu/hlsp/tess-spoc>

⁵<https://archive.stsci.edu/missions-and-data/tess>

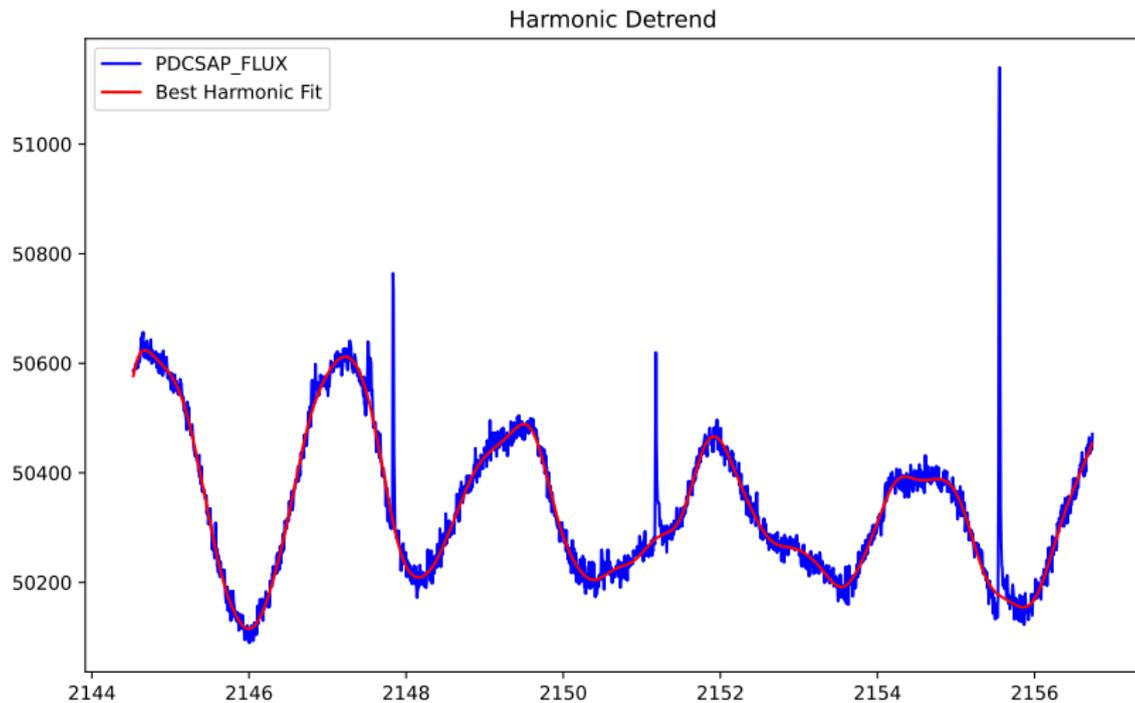
TIC13955147-s31



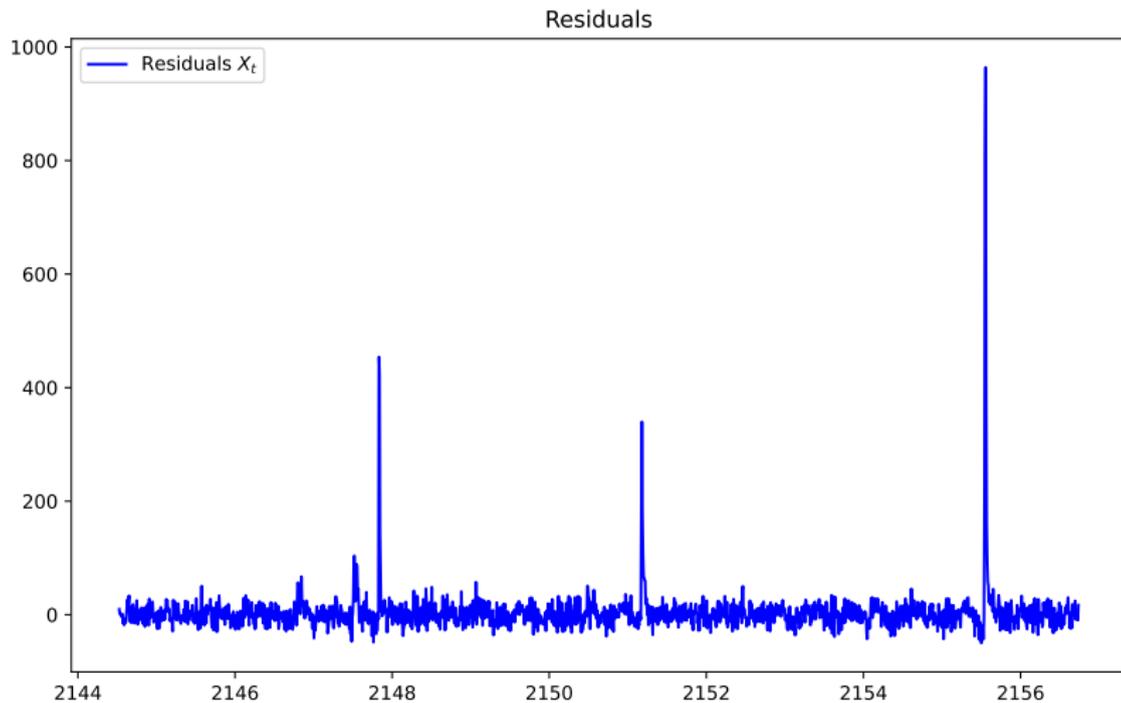
TIC13955147-s31-I



TIC13955147-s31-I – Harmonic Fit

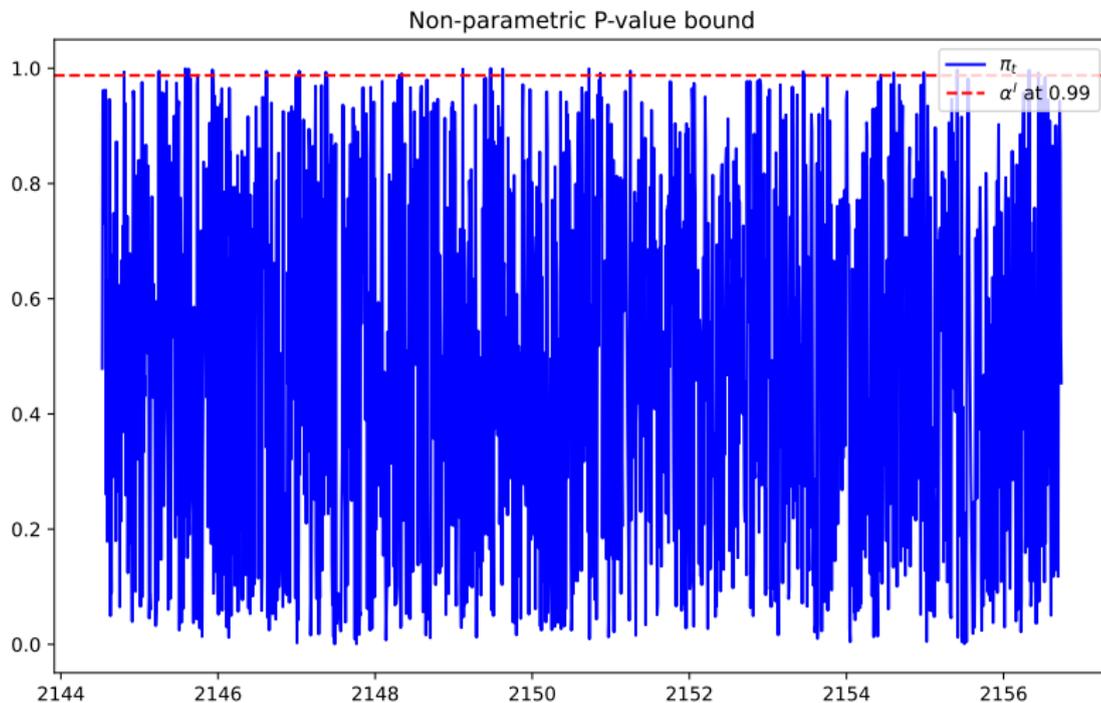


TIC13955147-s31-I – Residuals



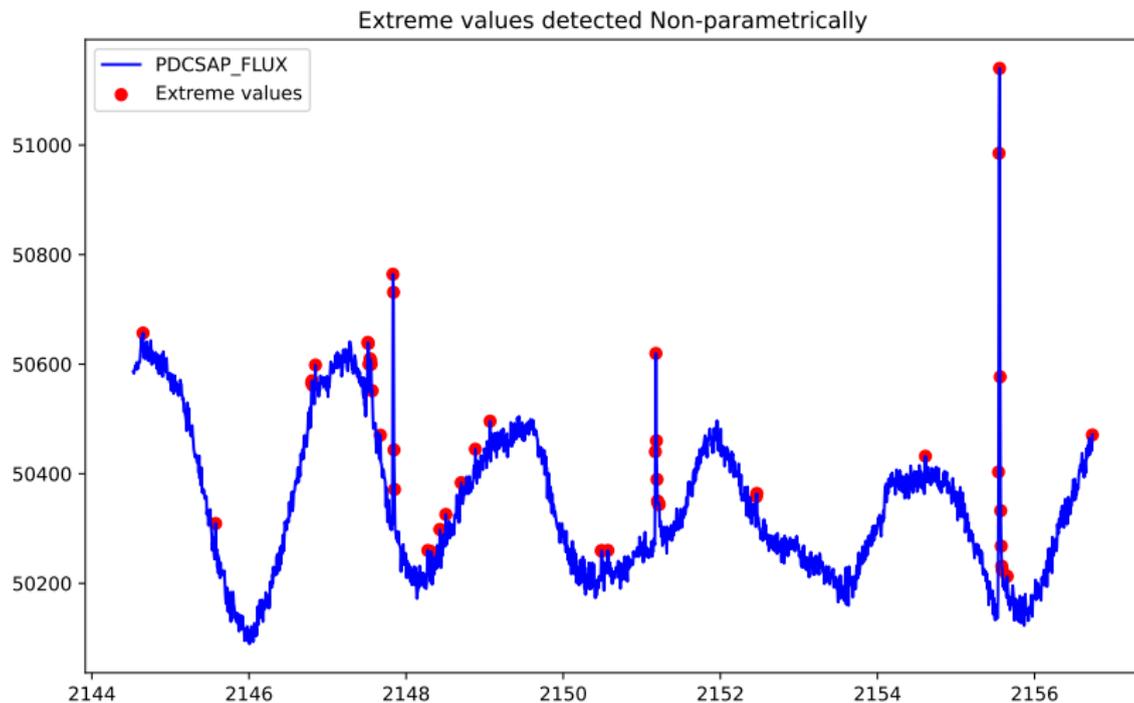
TIC13955147-s31-I – Non-parametric flares detection

$$\pi_t = \Pr(\mathcal{H} > \frac{|X_t|}{S_t}) < \alpha$$



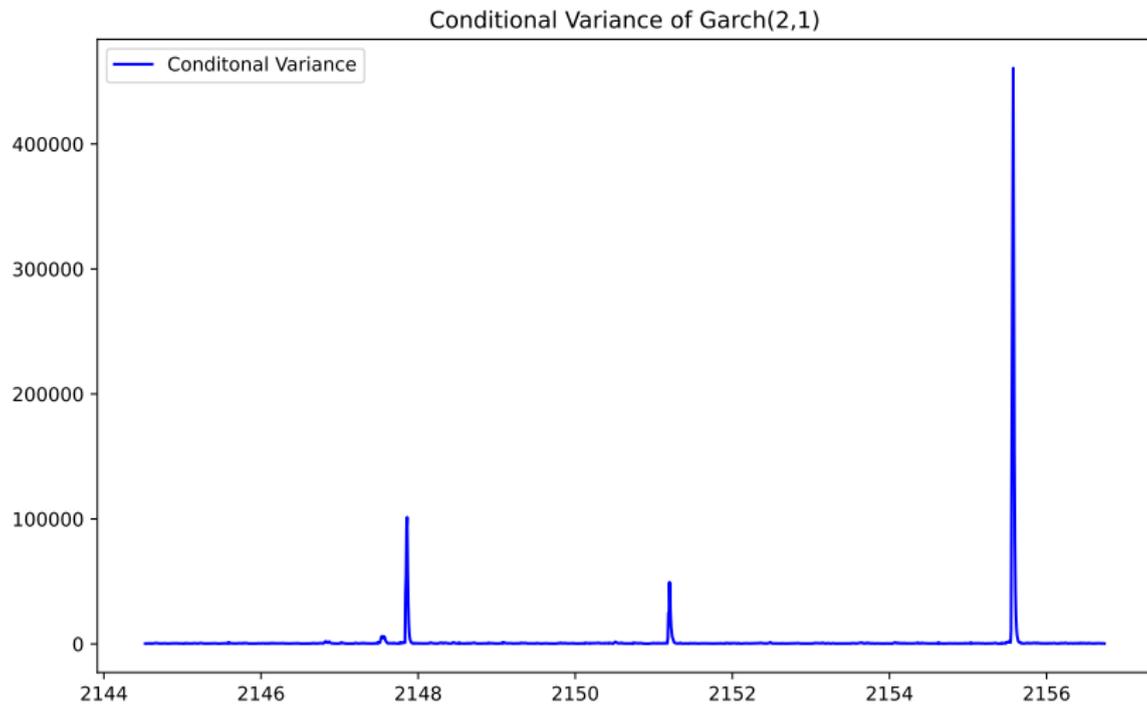
TIC13955147-s31-I – Non-parametric flares detection

$$\pi_t = \Pr(\mathcal{H} > \frac{|X_t|}{S_t}) < \alpha$$



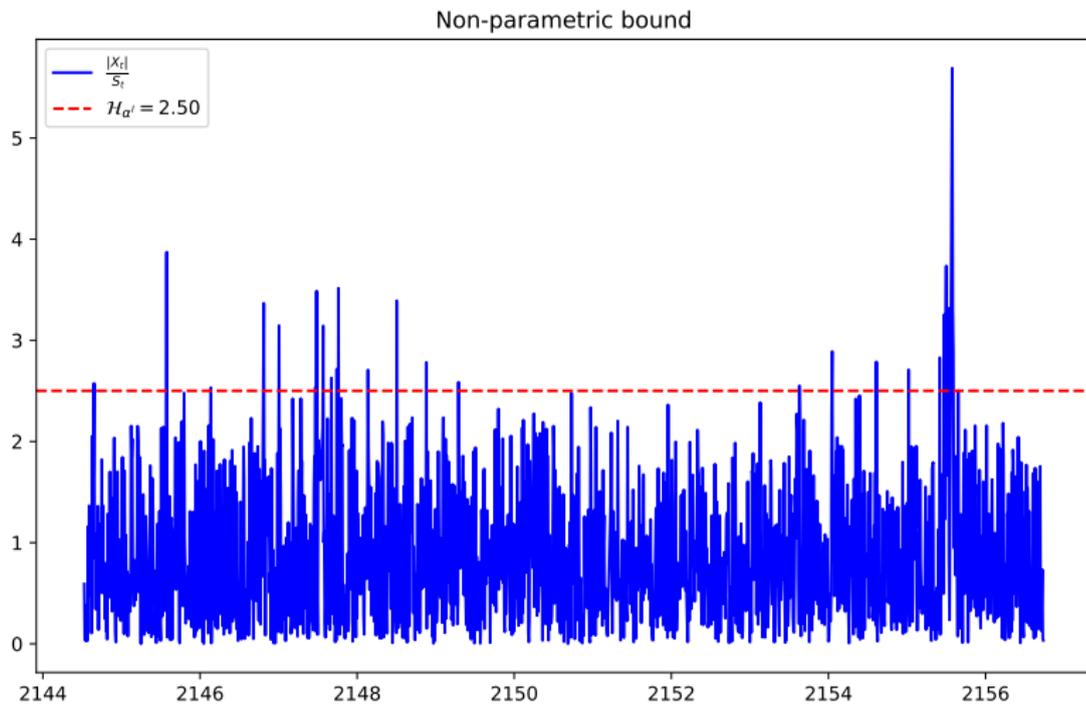
TIC13955147-s31-I – GARCH fitted to the residuals

$$X = Y^{(0)} - \mu^{(I)}$$



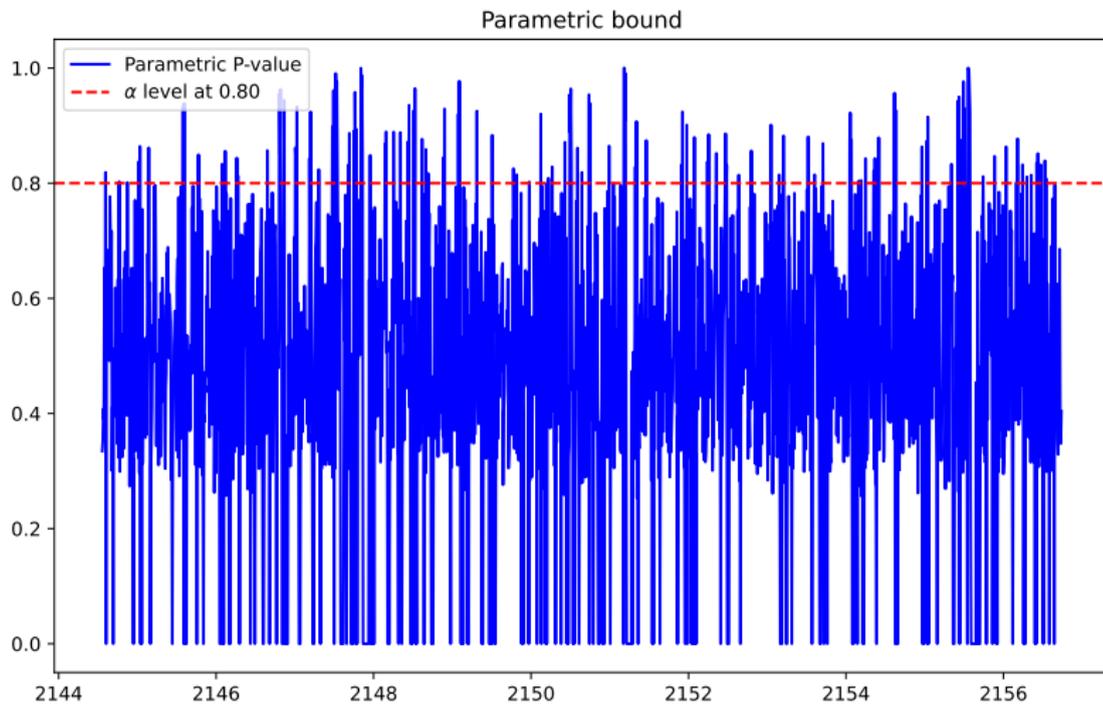
TIC13955147-s31-I – Non-parametric flares detection

$$X = Y^{(0)} - \mu^{(I)}$$



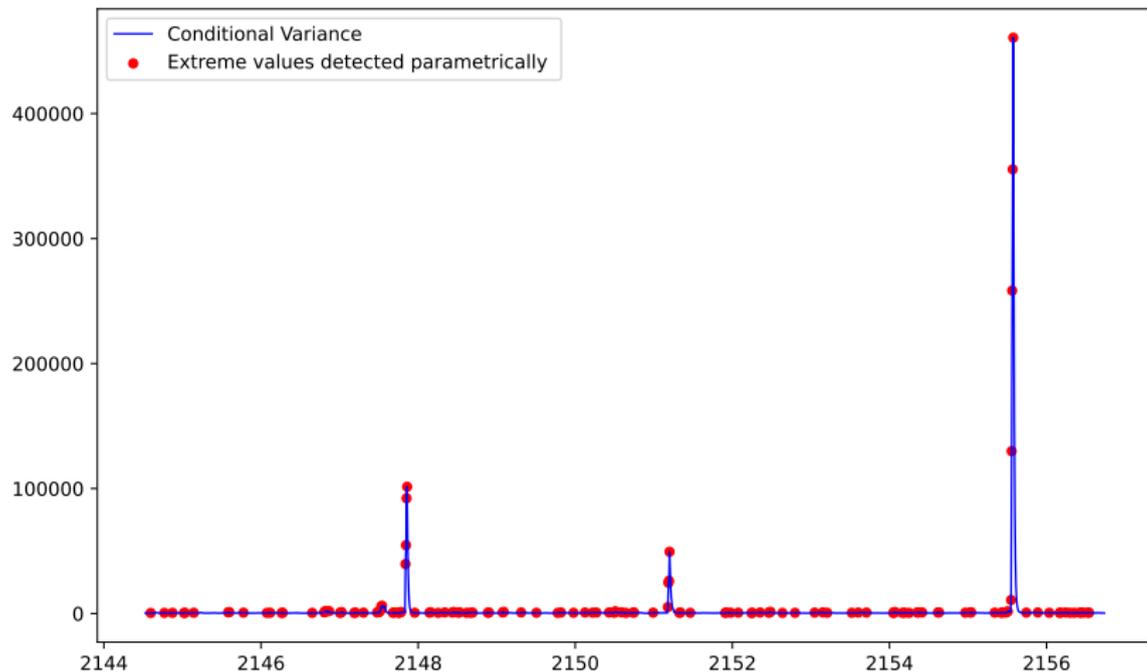
TIC13955147-s31-I – Parametric flares detection

$$X = Y^{(0)} - \mu^{(I)}$$



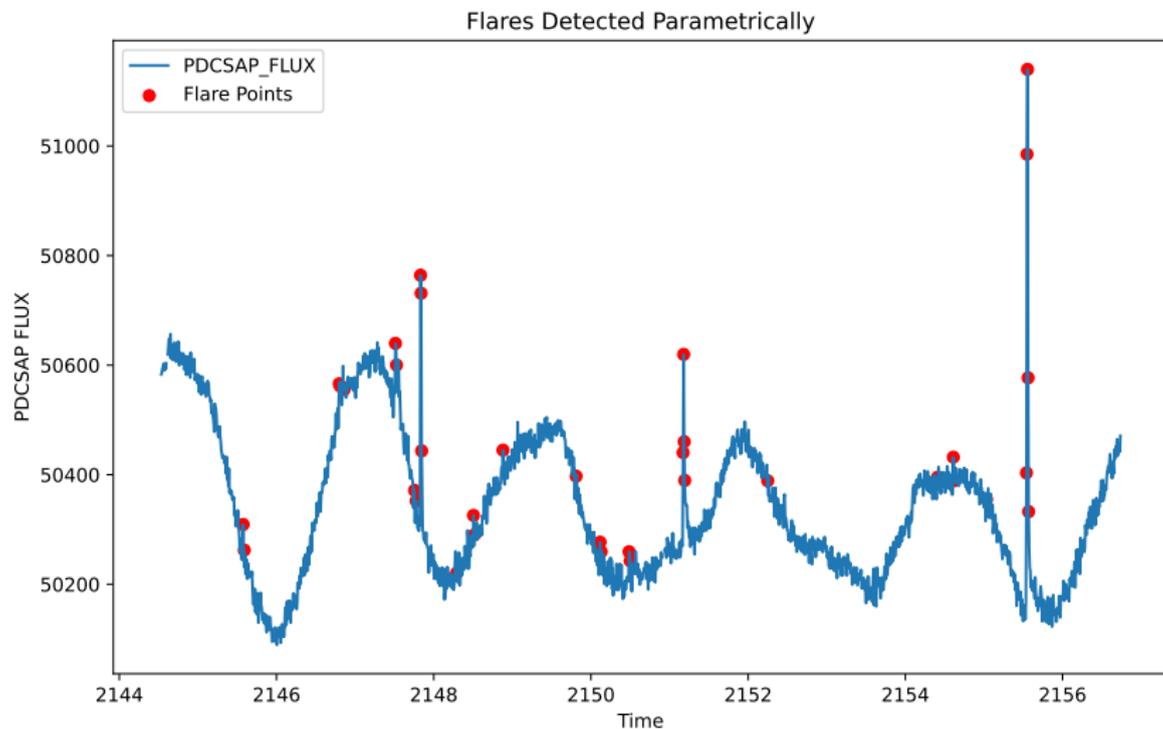
TIC13955147-s31-I – parametric flares detection

$$P_t = \Pr(G_t > \sigma_t^2) \leq \alpha$$



TIC13955147-s31-I – parametric flares detection

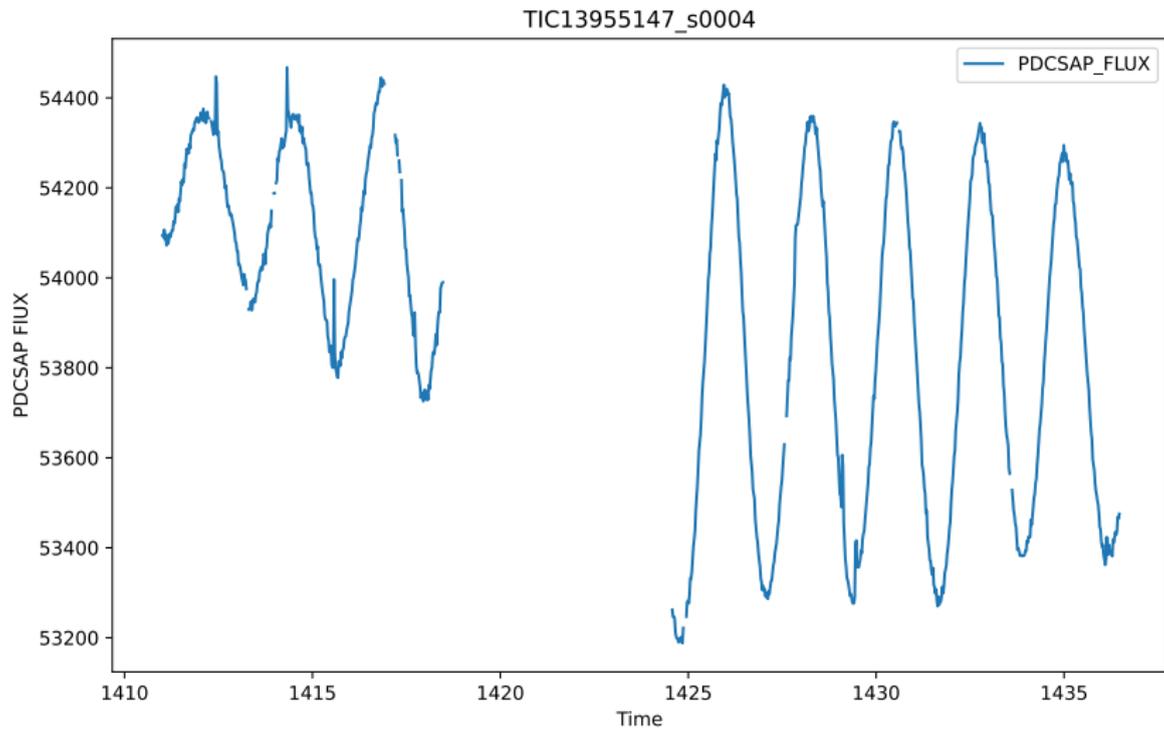
Flares are positive extreme values: $\mathbb{1}_{\{P_t \leq \alpha\}} \times \mathbb{1}_{\{X_t^{(I)} > 0\}} = 1$



That's it

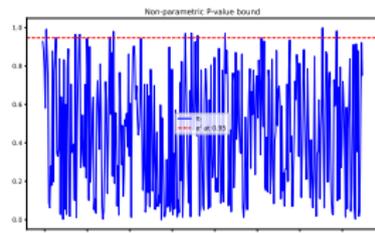
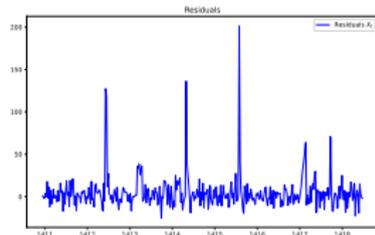
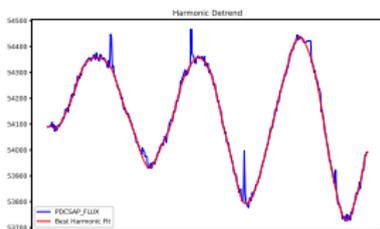
Thanks 😊

TIC13955147-s4

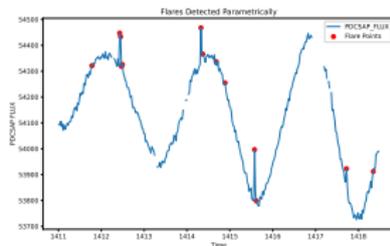
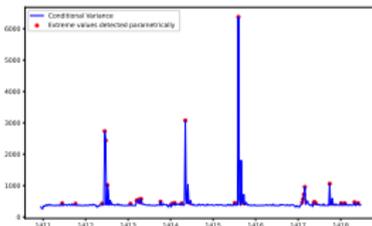
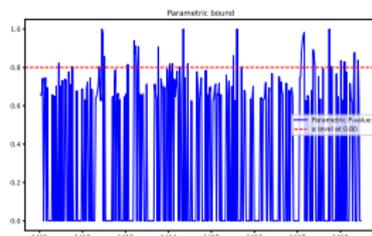


TIC13955147-s4-I

non-parametric detrend

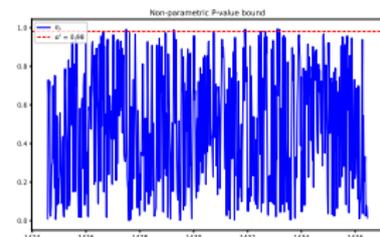
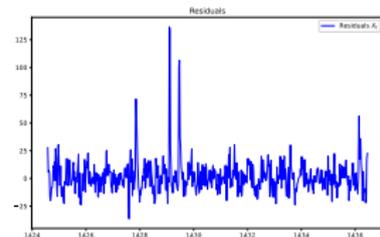
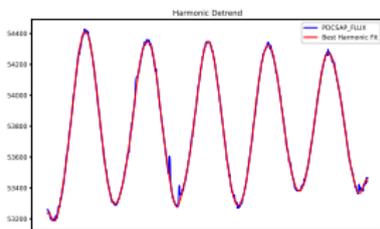


parametric flares detection

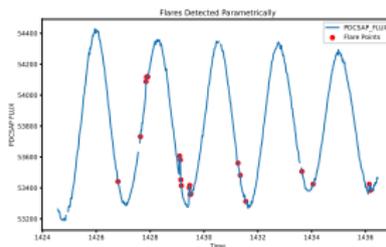
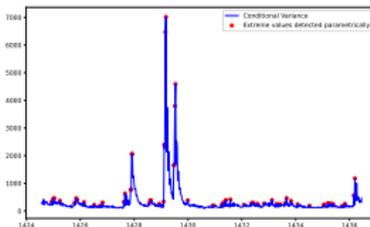
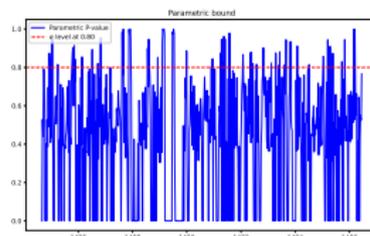


TIC13955147-s4-II

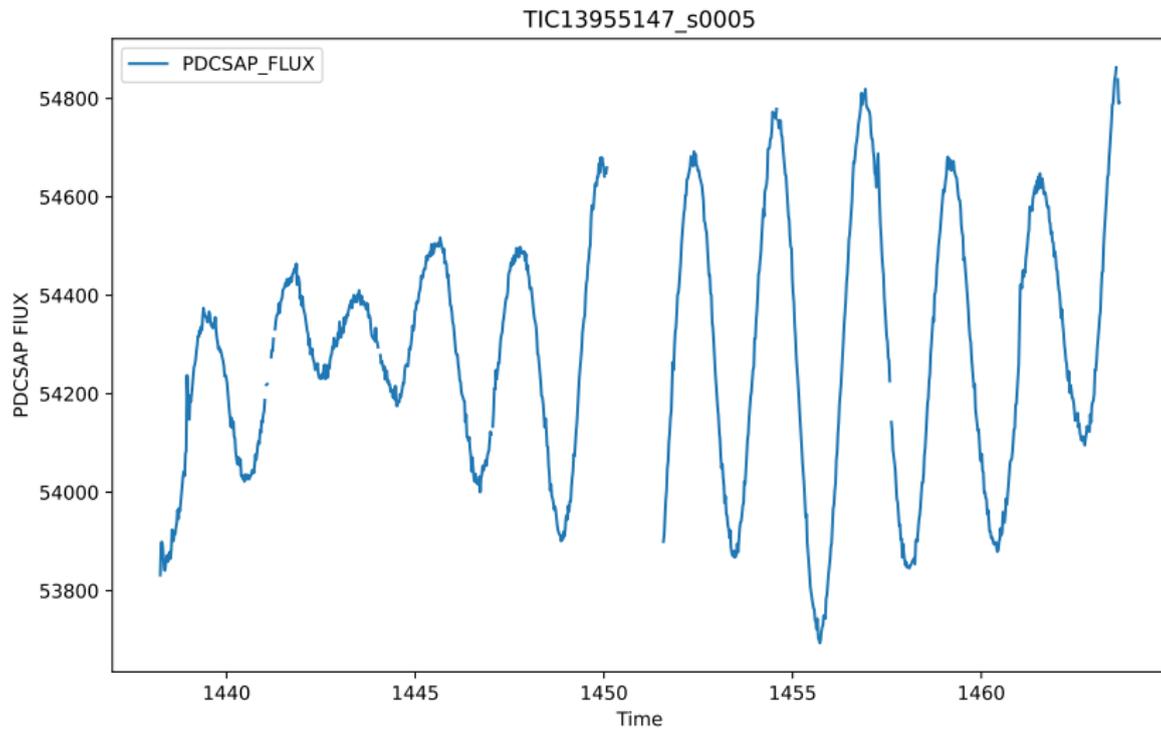
non-parametric detrend



parametric flares detection

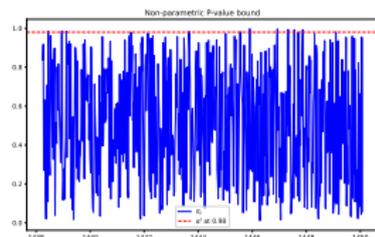
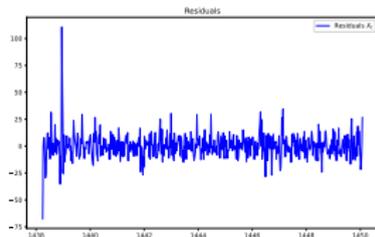
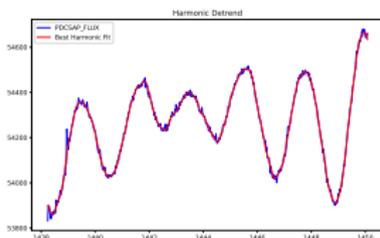


TIC13955147-s5

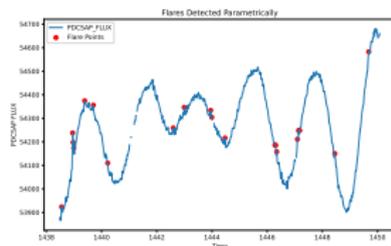
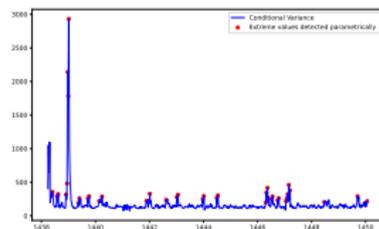
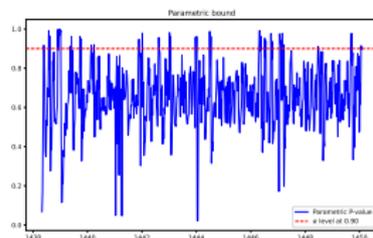


TIC13955147-s5-I

non-parametric detrend

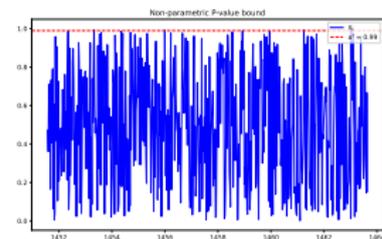
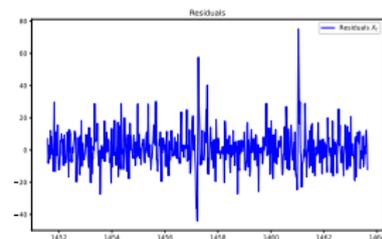
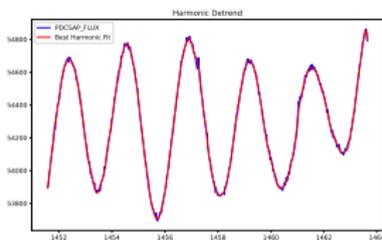


parametric flares detection

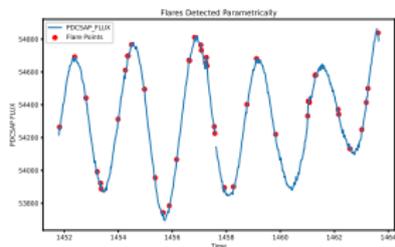
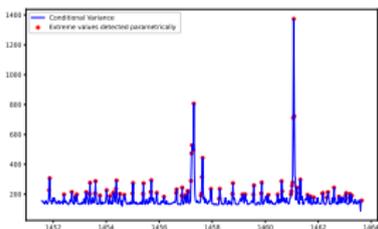
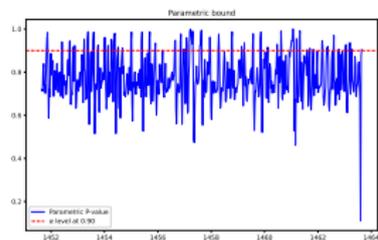


TIC13955147-s5-II

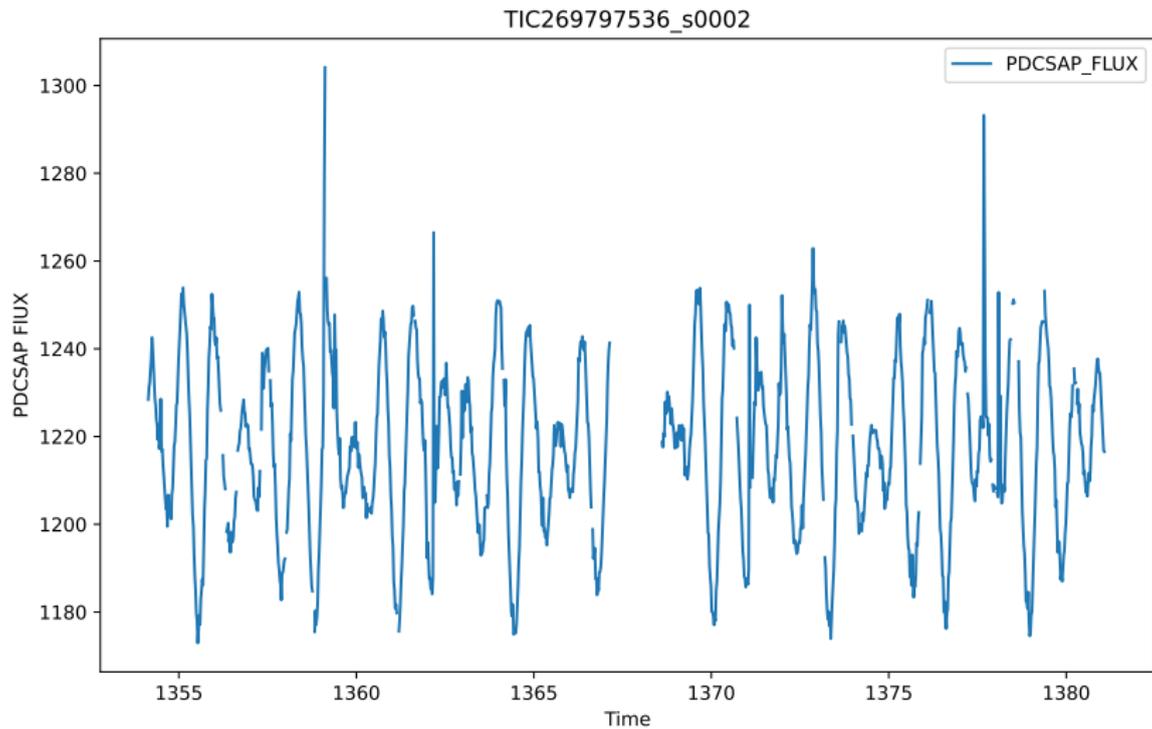
non-parametric detrend



parametric flares detection

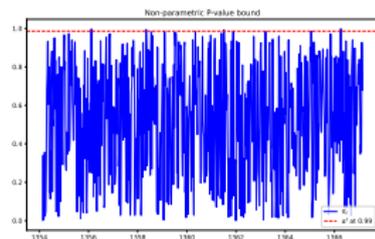
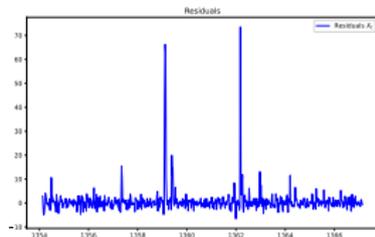
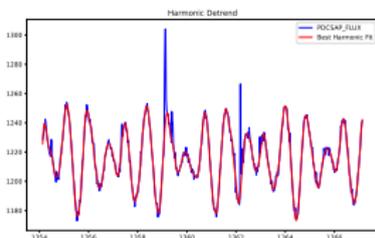


TIC269797536-s2

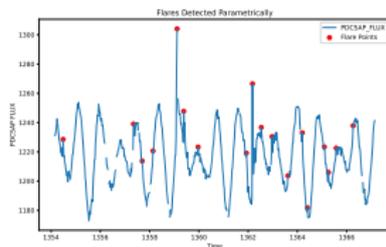
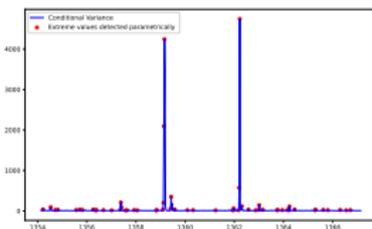
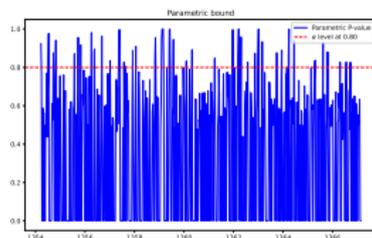


TIC269797536-s2-I

non-parametric detrend

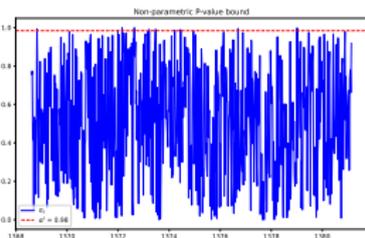
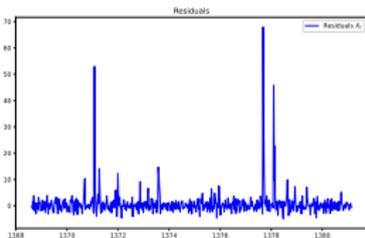
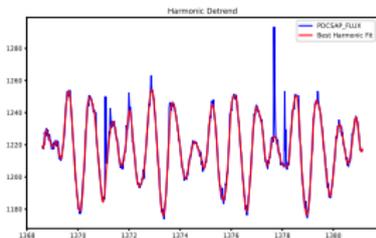


parametric flares detection

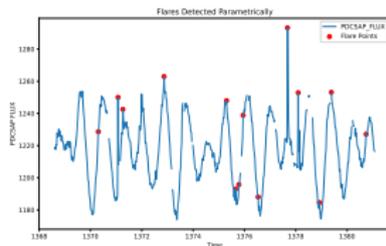
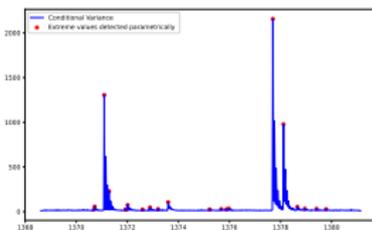
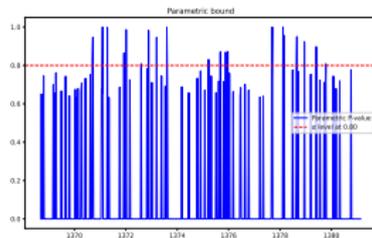


TIC269797536-s2-II

non-parametric detrend



parametric flares detection



A semi-parametric approach: robust estimation of σ_t

Hampel (1974)

$$MAD_n = 1.4826 \times \text{med}_{1 \leq i \leq n} |X_i - \text{med}_{1 \leq j \leq n} X_j|$$

- The MAD has the best possible breakdown point (50%, twice as much as the interquartile range)
- A breakdown point of 50% is the highest possible, because the estimate remains bounded when fewer than 50% of the data points are replaced by arbitrary numbers.
- The MAD also has some drawbacks.
 - 1) Its efficiency at Gaussian distributions is very low; whereas the location median's asymptotic efficiency is still 64%, the MAD is only 37% efficient.
 - 2) MAD takes a symmetric view on dispersion, because one first estimates a central value (the median) and then attaches equal importance to positive and negative deviations from it.

Flare distribution indices for each dataset

Dataset	α	Sample Size	Flare energy
TIC 269797536			
0005	1.4 ± 0.21	29	0.086
0039	2.3 ± 0.22	187	0.039
0027	2.0 ± 0.28	67	0.092
0008	1.8 ± 0.27	53	0.042
0032	1.6 ± 0.29	33	0.092
0013	1.7 ± 0.31	29	0.358
TIC 13955147			
0004	1.2 ± 0.27	28	0.303
0005	1.8 ± 0.30	58	0.386
0028	1.4 ± 0.29	88	0.113
0012	1.9 ± 0.38	30	0.091
0038	2.1 ± 0.30	80	0.081
0002	1.4 ± 0.18	38	0.053
0035	2.3 ± 0.26	99	0.037
0011	1.8 ± 0.37	31	0.076
0031	2.3 ± 0.20	154	0.178
0032	1.9 ± 0.20	128	0.218

A semi-parametric approach: robust estimation of σ_t

Rousseeuw & Croux (1993)

$$S_n = 1.1926 \times \operatorname{med}_{1 \leq i \leq n} \left\{ \operatorname{med}_{1 \leq j \leq n} |X_i - X_j| \right\}$$

- More efficient and not slanted towards symmetric distributions.
- Unlike the MAD, it does not need any location estimate of the data. Instead of measuring how far away the observations are from a central value, S_n looks at a typical distance between observations, which is still valid at asymmetric distributions.

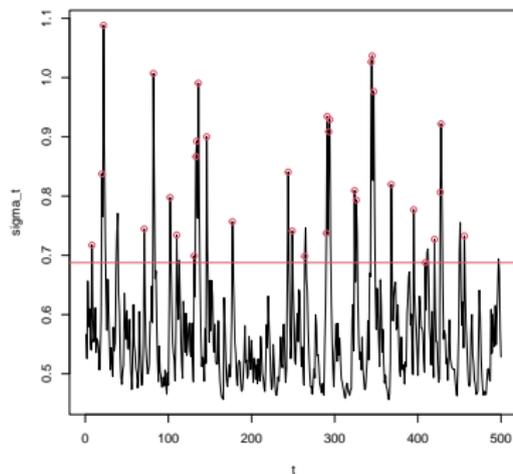
A semi-parametric approach: robust estimation of σ_t

Rousseeuw & Croux (1993)

Simulate

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

with $\alpha_0 = 0.1$, $\alpha_1 = 0.2$, and $\beta_1 = 0.5$.



ARFIMA

- The Autoregressive Fractionally Integrated Moving Average (ARFIMA) model extends the classical ARIMA model by incorporating a fractional differencing parameter, enabling the model to capture both short-term autocorrelation and long-range dependence within time series data.
- Particularly valuable for analyzing data sets where phenomena persist over time, such as in economics and finance.

$$\left(1 - \sum_{i=1}^p \phi_i L^i\right) (1 - L)^d Y_t = \left(1 + \sum_{j=1}^q \theta_j L^j\right) \varepsilon_t,$$

where:

L is the lag operator such that $LY_t = Y_{t-1}$,

p is the order of the autoregressive (AR) part,

d is the order of fractional differencing, allowing the model to handle processes with long memory,

q is the order of the moving average (MA) part,

ϕ_i are the coefficients of the AR part,

θ_j are the coefficients of the MA part,

ε_t is the error term, assumed to be white noise.

ARFIMA

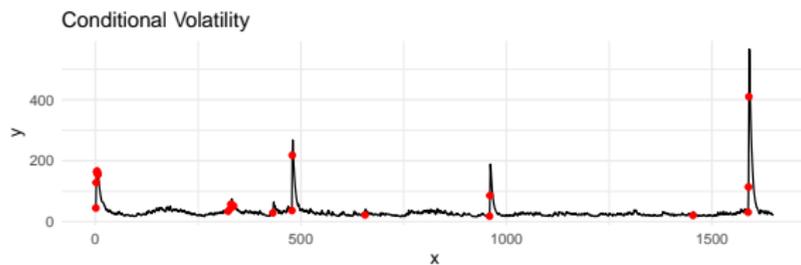
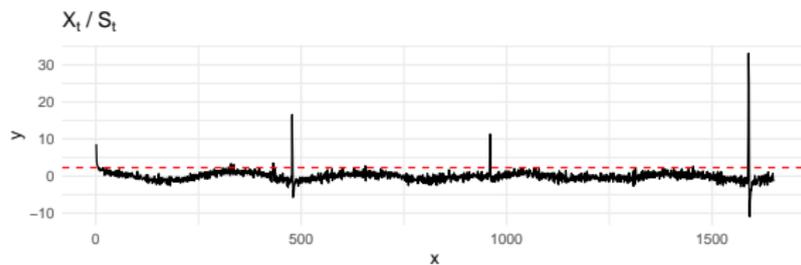
- The unconditional variance of the model is influenced by the parameters of both the AR and MA parts as well as the differencing parameter d , which affects the long-term dependence properties of the series.
- The conditional mean of the ARFIMA model is given by the AR part:

$$E(Y_t | Y_{t-1}, Y_{t-2}, \dots) = \sum_{i=1}^p \phi_i Y_{t-i}$$

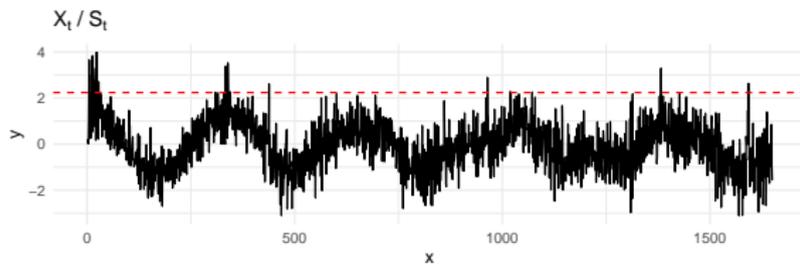
while the conditional variance remains constant.

- Primary motivation ARFIMA: their ability to model series with highly complex stochastic structures that exhibit both short-term and long-term dependencies.
- For instance, financial time series often display volatility clustering (short-term) and long memory (long-term dependencies in volatility), both of which can be effectively captured by the ARFIMA model.

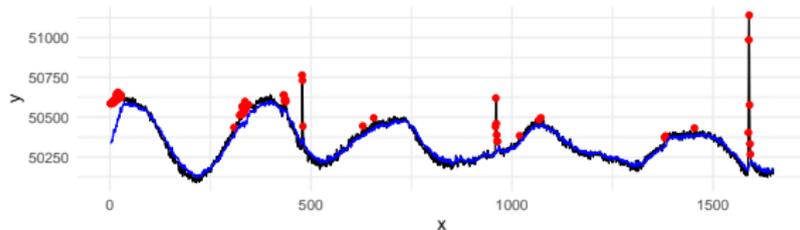
ARFIMA-GARCH fit with 1 iteration



ARFIMA-GARCH fit after 5 iteration



ARFIMA FIT AFTER 1 iteration



Conditional Volatility

