Higher Criticism Statistic: Theory and Applications in Cosmology and Astronomy

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Abstract

Higher Criticism is a recent statistic proposed by Donoho and Jin [1] where it has been shown to be effective at resolving a very subtle testing problem: test whether n normal means are all zero versus the alternative that a small fraction is non-zero.

Higher Criticism is also useful for non-Gaussian detection. Motivated by the recent problem of detecting cosmic strings, we consider a setting in which we test whether each of n tests is truly Gaussian, or a superposition of a Gaussian component with a very faint nonGaussian component, whose distribution is unknown but symmetric and has a power-law tail, e.g. the tail probability $\propto |x|^{-\alpha}$. We show that the Higher Criticism is asymptotically optimal when $\alpha < 8$, while the widely used statistic in cosmology - excess kurtosis - is asymptotically optimal when $\alpha > 8$.

We have also implemented the Higher Criticism in the first year data of the recent Wilkinson Microwave Anisotropy Project (WMAP, 2003), we found that the Higher Criticism is not only almost equally powerful as the kurtosis by reporting a nonGaussian detection with confidence $> 99\%$, but also suggests a way to track down the small portion of data corresponds to the nonGaussianity. We compare our result with the recent reported nonGaussianity detection (e.g. Vielva et al. 2004).

This talk includes some work collaborated with (alphabetically) Nabila Aghanim, David Donoho, Olivier Forni, and Jeau-Luc Starck, and some work collaborated with Laura Cayon.

References

[1] DONOHO, D. and JIN, J. (2004). Higher Criticism for Detecting Sparse Heterogeneous Mixtures. *Ann. Statist.*, Vol 32, **3**, 962-994.