Estimate Strong Lens Time Delay

multiple filters, multiple quasars

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Model: Intrinsic variation

Intrinsic variation of light source for multiple filters $\mathbf{X} = (X_1, X_2, ..., X_k)$ generated by correlated Ornstein-Uhlenbeck process (O-U process):

$$d\mathbf{X}(\mathbf{t}) = -\frac{1}{\tau}(\mathbf{X}(\mathbf{t}) - \mu)\mathbf{dt} + \sigma\mathbf{dB}(\mathbf{t}) \qquad (1)$$

$$dB_i(t)B_j(t) = \rho_{ij}dt$$

Model: Strong- and Micro- lensing

For each filter j, a pair of latent light curves $X_j(t)$ and $Y_j(t)$ are offset by δ because of strong lensing:

$$Y_j(t) = X_j(t - \delta) + \beta_0$$

Microlensing trends at different wavelengths modeled by low-order polynomial:

$$\tilde{X}_j(t) = X_j(t) + p_{m,\beta_{1,j}}(t)$$
$$\tilde{Y}_j(t) = X_j(t - \delta) + p_{m,\beta_{2,j}}(t - \delta) \qquad (2)$$

Observed data

Each pair of light curves $(\tilde{X}_j(t), \tilde{Y}_j(t))$ is sampled at irregular time intervals $(t_{j,1}, t_{j,2}, ..., t_{j,n_j})$.
With some measure errors, observed light curves are distributed as:

$$x(t_{j,i}) \sim N(\tilde{X}_j(t_{j,i}), \xi_j^2(t_{j,i}))$$
$$y(t_{j,i}) \sim N(\tilde{Y}_j(t_{j,i}), \eta_j^2(t_{j,i})), \; i = 1, 2, ...n_j \qquad (3)$$

$\xi_j(t_{j,i})$ and $\eta_j(t_{j,i})$ are given.

Combined Time Sequence

For simplicity, we drop the subscript j for each filter in this slide.
Let $\mathcal{T} = \{(t_1, t_2, ..., t_{2n}),\ t_1 \leq t_2 ... \leq t_{2n}\}$ be the ordered and combined time sequence of $\mathcal{T}_1 = \{t_i\}$ and $\mathcal{T}_2 = \{t_i - \delta\}$, $\{z(t_i)\}$ be the combined observed sequence of $\{x(t_i)\}$ and $\{y(t_i)\}$, and $\{Z(t_i)\}$ be the combined latent light curve:

$$z(t_i) = \begin{cases} x(t_i) & t_i \in \mathcal{T}_1 \\ y(t_i + \delta) & t_i \in \mathcal{T}_2 \end{cases}$$

$$Z(t_i) = \begin{cases} \tilde{X}(t_i) = X(t_i) + p_{m,\beta_1}(t_i) & t_i \in \mathcal{T}_1 \\ \tilde{Y}(t_i + \delta) = X(t_i) + p_{m,\beta_2}(t_i) & t_i \in \mathcal{T}_2 \end{cases}$$

Likelihood: combine multiple filters

Let $\{z(t_i)\}$ be the combined observed sequences of all filters, $\{\mathbf{Z}(t_i)\}$ is the latent light curves, which is k-dim.

$z(t_i)|\mathbf{Z}(t_i) \sim N(\mathbf{Z}_j(t_i), \eta_j^2(t_i))$, filter j is observed at t_i

$$Z_j(t_i) = X_j(t_i) + p_{m,\beta_{j1}}(t_i)I(t_i \in \mathcal{T}_{j1}) + p_{m,\beta_{j2}}(t_i)I(t_i \in \mathcal{T}_{j2}), \quad (4)$$

$$\mathbf{X}(t)|\mathbf{X}(s) \sim \mathrm{MVN}(\boldsymbol{\mu} + e^{-(t-s)/\tau}(\mathbf{X}(s) - \boldsymbol{\mu}), Q(t-s))$$

where $Q_{ij}(t-s) = \frac{\sigma_i\sigma_j\rho_{ij}\tau_i\tau_j}{\tau_i+\tau_j}(1 - e^{-(1/\tau_i+1/\tau_j)(t-s)})$.
For identification, we assumed $\mu_1 = 0$ and absorbed the mean into the constant term in the polynomial.

Prior for Microlensing

Microlensing is partially coherent for multiple bands and indepndent for (un-)shift curve:

$$\boldsymbol{\beta}_{j1} \sim N(\boldsymbol{\beta}_1, \boldsymbol{\sigma}^2), \boldsymbol{\beta}_{j2} \sim N(\boldsymbol{\beta}_2, \boldsymbol{\sigma}^2),\ j \in \{1..k\}$$
$$\boldsymbol{\beta}_1,\ \boldsymbol{\beta}_2 \sim N(\boldsymbol{\mu}, \boldsymbol{\sigma}^2/\boldsymbol{\kappa}) \tag{5}$$
$$p(\boldsymbol{\mu}) \propto 1,\ p(\sigma_m^2) \propto 1/\sigma_m^2,\ p(\kappa_m) \sim \mathrm{Gam}(1,1)\ \forall m \tag{6}$$

Prior for parameters in OU process

$$\sigma_j^2 \sim \text{IG}(\alpha_\sigma, \beta_\sigma),\ \tau_j \sim \text{IG}(\alpha_\tau, \beta_\tau) \tag{7}$$
$$\alpha_\sigma, \alpha_\tau, \beta_\tau \sim \text{Gamma}(1, 1)$$
$$\beta_\sigma \sim \text{Gamma}(10^{-6}, 1)$$

And $\rho_{ij} \sim \text{Uniform}(0, 1)$

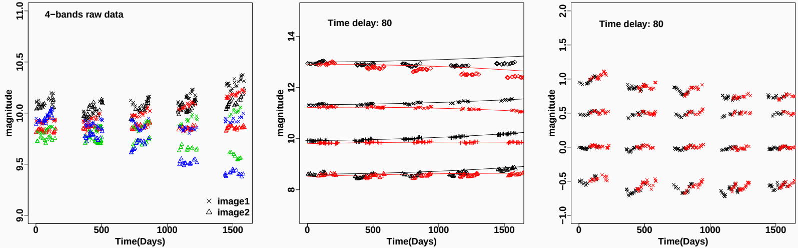
Inference: MCMC

Kalman Filter to sample the latent light curves ($\{\mathbf{Z}(t)\}$) given other parameters.

Sample from conditional distribution of βs given $\mathbf{Z}(t)$ which is Gaussian.

Metropolis-Hastings within Gibbs sampling to update $\tau,\ \sigma,\ \rho,\ \delta$.

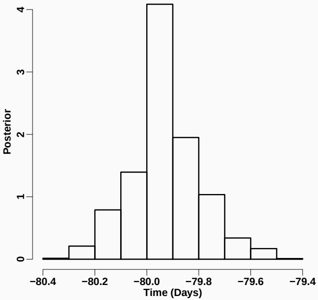
Simulations



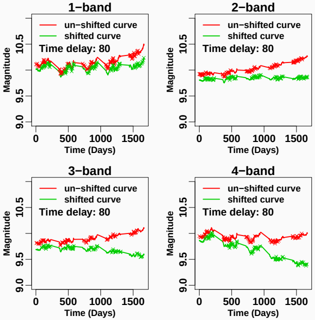
(a) Raw Data (b) Shifted Data (c) OU-process

Figure: Simulation Data: 5 years, Season = 5 months, Cadence = 2 days, n≈ 90 for each band.

Simulation Result



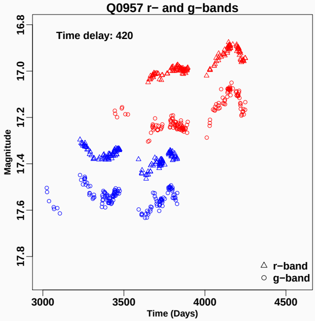
(a) Posterior of δ



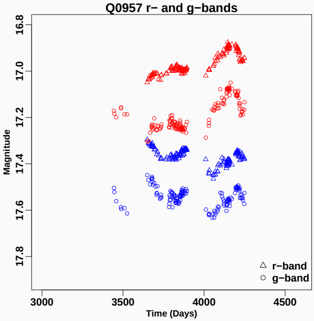
(b) Fitted Data

Real Data

Doubly-lensed Q0957+561 quasar r- and g-bands data. $132 \times 2 = 264$ samples from r-band and $142 \times 2 = 284$ from g- band in $\sim$5 years.



(a) shifted



(b) un-shifted

Result: Profile log-likelihood

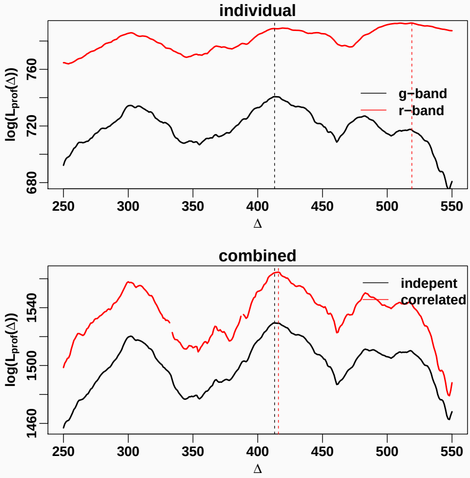


Figure: Individually and combined profile log -likelihood. For combined model, either independent or correlated O-U processes.

Result: Full Bayesian Model

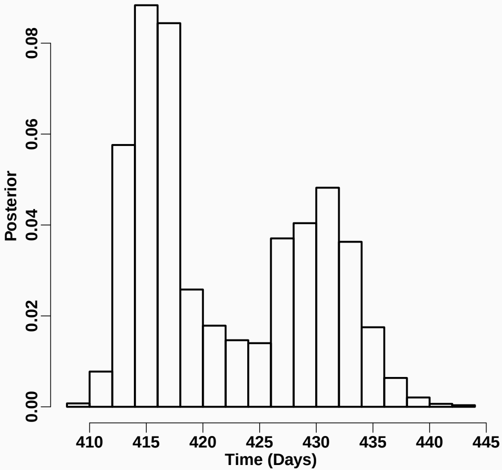


Figure: Posterior distribution of δ.

Result: Full Bayesian Model

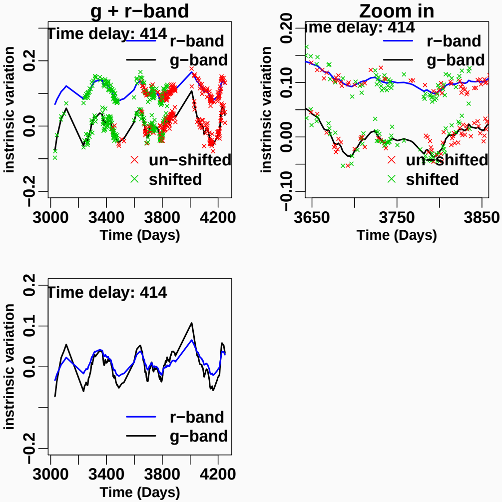


Figure: Posterior mean of intrinsic brightness by O-U process (remove microlensing with $m = 3$).

Result: Full Bayesian Model

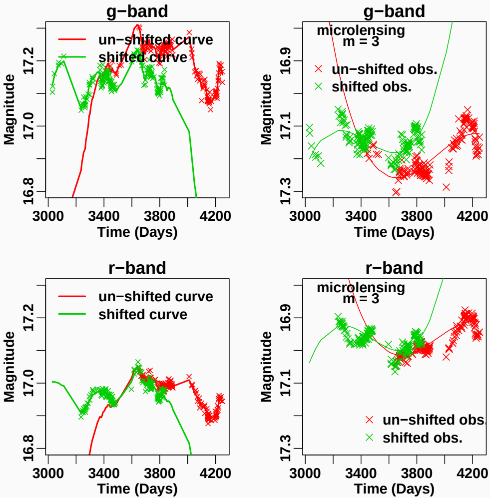


Figure: Posterior mean of latent light curve (including microlensing with $m = 3$).

Future work

With additional flux errors, model the error by heavier tail distribution (Student's t). More computational cost.

Different resolution of OU process for intra-night and inter-night variation of light source

Model Microlensing as hierarchical Gaussian Process or Choose other basis function

Combine different quasars to estimate Hubble constant.