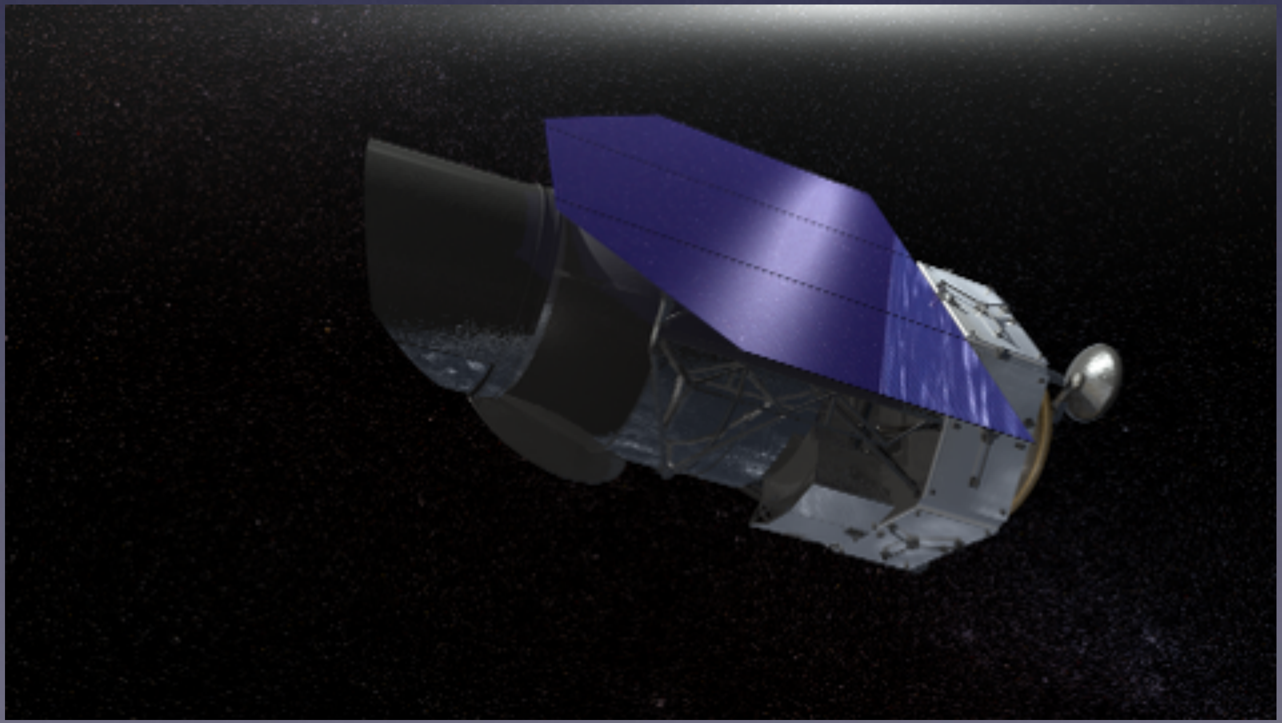
The Type Ia Supernova Color-Magnitude Relation and Host Galaxy Dust:

A Simple Hierarchical Bayesian Model


SN 2014J KAIT/LOSS color image

Kaisey Mandel
Harvard-Smithsonian
Center for Astrophysics

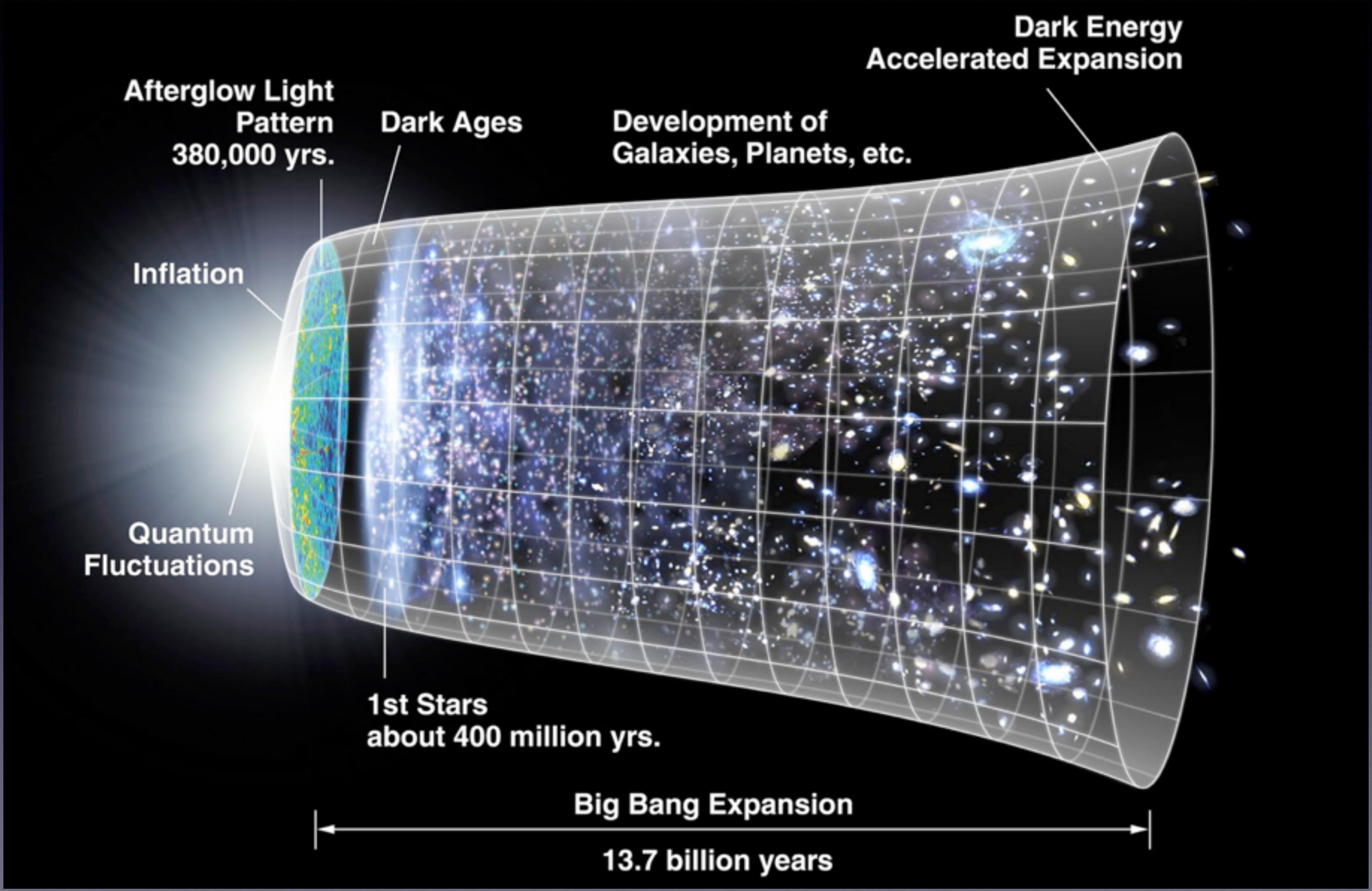


Astrostatistics Seminar
22 Nov 2016

Outline

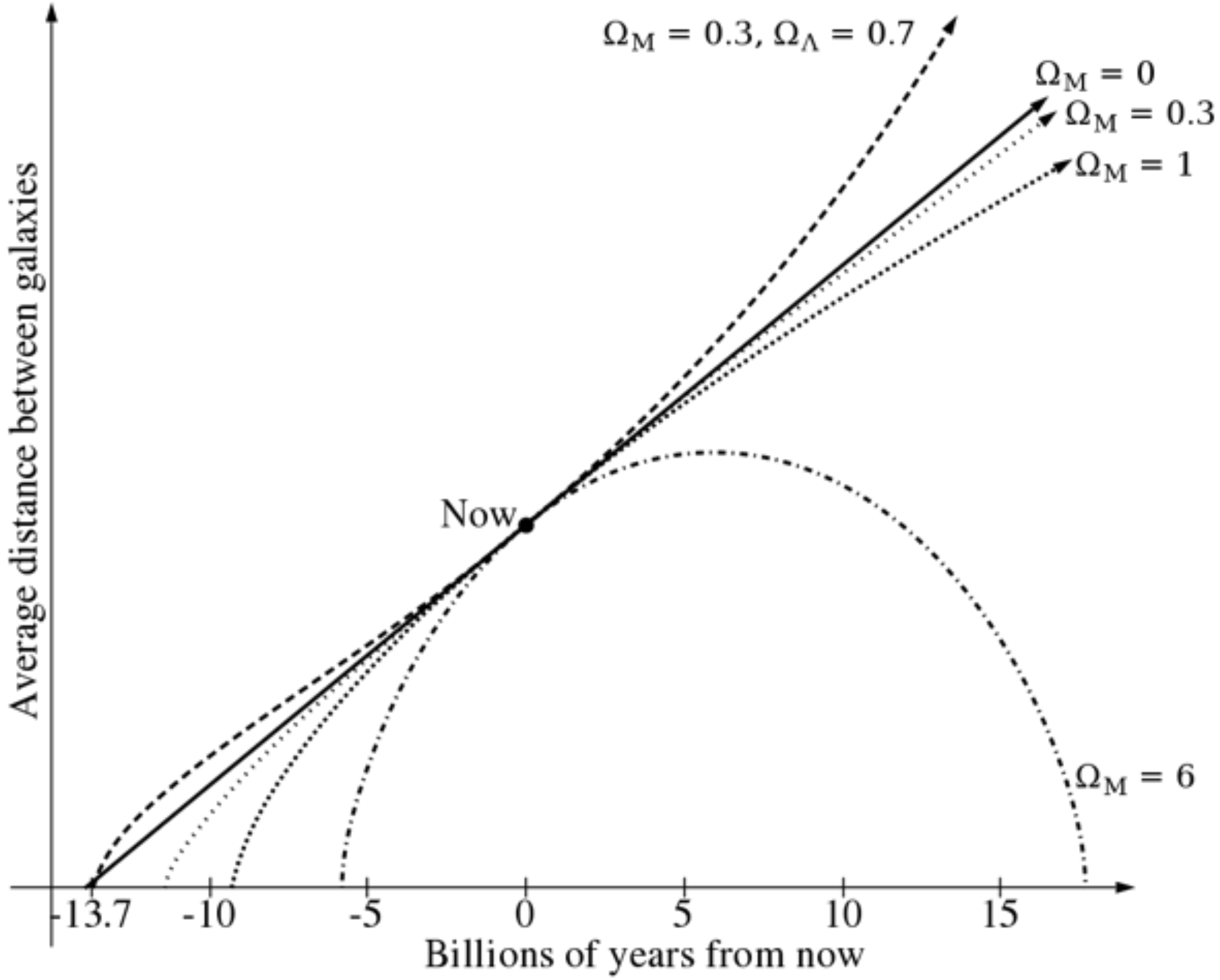
- Introductory Background and Scientific Motivation:
  Type Ia Supernova (SN Ia) Cosmology
- Simple-BayeSN: a new hierarchical Bayesian statistical model for SN Ia data to incorporate multiple random effects (intrinsic and dust), individuals & populations, distances & cosmology
    - K. Mandel, D. Scolnic, H. Shariff, R. Foley & R.P. Kirshner (2016, submitted to ApJ, arXiv:1609.04470)
- Application to SN Ia cosmology data
- Future Developments

The History of Cosmic Expansion

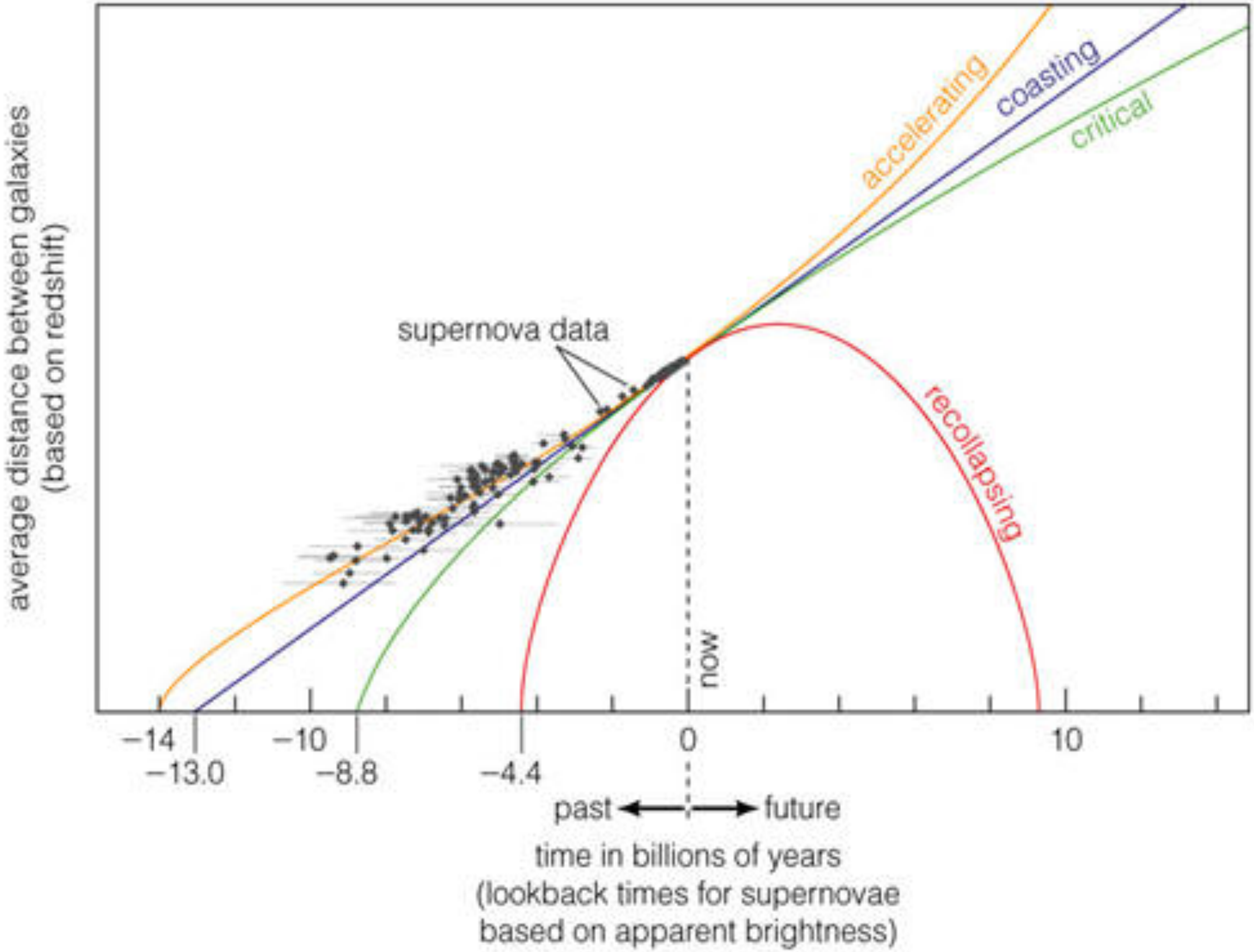


Expansion History (and Future) of the Universe: Determined by its Physical Energy Content

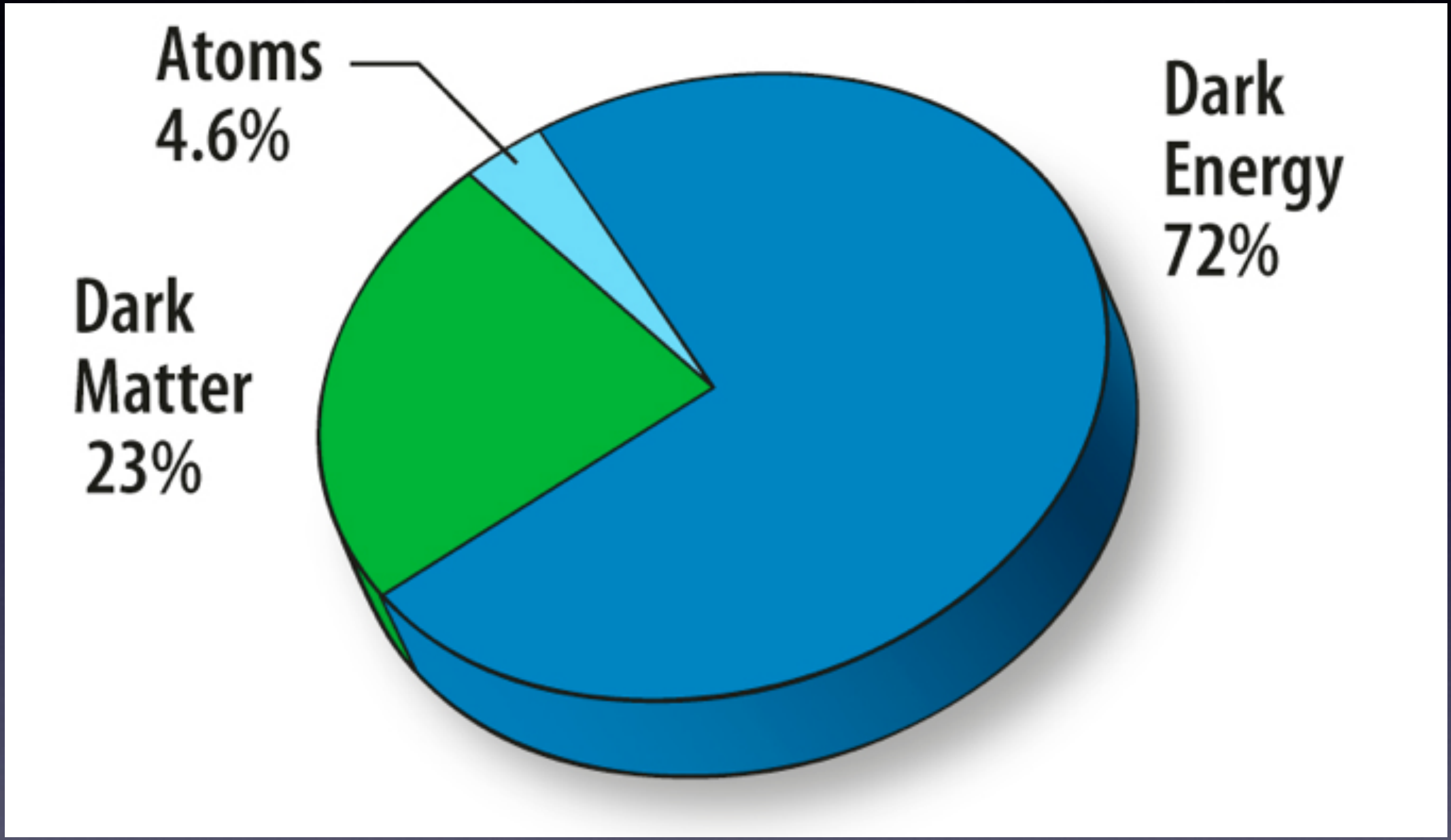
Ω_M = Matter Density; Ω_Λ = Dark Energy Density



Supernovae Trace the History of Cosmic Expansion



Cosmological Energy Content

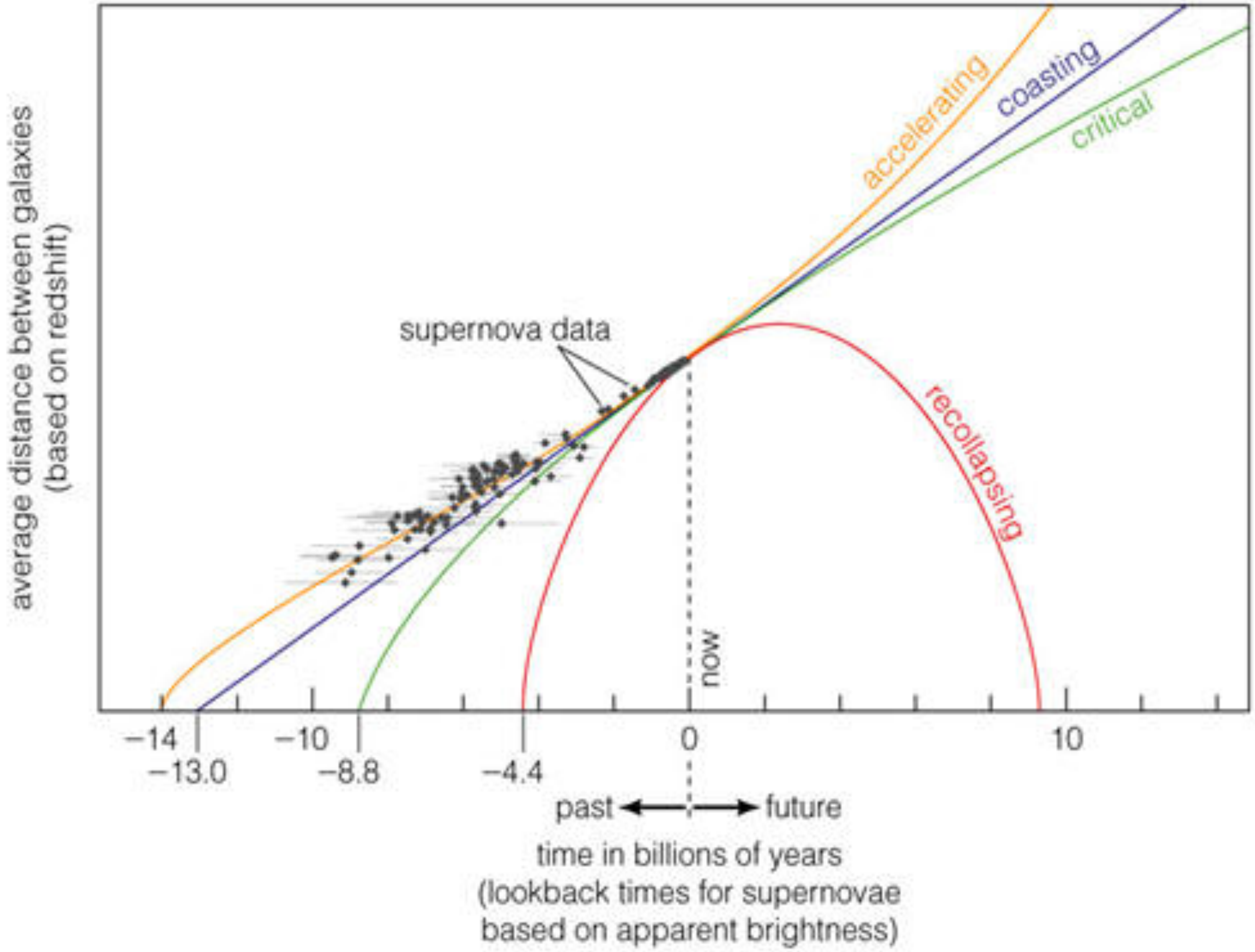


What is Dark Energy?

Dark Energy Equation of state $P = w\rho$

Is $w + 1 = 0$? (Cosmological Constant: $w = -1$)

Supernovae Trace the History of Cosmic Expansion

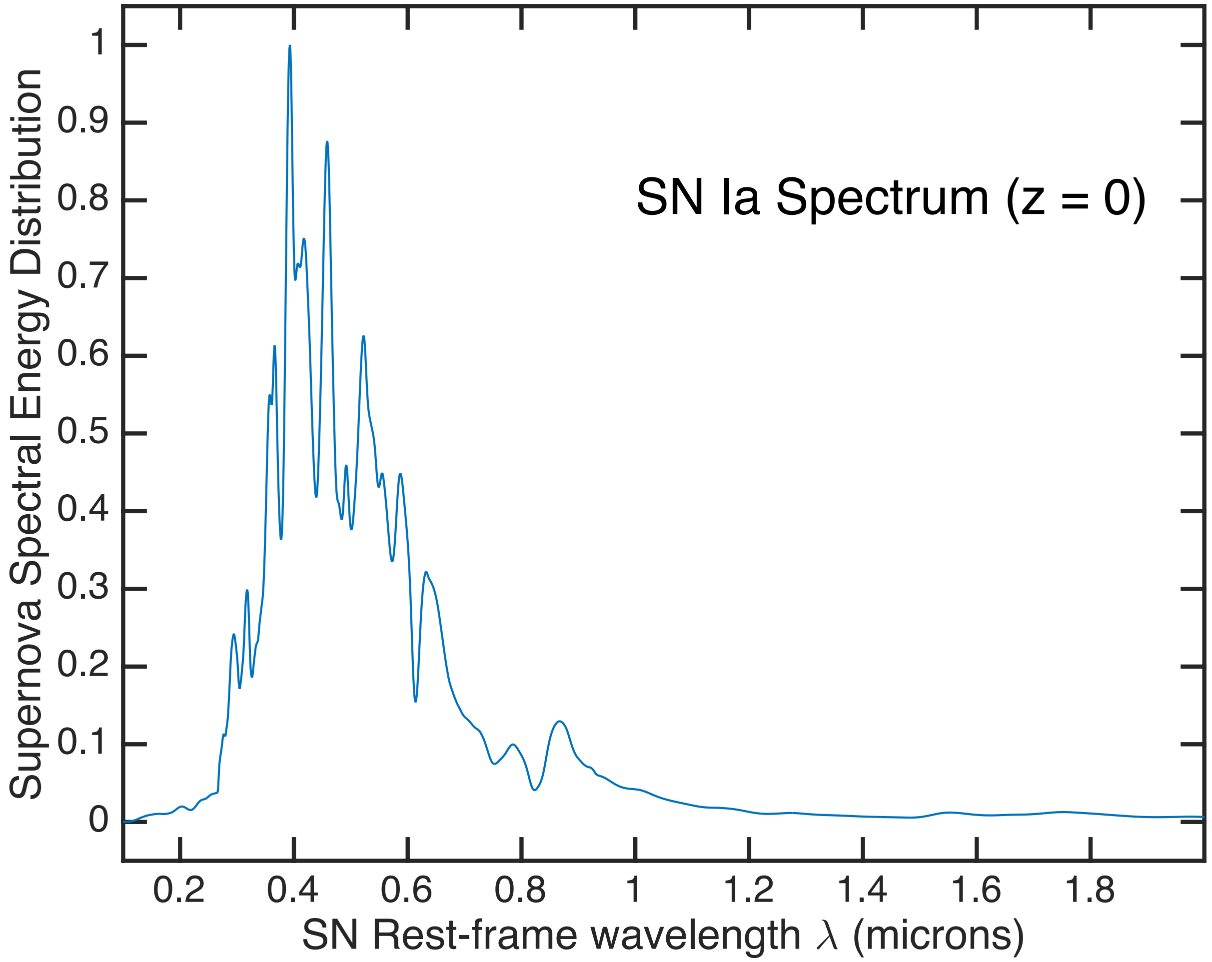


But we don't actually measure these things!

Time → Distance (μ)

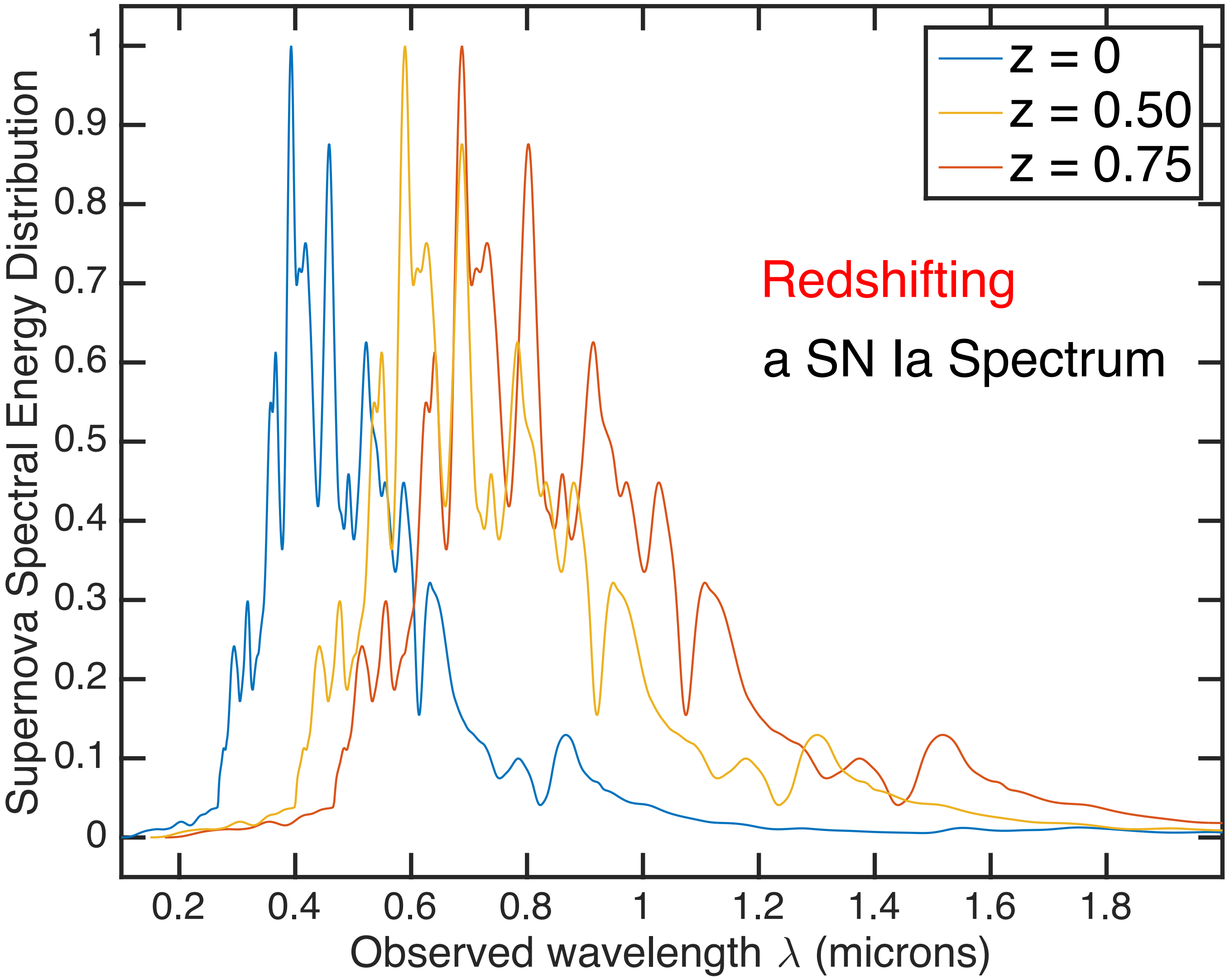
Relative Size of Universe → Redshift (z)

Expansion of the Universe: Redshifts (z)



Expansion of Universe over time "stretches" out wavelengths of light

Measure of speed of expansion between observer and SN event



Spectral Lines are observed at longer wavelengths than originally emitted by the supernova: redshift (z)

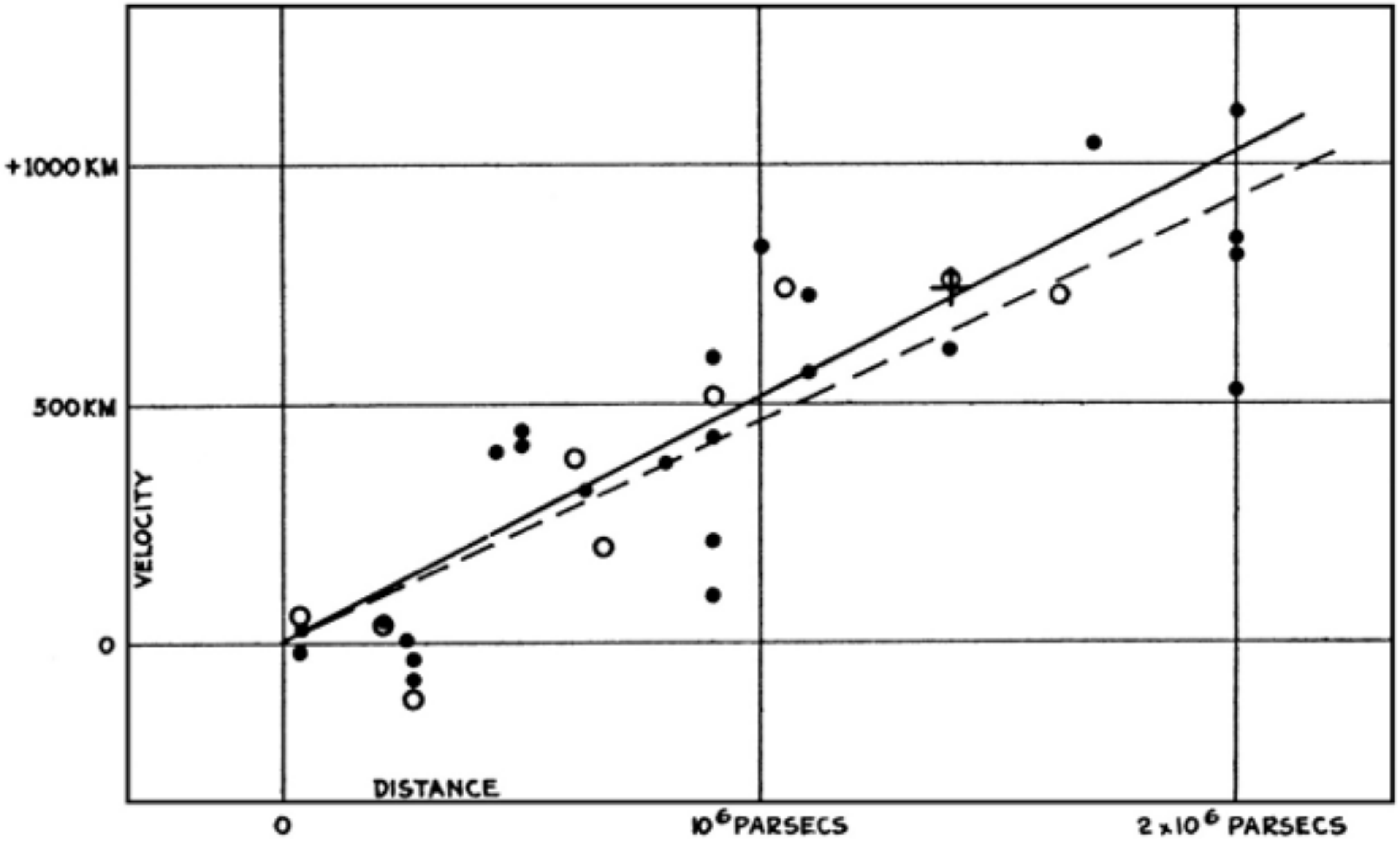
Determining Astronomical Distances using Standard Candles

1. Estimate or model Luminosity L of a Class of Astronomical Objects
2. Measure the apparent brightness or flux F
3. Derive the distance D to Object using Inverse Square Law: $F = L / (4\pi D^2)$
4. Optical Astronomer's units: $\mu = m - M$

(m = apparent magnitude, M = absolute magnitude,

μ = distance modulus [log distance])

The Expanding Universe:
Galaxies are moving apart! Hubble's Law (1929)



Einstein & Hubble

Distance $\propto$ Velocity (Redshift)

But what is the rate of change of the expansion?

(the deceleration parameter) Need better distances!

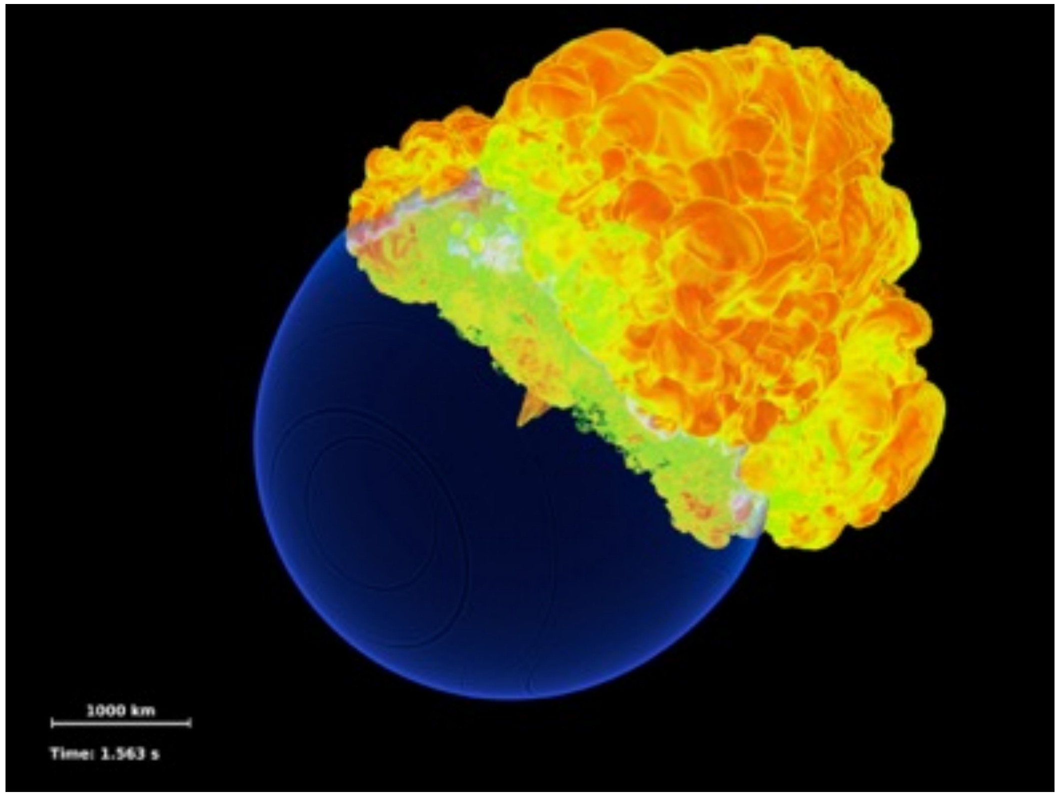
Type Ia Supernovae (SN Ia) are Almost Standard Candles

- Progenitor: C/O White Dwarf Star accreting mass leads to instability

- Thermonuclear Explosion: Deflagration/Detonation

- Nickel to Cobalt to Iron Decay + radiative transfer powers the light curve

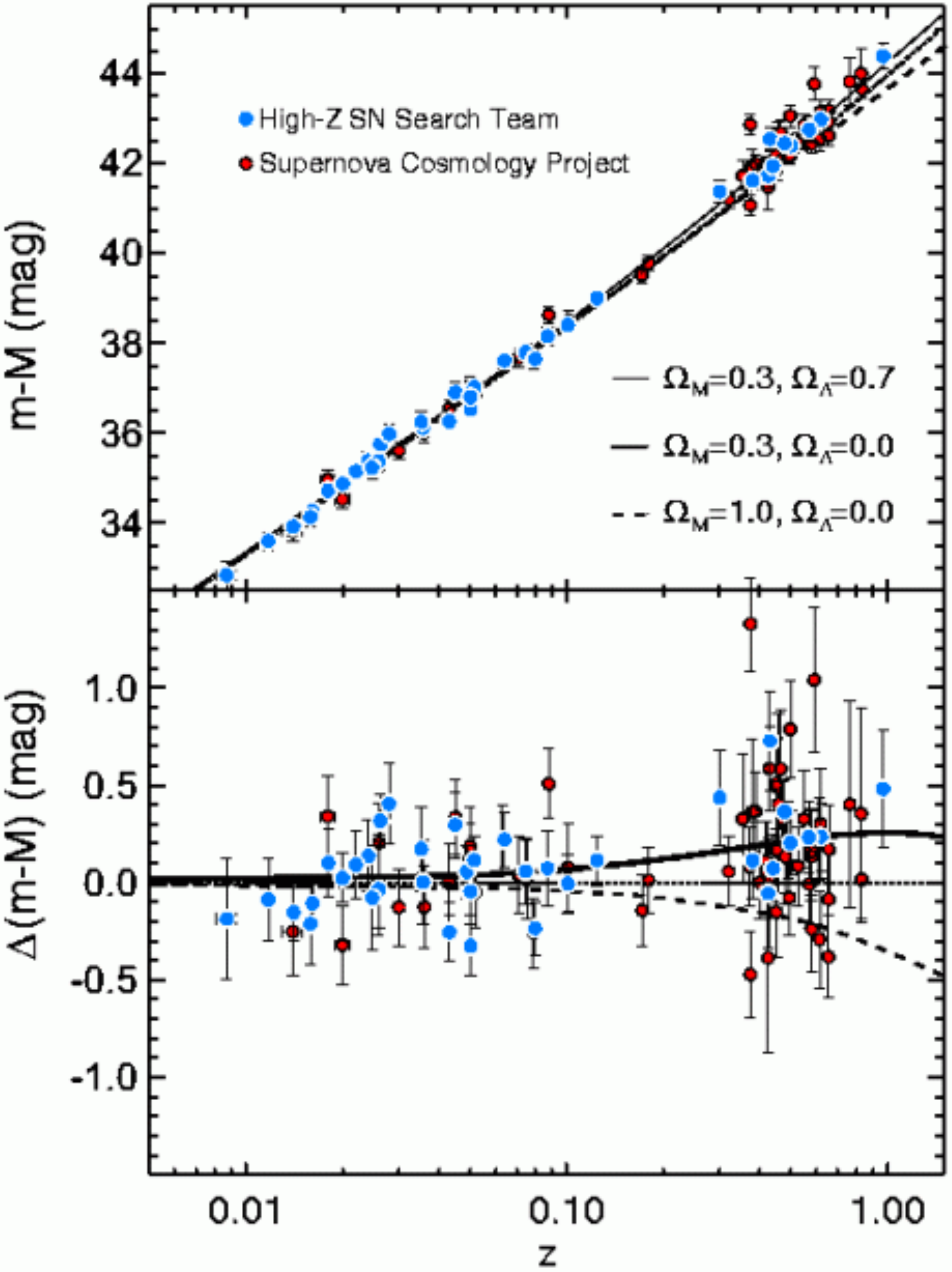
- General Idea, but Theoretical Astrophysics simulations cannot quantitatively reproduce realistic observations (use empirical models)



Credit: FLASH Center

SN Ia Hubble Diagram (Distance Moduli vs. z):

The Universe is accelerating ($\Omega_\Lambda > 0$)!



The Accelerating Universe
2011 Nobel Prize in Physics


Photo: U. Montan

Saul Perlmutter

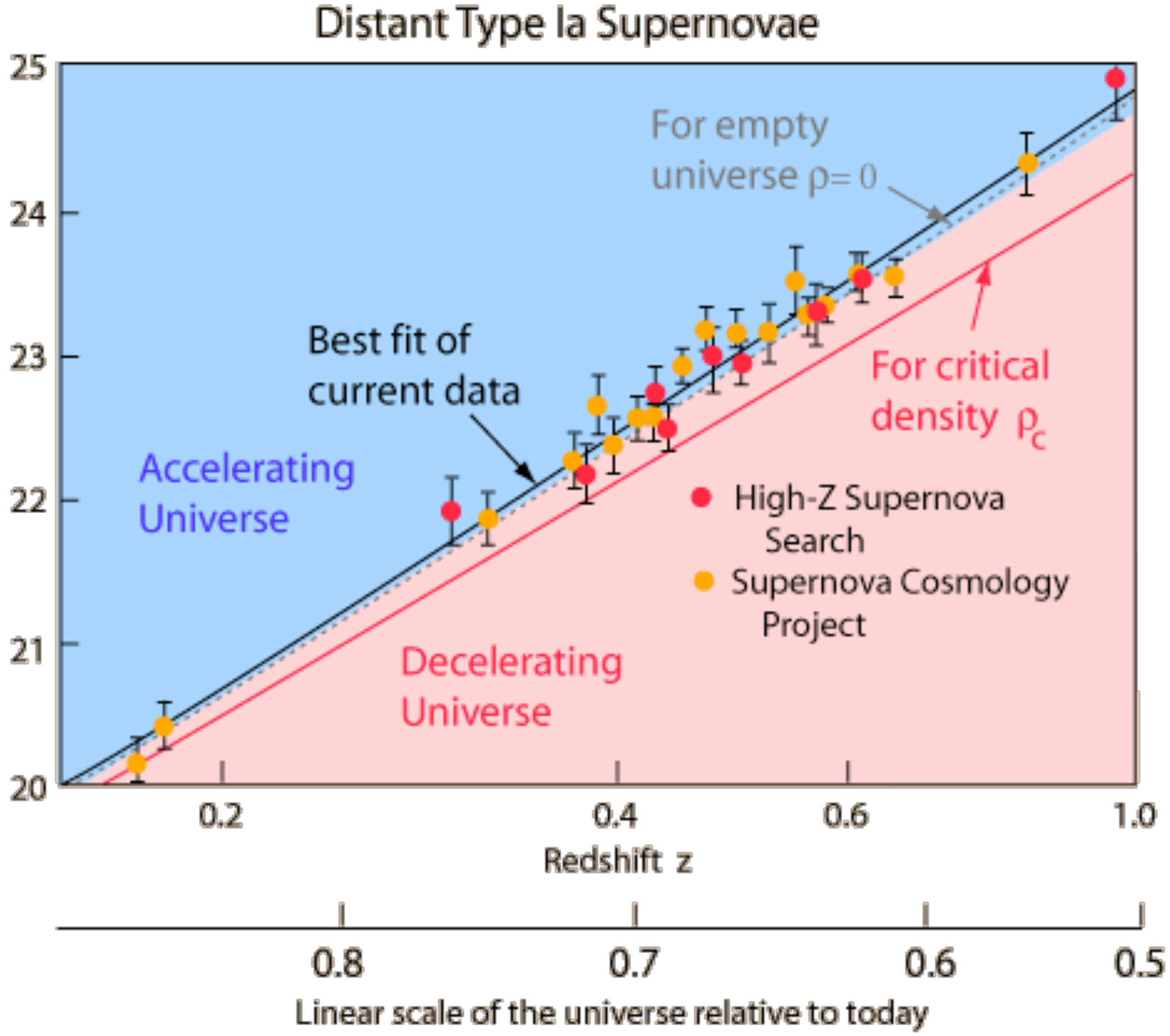

Photo: U. Montan

Brian P. Schmidt


Photo: U. Montan

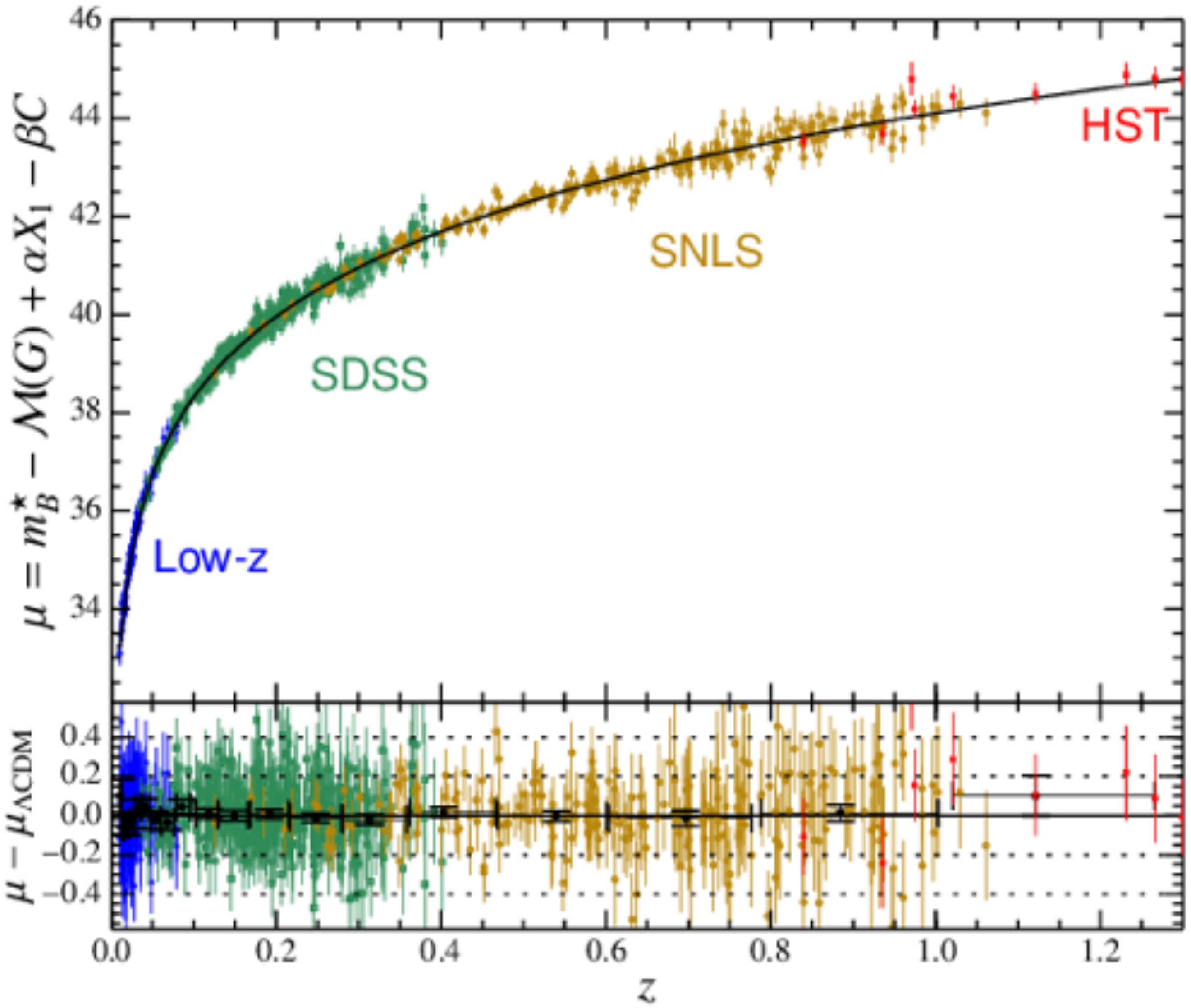
Adam G. Riess

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

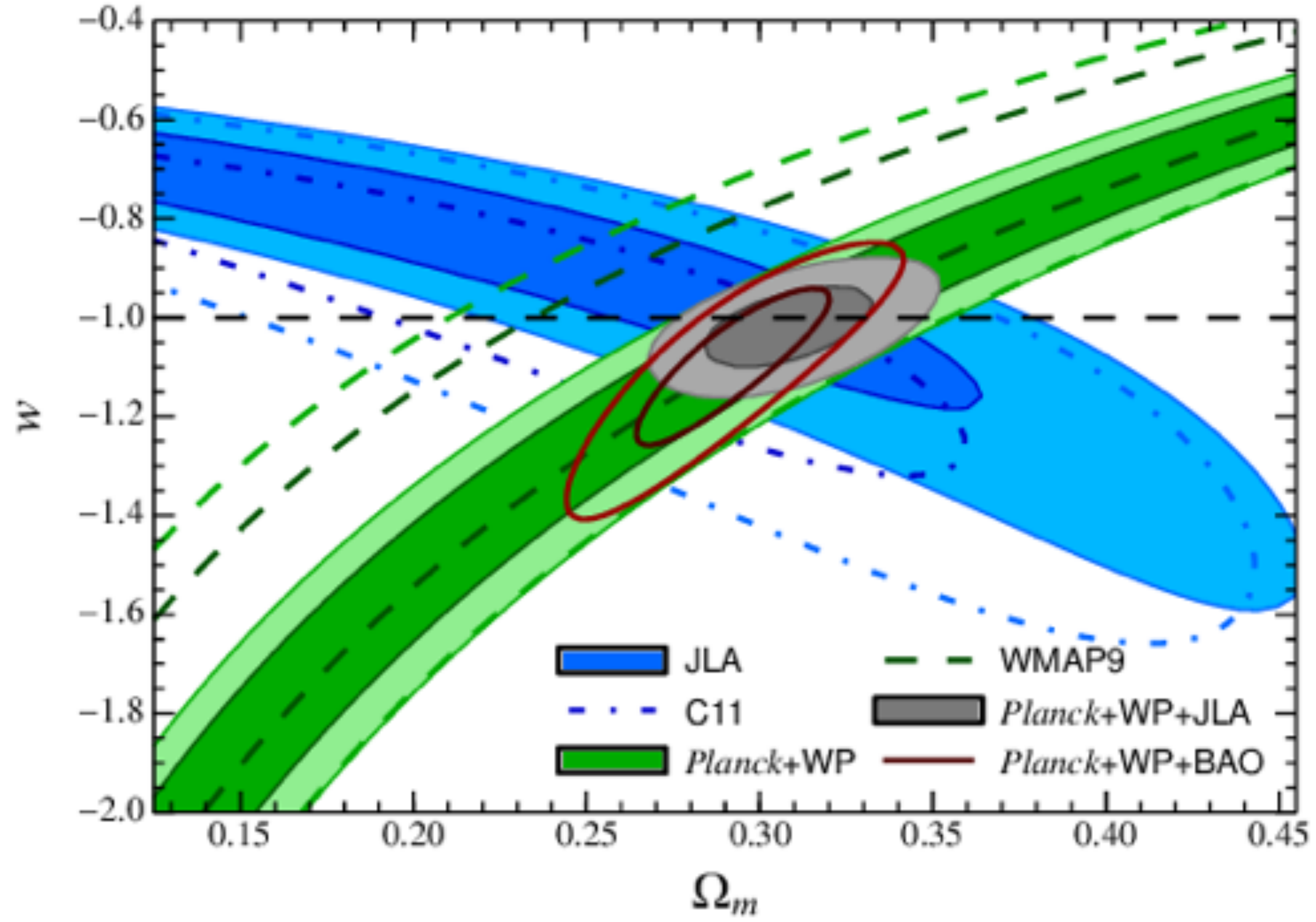


18 Years Later…

Hubble Diagram
Modern SN Ia Surveys



Cosmological Constraints



Joint Lightcurve Analysis
(JLA, Betoule et al. 2014)

Example of SN Ia cosmology in practice

PanSTARRS: A Supernova Discovery Machine

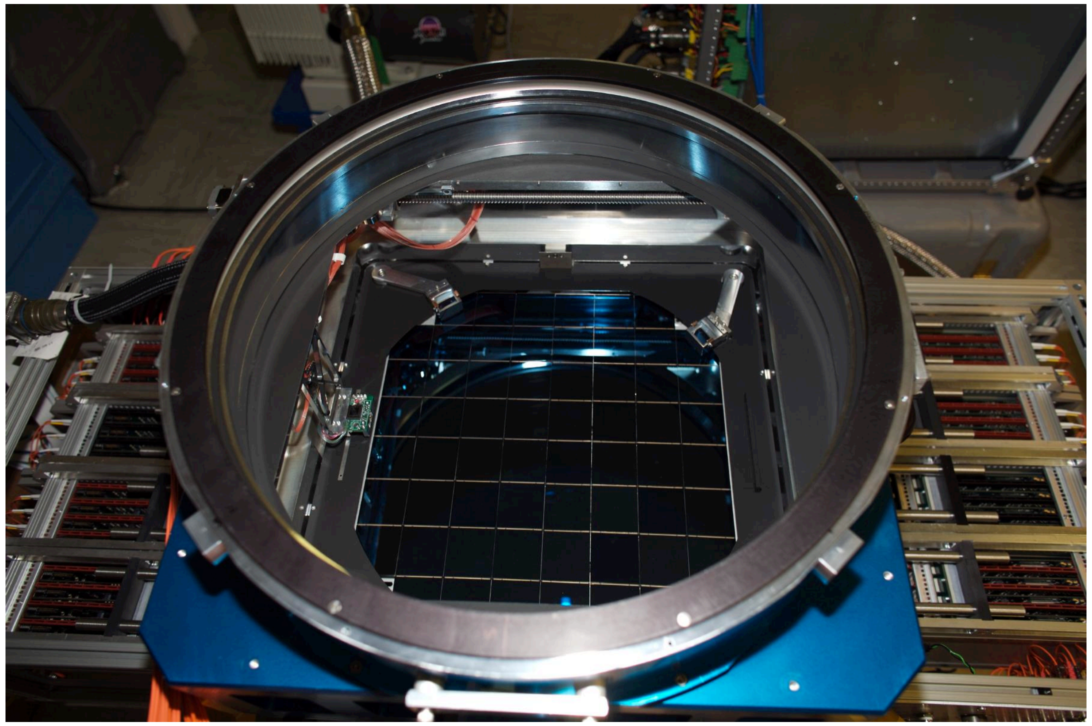
Medium-Deep Fields

Good light curves at z~0.4

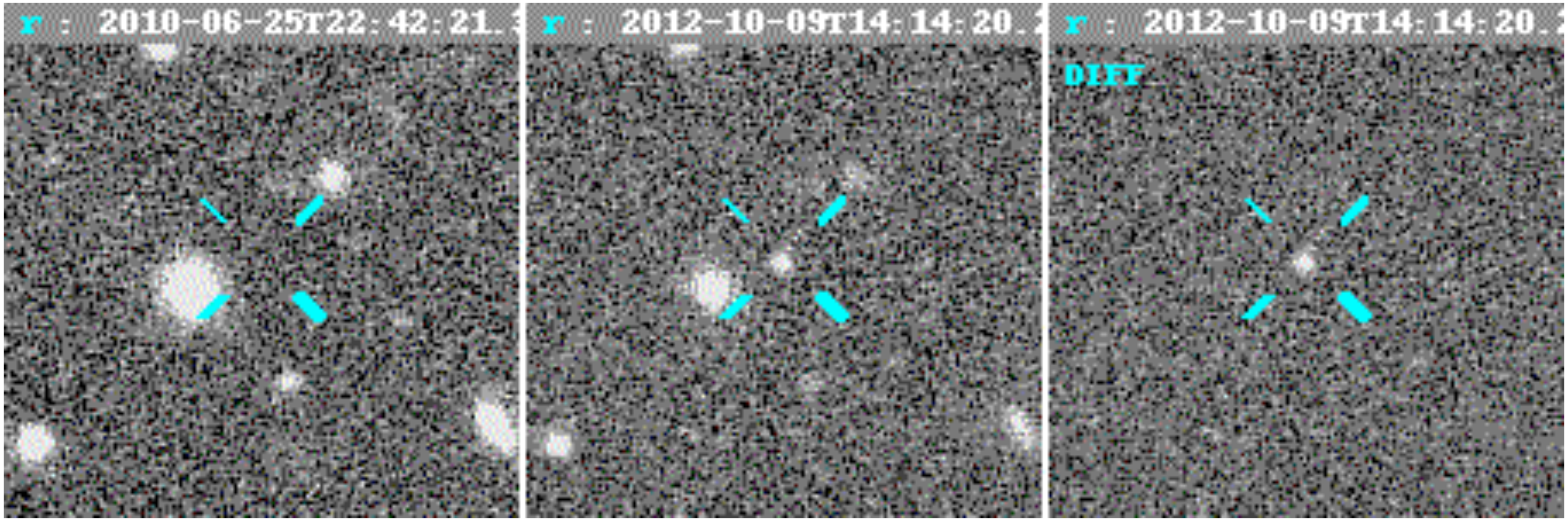
Every 4 days griz

7 square degrees 0.26”/pixel

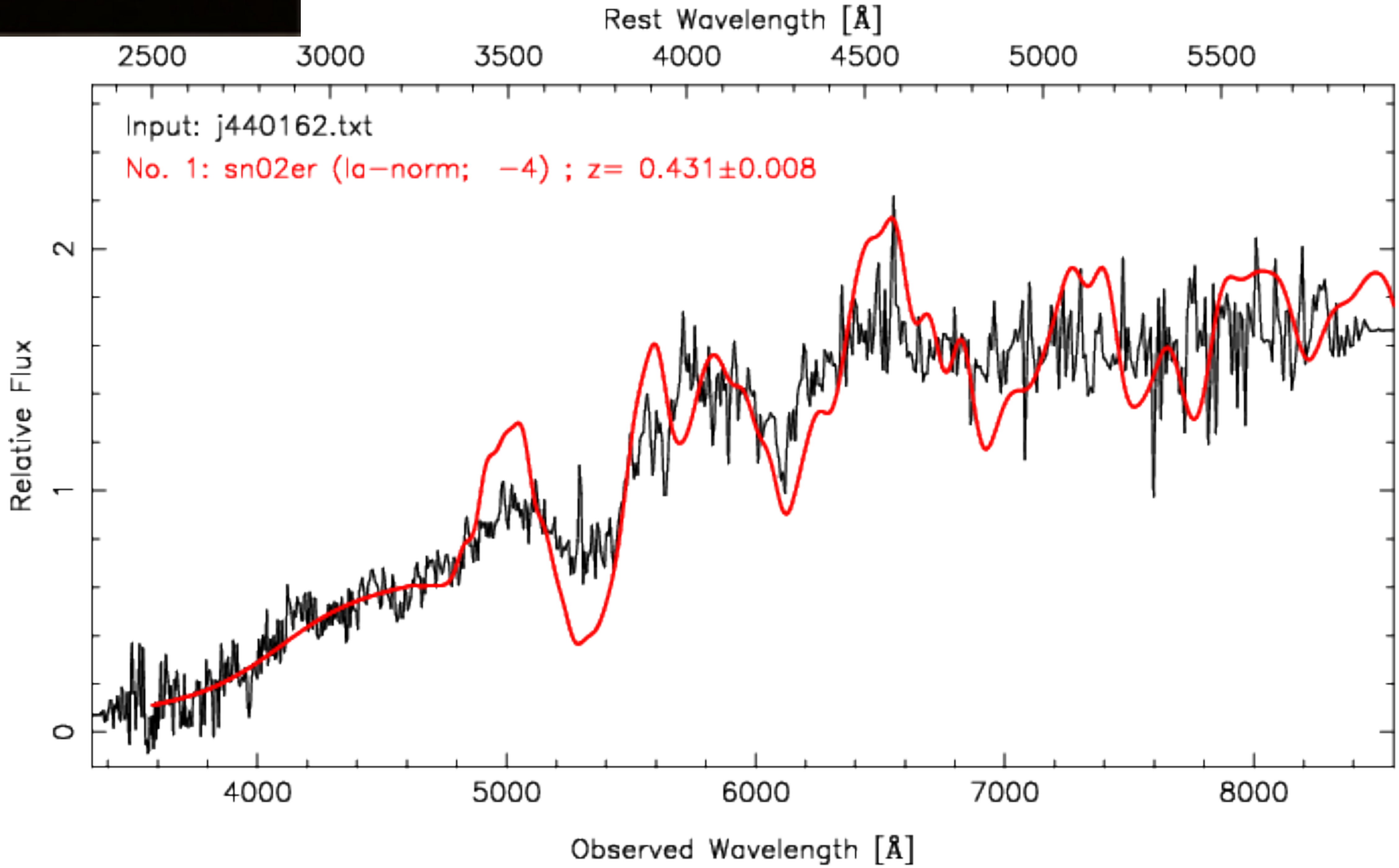
Dozens of supernova candidates every month!



Discovering Supernovae with Pan-STARRS and Difference Imaging

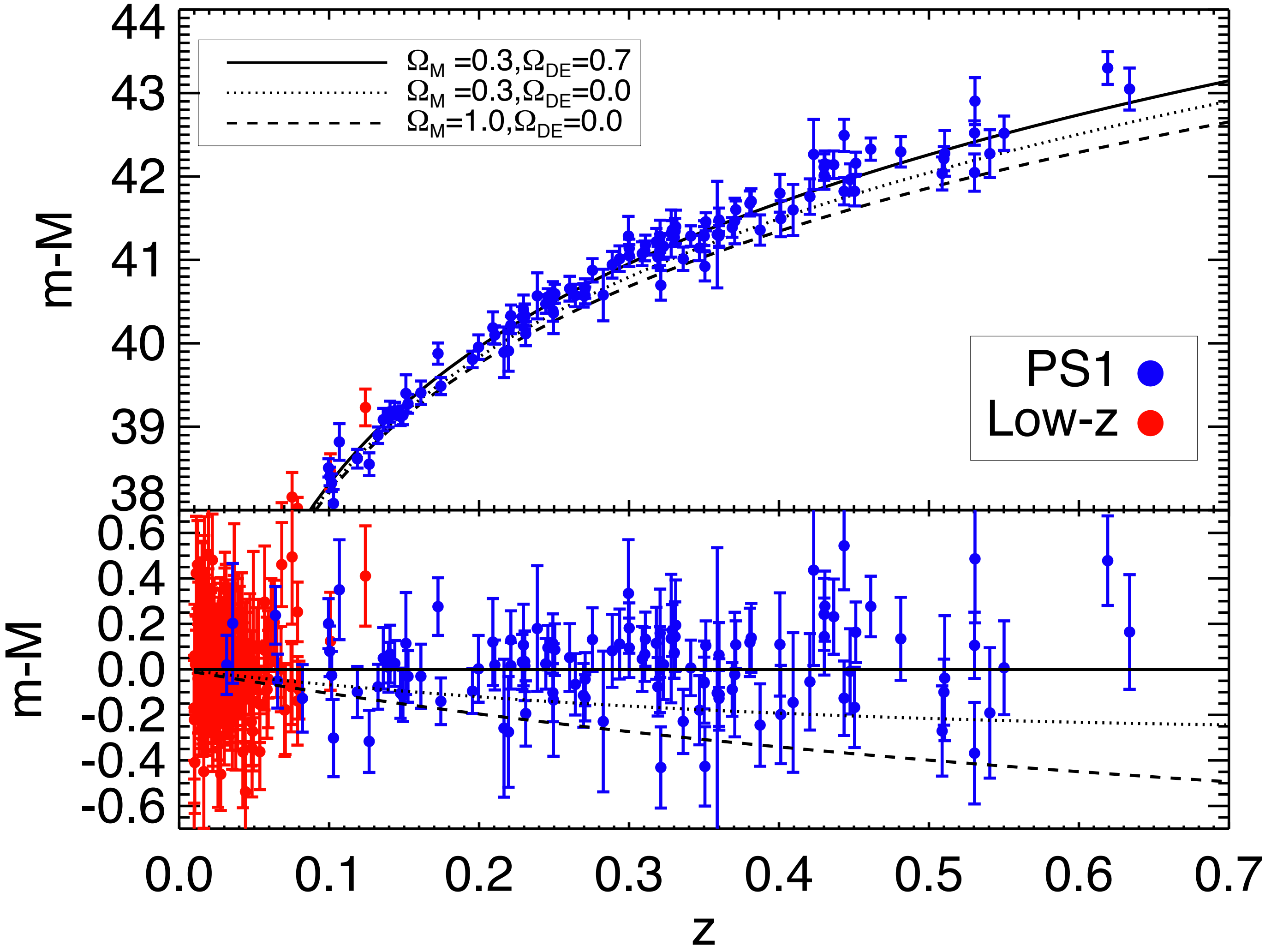


Get spectrum with MMT
(or Magellan, Gemini or Keck)
358 Spectroscopic SN Ia



First Pan-STARRS PS1 results

(Rest et al., 2014, Scolnic et al., 2014)

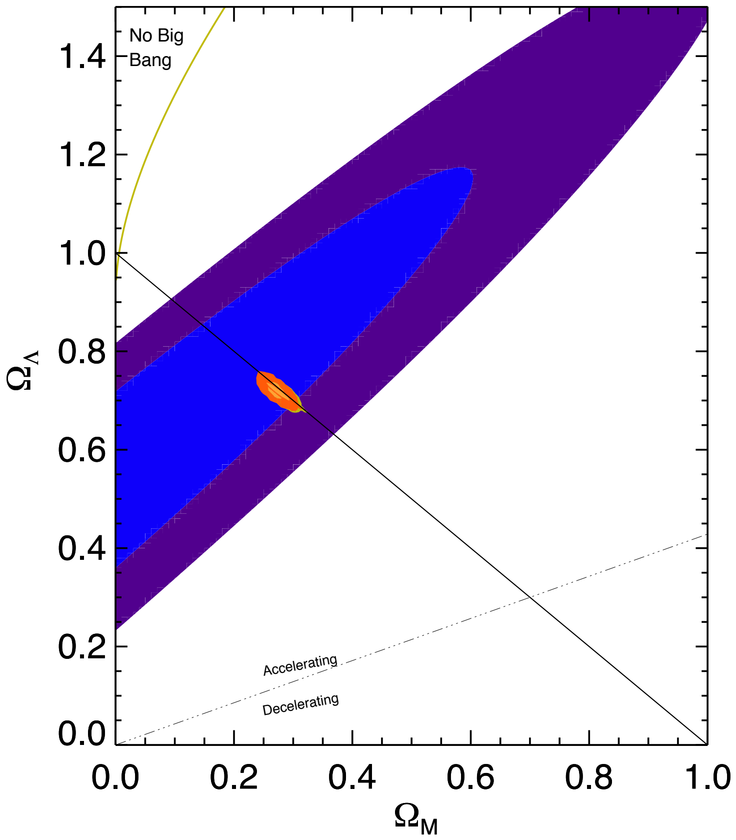


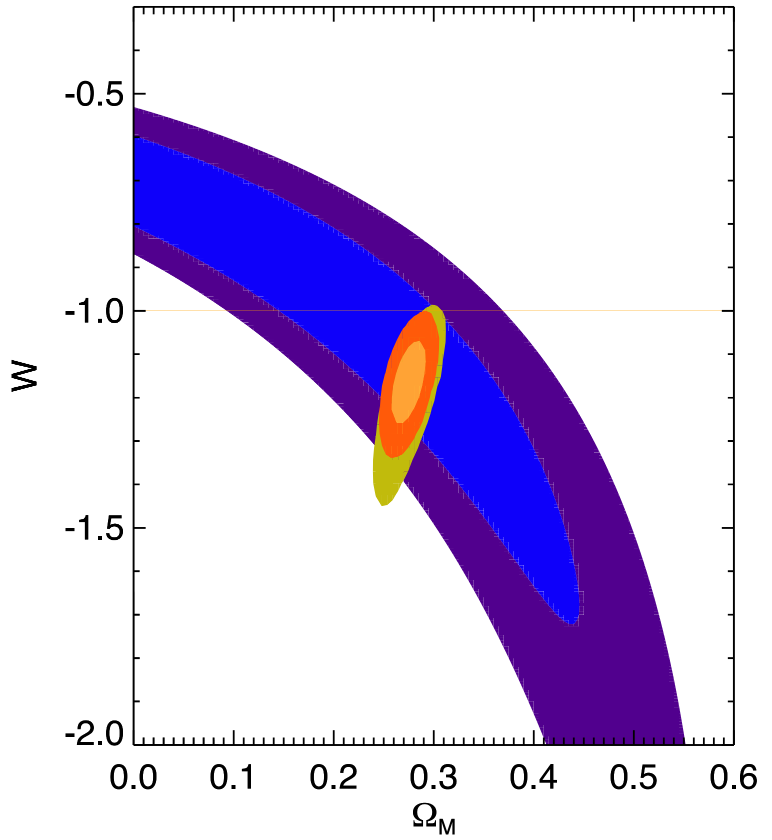
Combined PS1+Low-z + Planck+BAO+H$_o$:

Ω_M = 0.280 (0.013)

w = −1.166 (0.07)

Real or Systematic Error?



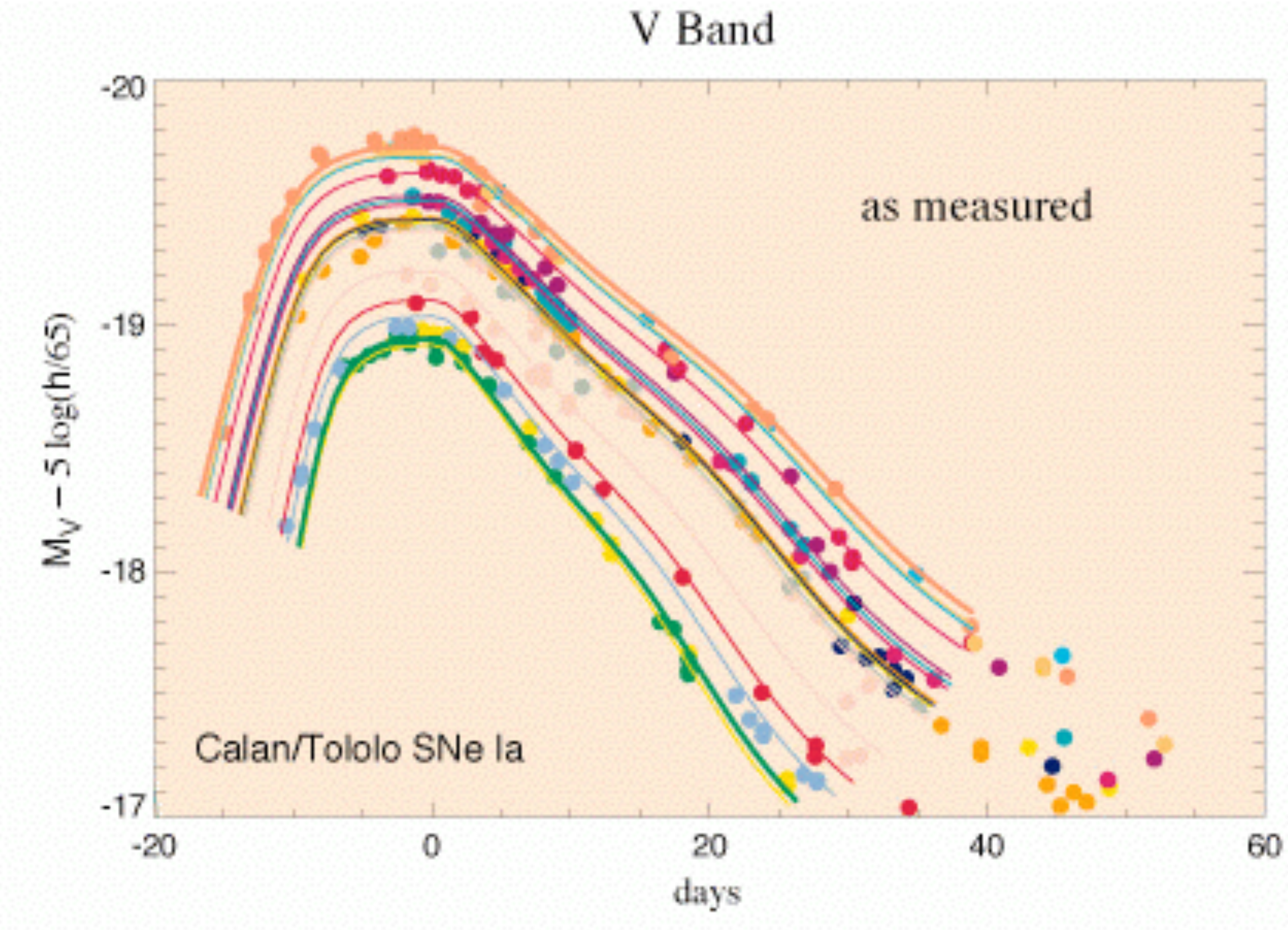


Current State of Play

- Current optical surveys are now limited by "systematic" uncertainties, e.g. photometric calibration error and modeling error, rather than "statistical" (number of supernovae).
- Conventional analysis method does not distinguish between intrinsic SN variations and extrinsic effects of host galaxy dust extinction and reddening
- Incorrect color modeling interpretation of the Hubble Diagram scatter can result in bias in cosmological parameter inferences (Scolnic et al. 2014)
- Confounding of host galaxy dust extinction/reddening with intrinsic SN Ia optical color variations systematically limits the accuracy and precision of SN Ia distances & cosmological constraints

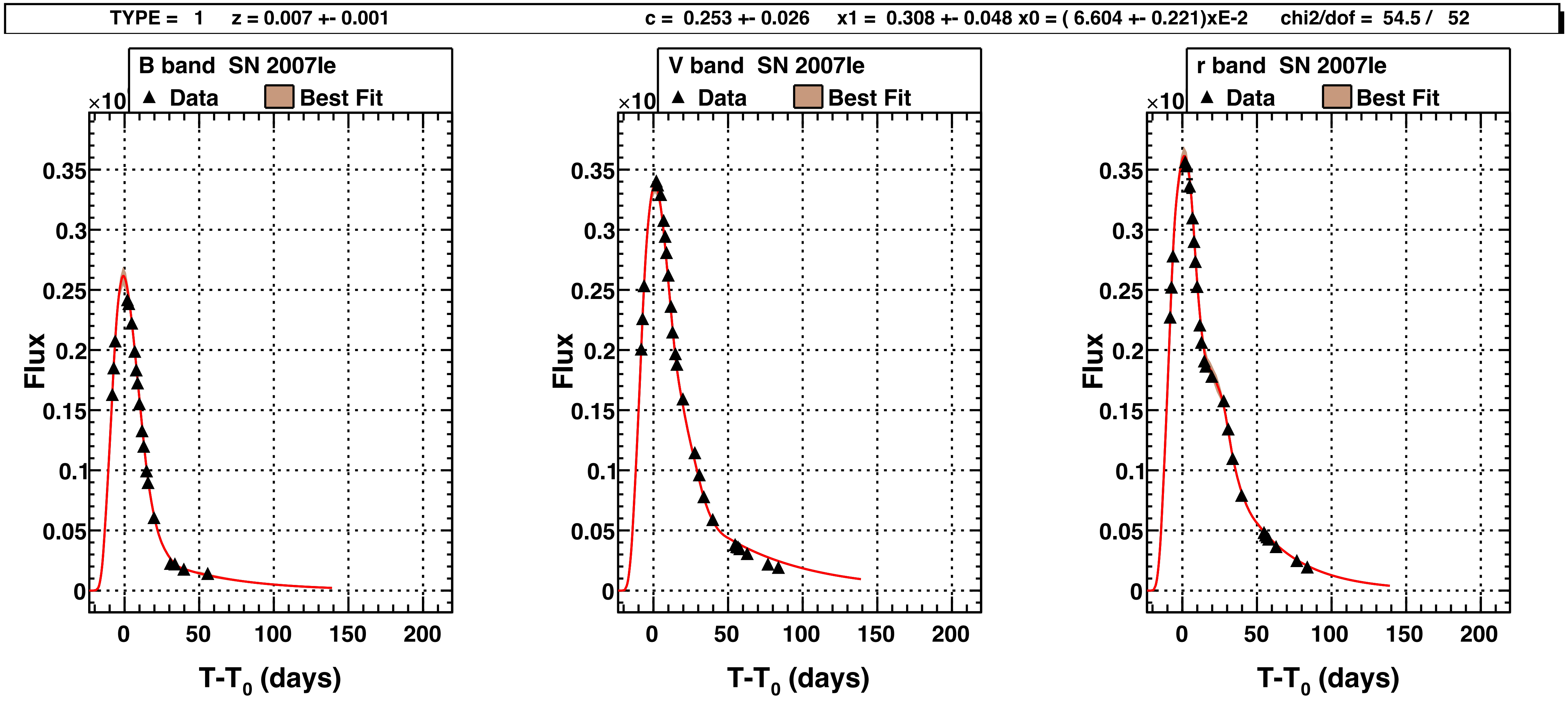
Conventional Approach: Reading the Wattage of a SN Ia: Empirical Correlations "Standardize" the Candle (infer Luminosity)

- Width-Luminosity Relation: an observed correlation (Broader-Brighter, Mark Phillips)
- Observe optical SN Ia Light Curve Shape to estimate the peak luminosity of SN Ia: ~0.15 mag (8% in distance)
- Color-Luminosity Relation (Redder-Dimmer)



Intrinsically Brighter SN Ia have broader light curves and are slow decliners

Conventional Approach



- SALT2 continuous light curve model fit to irregularly samples, noisy optical data (SN2007le, BVR, CfA4)
- Estimates peak apparent magnitude m_B, peak apparent color c = (B-V), and light curve shape x

Conventional Tripp Formula

$$\text{Abs Mag} = m_B - \mu = M_0 + \alpha \cdot x + \beta \cdot c$$

- A Simplistic Linear Model for Absolute Magnitude with width-luminosity (α) and color-luminosity trends (β)
- Typically find β ≈ [ΔMag in B / ΔColor B-V] ≈ 3 Unusually low β compared to normal MW interstellar dust c.f. $R_B \approx 4.1$ ($R_V = R_B - 1 \approx 3.1$).
- Problem: Regresses dust-extinguished magnitude $M^{\text{ext}}_{B,s}$ vs dust-reddened apparent color c^{app}_s
  $$m^s_B - \mu_s = M^{\text{ext}}_{B,s} = M^{\text{ext}}_0 + \alpha \cdot x_s + \beta \cdot c^{\text{app}}_s$$
- Does not distinguish between intrinsic SN Ia variations and host galaxy dust (only one β for all color-mag effects)
- More realistically, magnitudes and colors are composed simultaneously of both intrinsic SN Ia variations and host galaxy dust reddening/extinction

Words (and Notation) Matter!

“Intrinsic” : Latent parameters of SN in absence of host galaxy dust

- Intrinsic Abs. Mag: M_s^{int}
- Intrinsic Color: c_s^{int}

Effects of Host Galaxy Dust for each SN (only positive!)

- Reddening $E_s \equiv E(B-V)_s$
- Extinction (dimming) $A_B^s = R_B \times E(B-V)_s$

“Dusty” : Latent parameters of SN including effects of host galaxy dust

- Extinguished Abs. Mag $M_s^{\text{ext}} = M_s^{\text{int}} + A_B^s$
- Apparent Color $c_s^{\text{app}} = c_s^{\text{int}} + E(B-V)_s$

What about the host galaxy dust?

Dust Absorption vs. Wavelength of Light

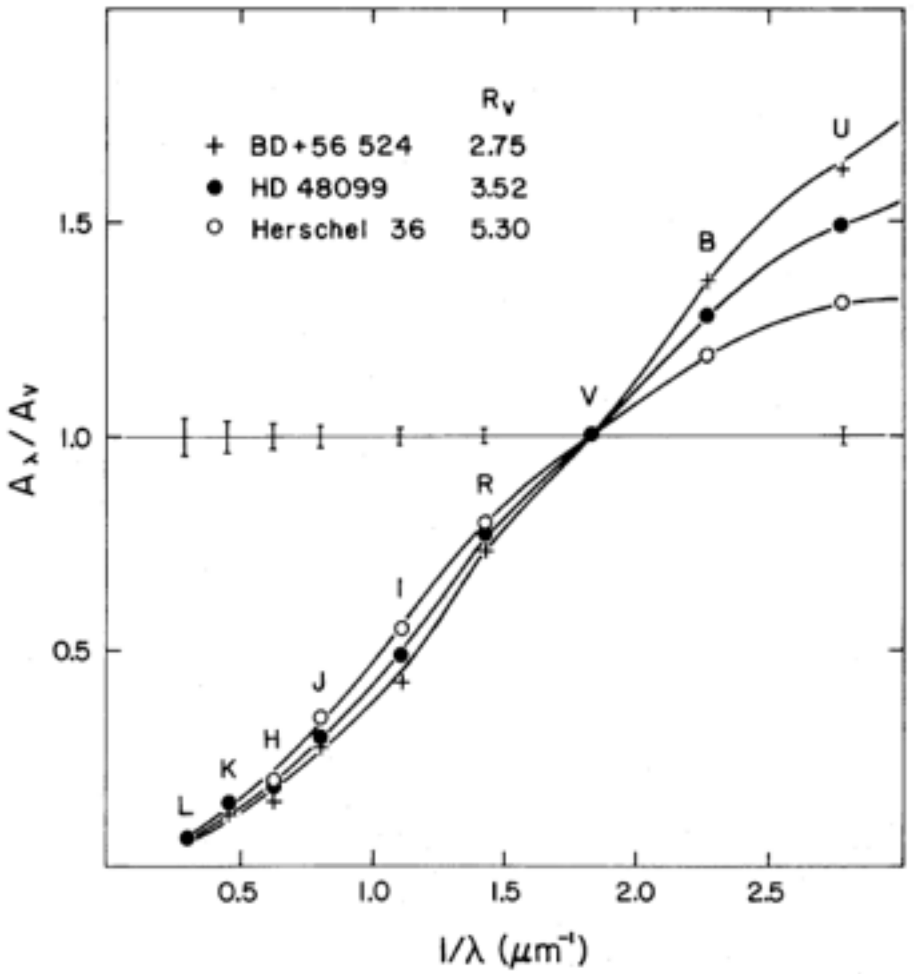
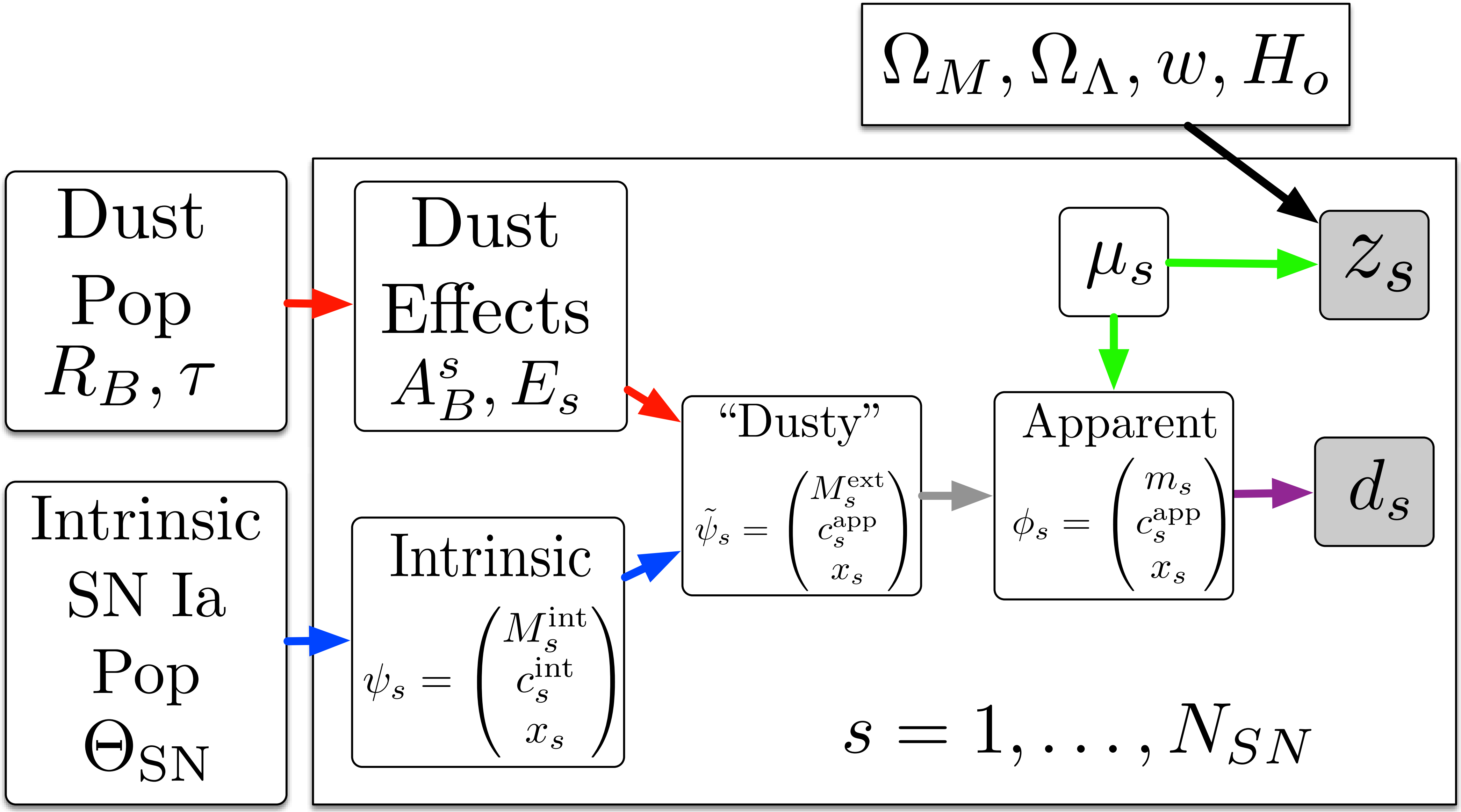


FIG. 3.—Comparison between the mean optical/NIR R_V-dependent extinction law from eqs. (2) and (3) and three lines of sight with largely separated R_V values. The wavelength position of the various broad-band filters from which the data were obtained are labeled (see Table 3). The "error" bars represent the computed standard deviation of the data about the best fit of $A(\lambda)/A(V)$ vs. R_V^{-1} with $a(x) + b(x)/R_V$ where $x \equiv \lambda^{-1}$. The effect of varying R_V on the shape of the extinction curves is quite apparent, particularly at the shorter wavelengths.

- Absorption of light (dimming) depends on λ, causing reddening
- Interstellar lines of sight to SN in different galaxies can pass through different random amounts of dust
- Key Parameters of Interstellar Dust (different for each SN)
  - A_B ~ Amount of Dust Absorption (dimming)
  - $R_B = A_B/E(B-V)$ ~ Wavelength Dependence of Dust Absorption
- Don't really know a priori which SN are unaffected by dust; must model probabilistically

My Approach (Mandel+09,11,14,16):

Hierarchical Bayesian / Probabilistic Generative Model



Observed SN Ia Data = Sum of latent random effects:

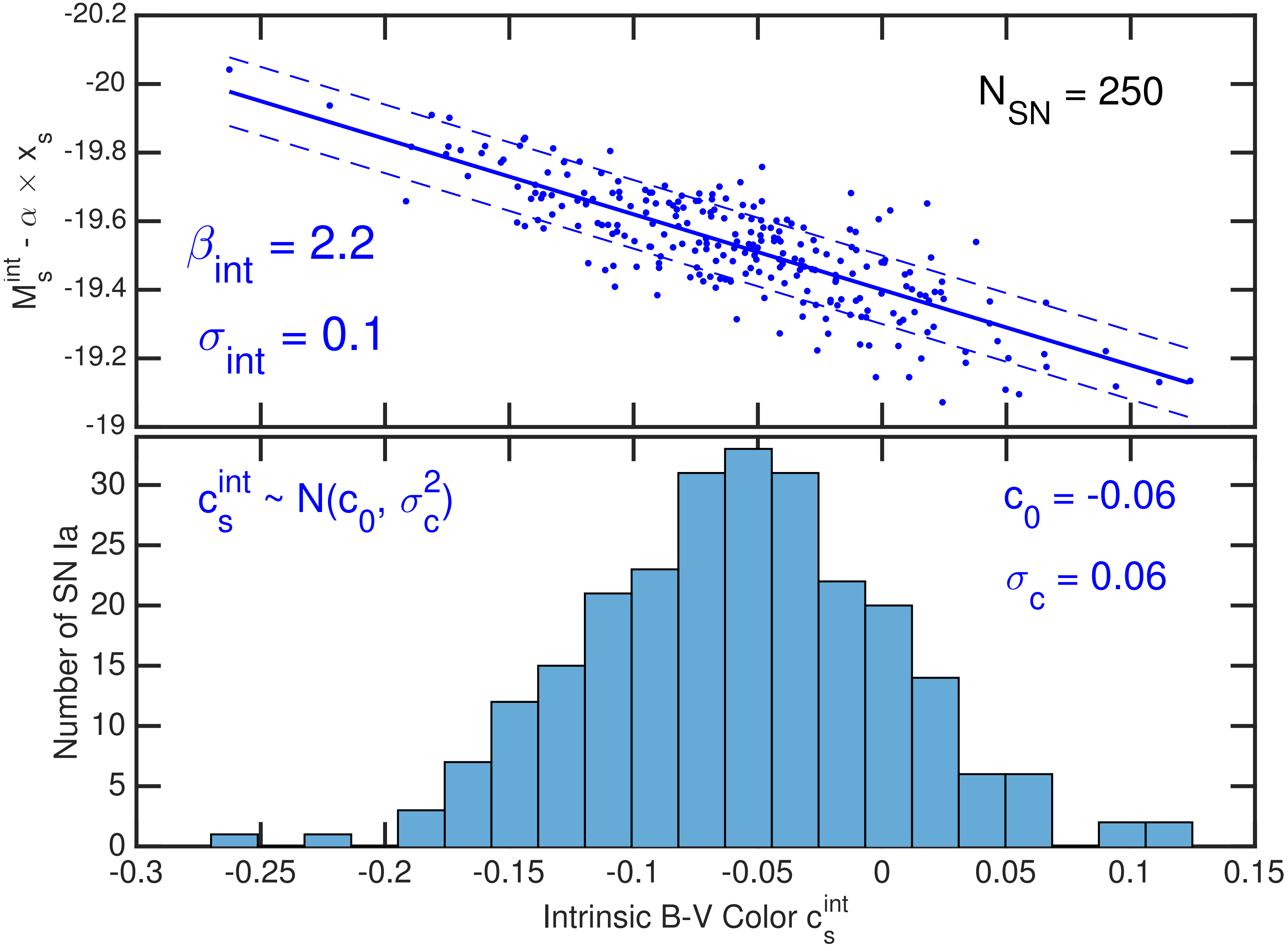
intrinsic variation, dust, measurement error

(Simple-BayeSN)

Understanding the Probabilistic Generative Model via Forward Simulation

Intrinsic Color-Luminosity Variations

Intrinsic Absolute Mag



Intrinsic Color

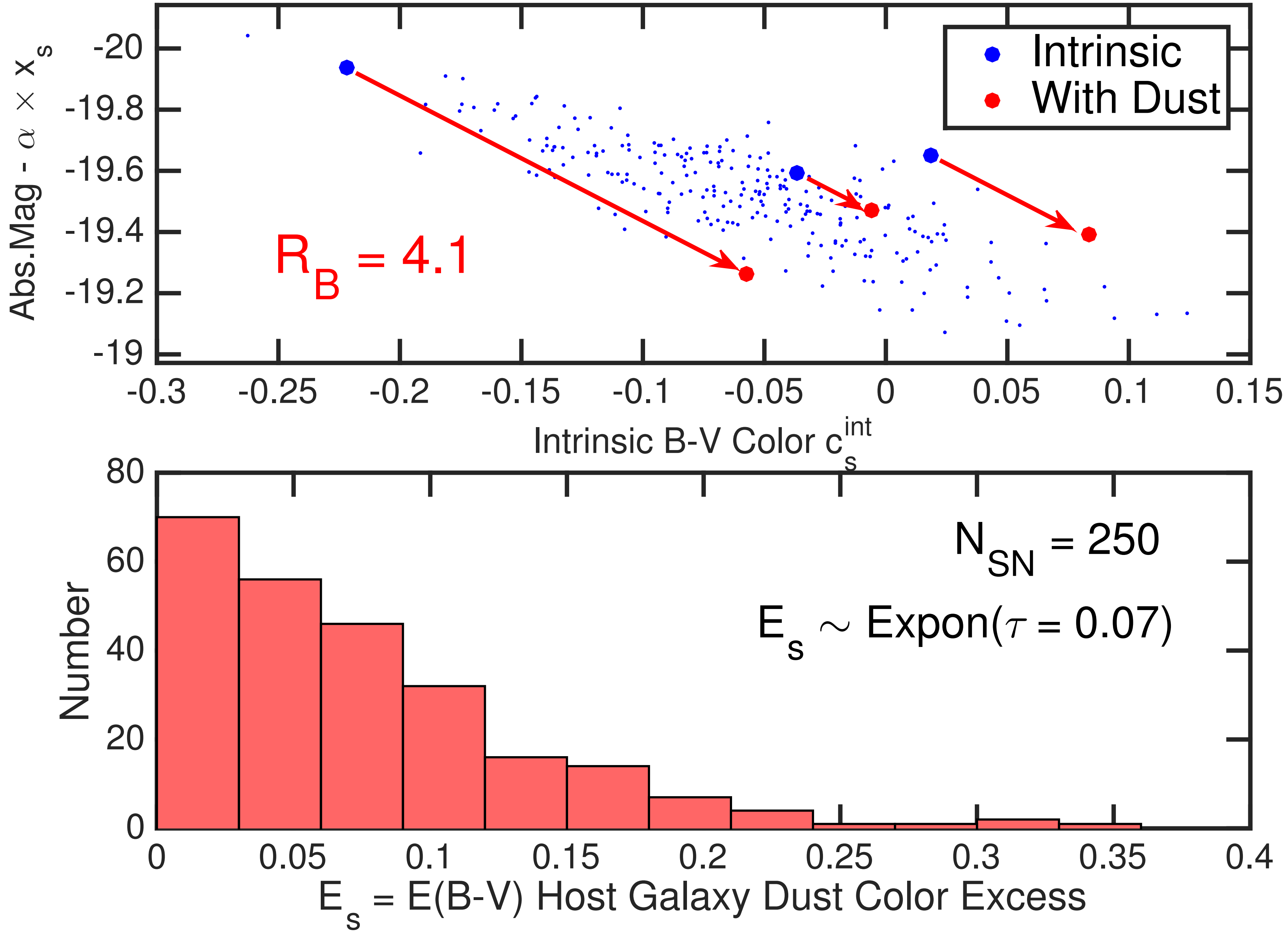
Host Galaxy Dust Effects:

Reddening: $c_s^{\text{app}} = c_s^{\text{int}} + E(B-V)_s$

Extinction: $M_s^{\text{ext}} = M_s^{\text{int}} + A_B^s$

Dust Law: $R_B = R_B + 1 = A_B/E(B-V)$

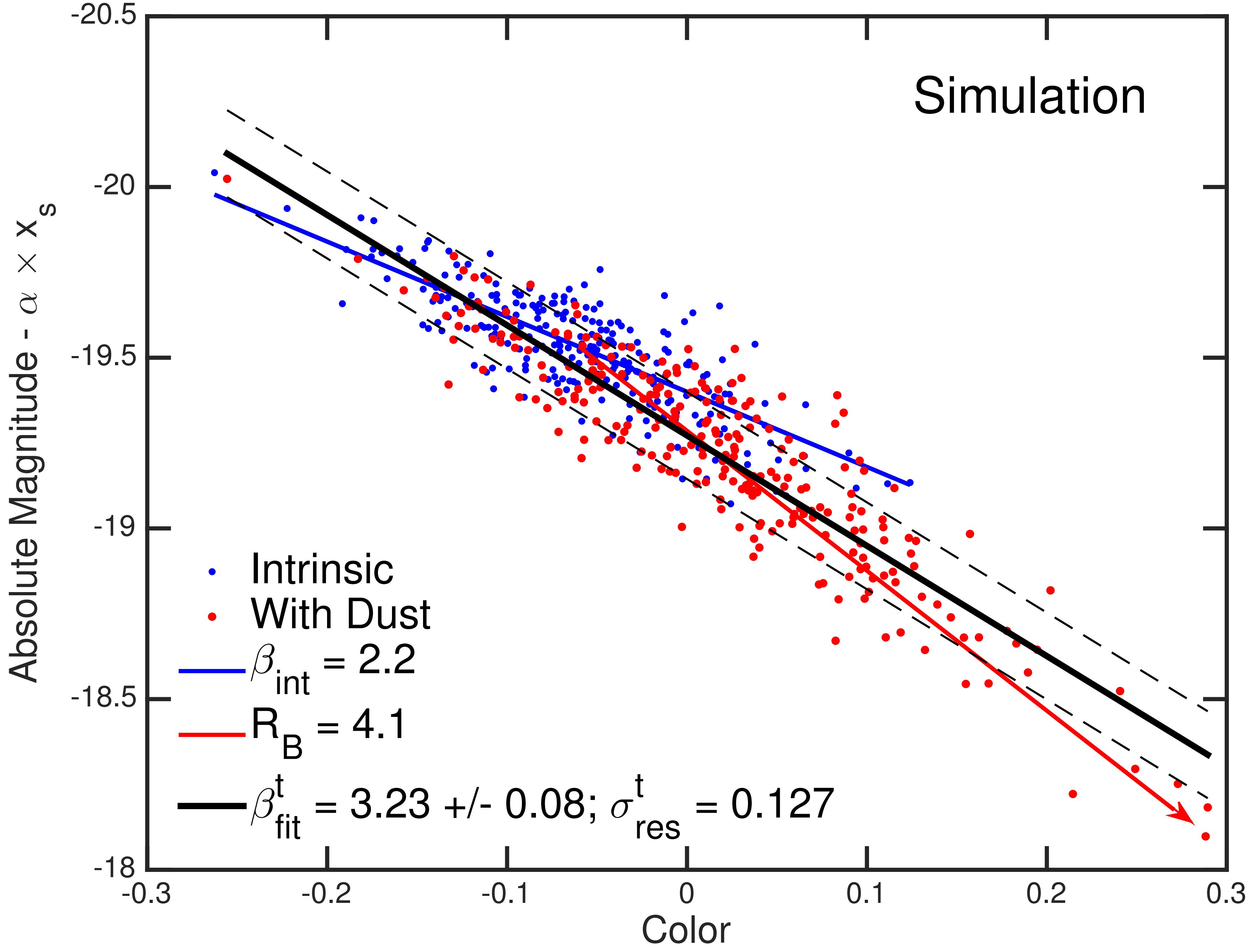
(Simulation)



Dust Extinction & Reddening are Only Positive! ($E_s > 0$)

SN Ia Color-Mag Distribution (intrinsic vs dusty)

(Simulation)



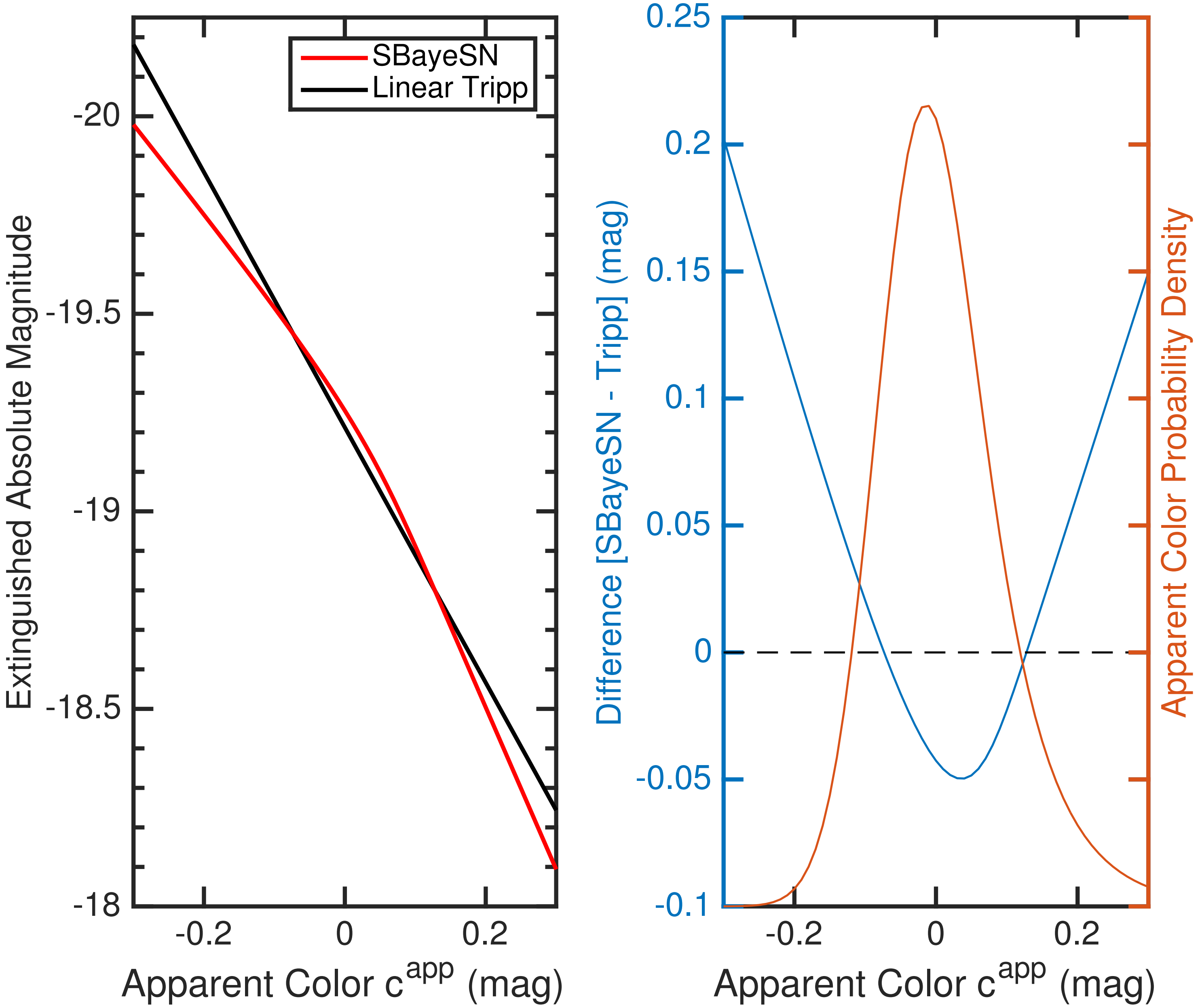
(Black: Conventional Tripp Fit)

$$m_B^s - \mu_s = M_B^{\rm ext} = M_0^{\rm ext} + \alpha \cdot x_s + \beta \cdot c_s^{\rm app}$$

Effective "Dusty" Color-Magnitude Distribution is a Convolution of the Intrinsic & Dust Distributions:

Effective Color-Mag Trend is a Curve!

(Simulation)



Model Predicts Positive Distance Bias for Linear Tripp Fit in the tails of apparent color distribution

Tripp Fit is a linear approx. to curve near mean apparent color

Inverse Problem: Statistical inference with SN Ia

- SN Ia cosmology inference based on empirical relations
- Statistical models for SN Ia are learned from the data
- Several Sources of Randomness & Uncertainty
  1. Photometric (Measurement) & LC Fitting errors
  2. "Intrinsic Variation" = Population Distribution of SN Ia
  3. Random Peculiar Velocities in Hubble Flow
  4. Host Galaxy Dust: extinction and reddening.
- Observed Distributions are convolutions of these effects
- How to incorporate this all into a coherent statistical model? (How to "de-convolve"?) - Hierarchical Bayes!

Advantages of Hierarchical Models

- Incorporate multiple sources of randomness & uncertainty underlying the observed data
- Express structured probability models adapted to conceptual / physical data-generating forward process
- Hierarchically Model (Physical) Populations and Individuals simultaneously: e.g. intrinsic SN Ia properties and Dust Reddening/Absorption
- Inference = probabilistically de-convolves multiple latent effects underlying data
- Full Posterior probability distribution = Global, coherent quantification of uncertainties at individual and population levels

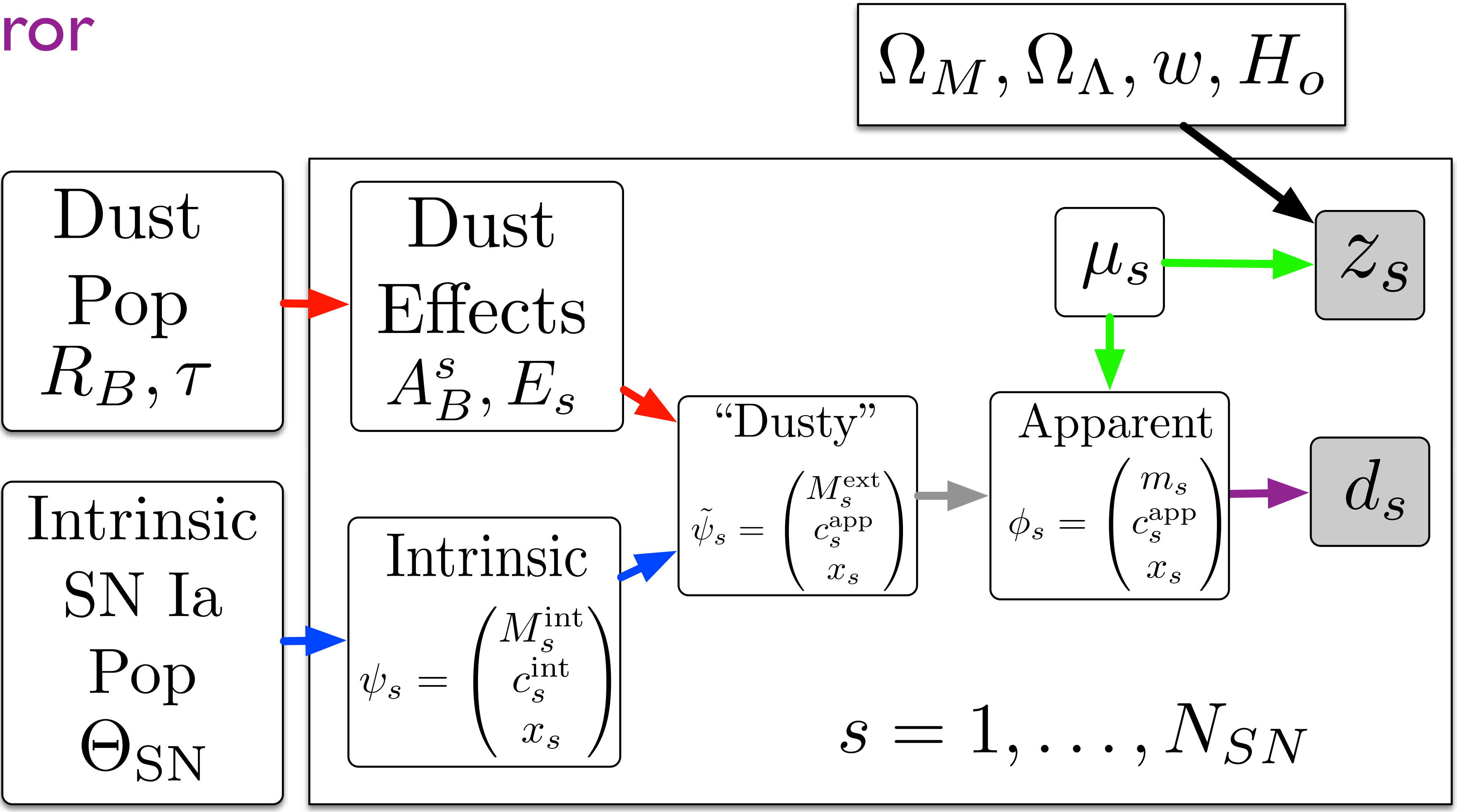
Directed Acyclic Graph for SN Ia Inference with Hierarchical Bayesian Model (Simple-BayeSN)

(Mandel et al. 2016)

- Intrinsic Variation of SN Ia
- Dust Extinction & Reddening
- Peculiar Velocities
- Measurement Error

Probabilistic Graphical Model

Global Joint Posterior Probability Density Conditional on all SN Data



Some Math:
Intrinsic SN Ia Population Distribution Model

Conditional Factorization

$$\begin{aligned} P(\boldsymbol{\psi}_s |\, \boldsymbol{\Theta}_{\text{SN}}) &= P(M_s^{\text{int}}, c_s^{\text{int}}, x_s |\, \boldsymbol{\Theta}_{\text{SN}}) \\ &= P(M_s^{\text{int}} |\, c_s^{\text{int}}, x_s; \boldsymbol{\Theta}_{\text{SN}}) \\ &\times P(c_s^{\text{int}} | x_s; \boldsymbol{\Theta}_{\text{SN}}) \\ &\times P(x_s |\, \boldsymbol{\Theta}_{\text{SN}}) \end{aligned}$$

Simplest Linear Model:

$$M_s^{\text{int}} = M_0^{\text{int}} + \alpha x_s + \beta_{\text{int}} c_s^{\text{int}} + \epsilon_s^{\text{int}}$$

$$c_s^{\text{int}} = c_0^{\text{int}} + \alpha_c x_s + \epsilon_s^{c,\text{int}}$$

$$x_s = x_0 + \epsilon_s^x$$

Gaussian scatter

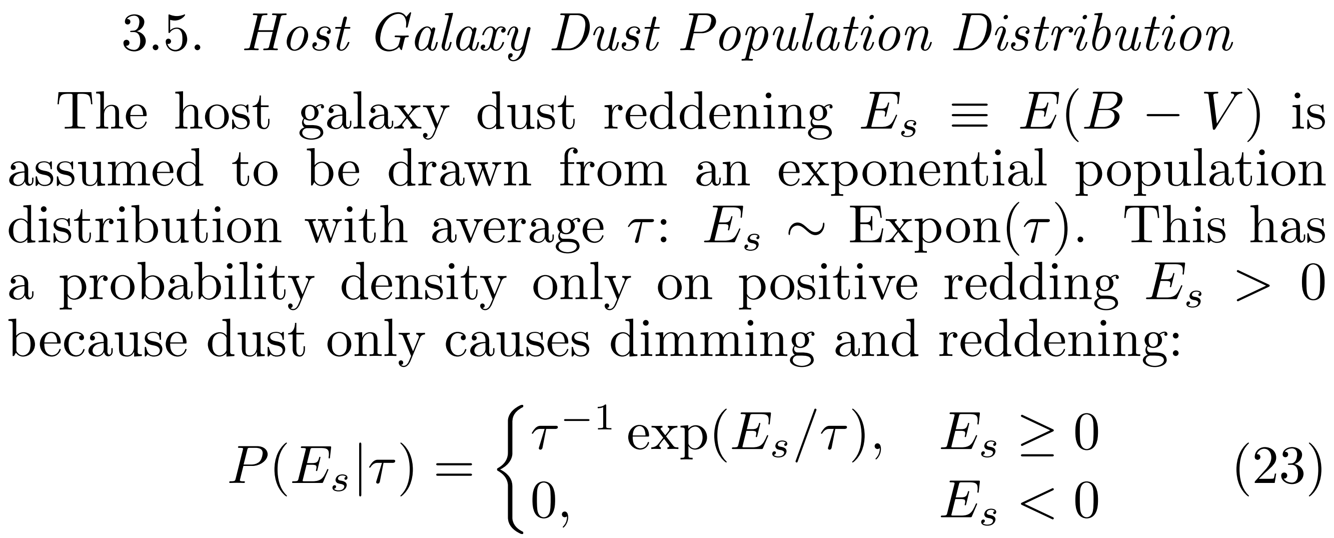
$$\epsilon_s^x \sim N(0, \sigma_x^2),\ \epsilon_s^{c,\text{int}} \sim N(0, \sigma_{c,\text{int}}^2),\ \epsilon_s^{\text{int}} \sim N(0, \sigma_{\text{int}}^2).$$

To summarize, the nine hyperparameters governing the structure of the population distribution for the intrinsic SN Ia parameters are

$$\boldsymbol{\Theta}_{\text{SN}} = (M_0^{\text{int}}, \alpha, \beta_{\text{int}}, \sigma_{\text{int}}^2, c_0^{\text{int}}, \alpha_c^{\text{int}}, \sigma_{c,\text{int}}^2, x_0, \sigma_x^2) \quad (22)$$

- M_0^{int} : the intrinsic absolute magnitude constant is the expected intrinsic absolute magnitude for a SN with light curve shape $x_s = 0$ and intrinsic color $c_s^{\text{int}} = 0$,
- α : the slope of the trend of intrinsic absolute magnitude vs. light curve shape,
- β_{int} : the slope of the trend of intrinsic absolute magnitude vs. intrinsic color,
- σ_{int}^2 : the intrinsic variance around the mean trend of intrinsic absolute magnitude vs. light curve shape and color,
- c_0^{int} : the expected intrinsic color for a SN Ia with light curve shape $x = 0$. If $\alpha_c^{\text{int}} = 0$, then c_0^{int} is the population mean intrinsic color,
- α_c^{int} : the slope of the trend of intrinsic color vs. light curve shape,
- $\sigma_{c,\text{int}}^2$: the intrinsic variance around the mean trend of intrinsic color vs. light curve shape,
- x_0 : the mean of the x light curve shape population distribution,
- σ_x^2 : the variance of the x light curve shape population distribution.

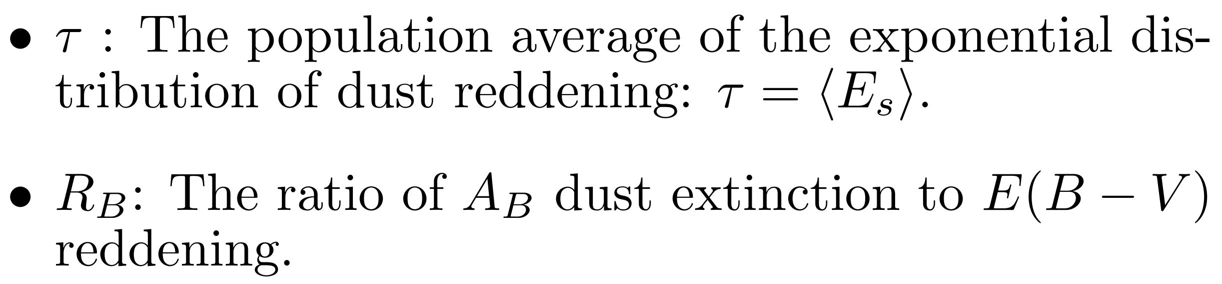
The Host Galaxy Dust
Population Distribution Model



3.5. *Host Galaxy Dust Population Distribution*

The host galaxy dust reddening $E_s \equiv E(B-V)$ is assumed to be drawn from an exponential population distribution with average τ: $E_s \sim \text{Expon}(\tau)$. This has a probability density only on positive redding $E_s > 0$ because dust only causes dimming and reddening:

$$P(E_s|\tau) = \begin{cases} \tau^{-1}\exp(E_s/\tau), & E_s \geq 0 \\ 0, & E_s < 0 \end{cases} \tag{23}$$



- τ : The population average of the exponential distribution of dust reddening: $\tau = \langle E_s \rangle$.
- R_B: The ratio of A_B dust extinction to $E(B-V)$ reddening.

Effects of Host Galaxy Dust for each SN (only positive!)

- Reddening $E_s \equiv E(B-V)_s$
- Extinction (dimming)

$$A_B^s = R_B \times E(B-V)_s$$

"Dusty" : Latent parameters of SN including effects of host galaxy dust

- Extinguished Abs. Mag

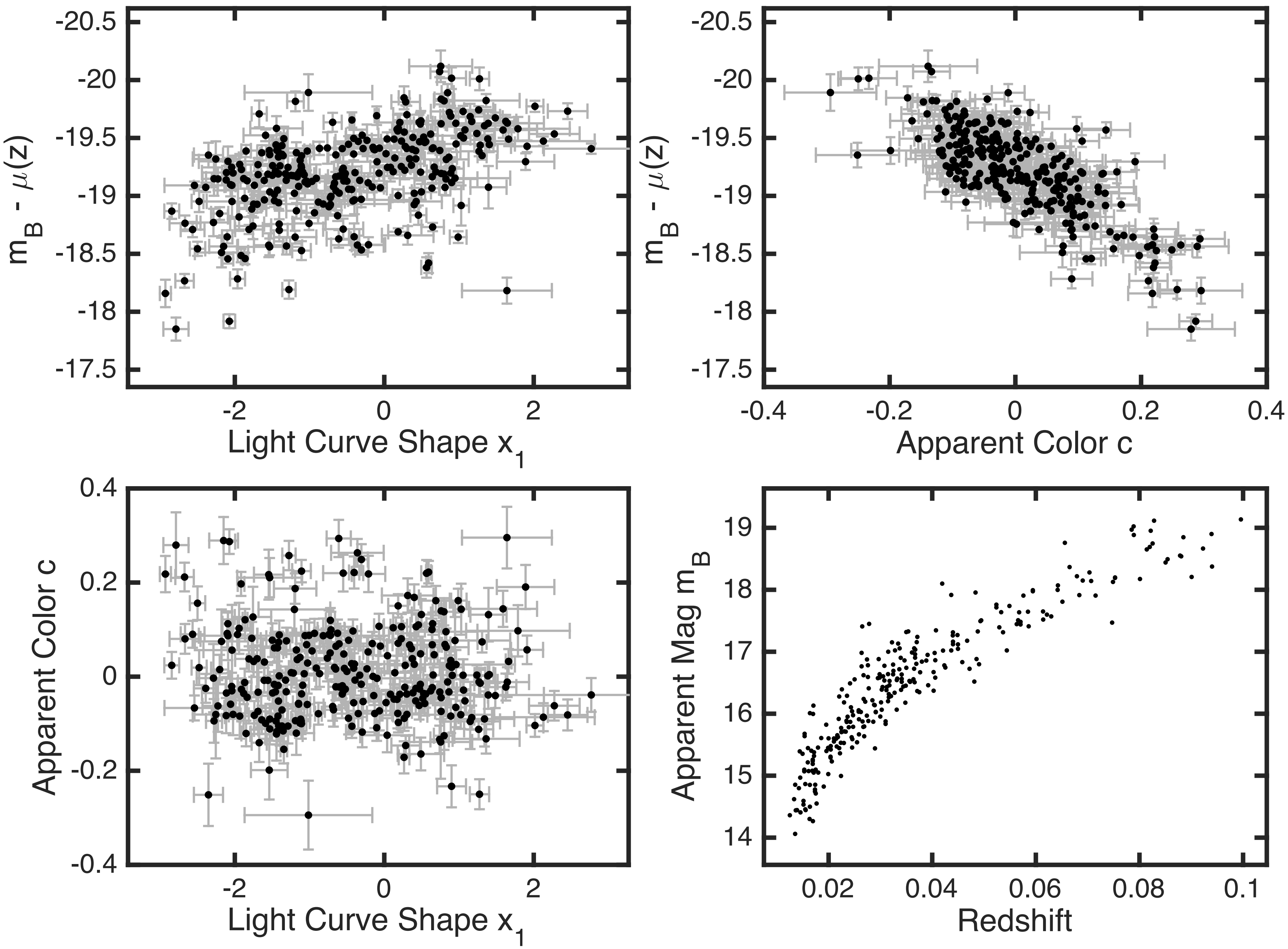
$$M_s^{\text{ext}} = M_s^{\text{int}} + A_B^s$$

- Apparent Color

$$c_s^{\text{app}} = c_s^{\text{int}} + E_s$$

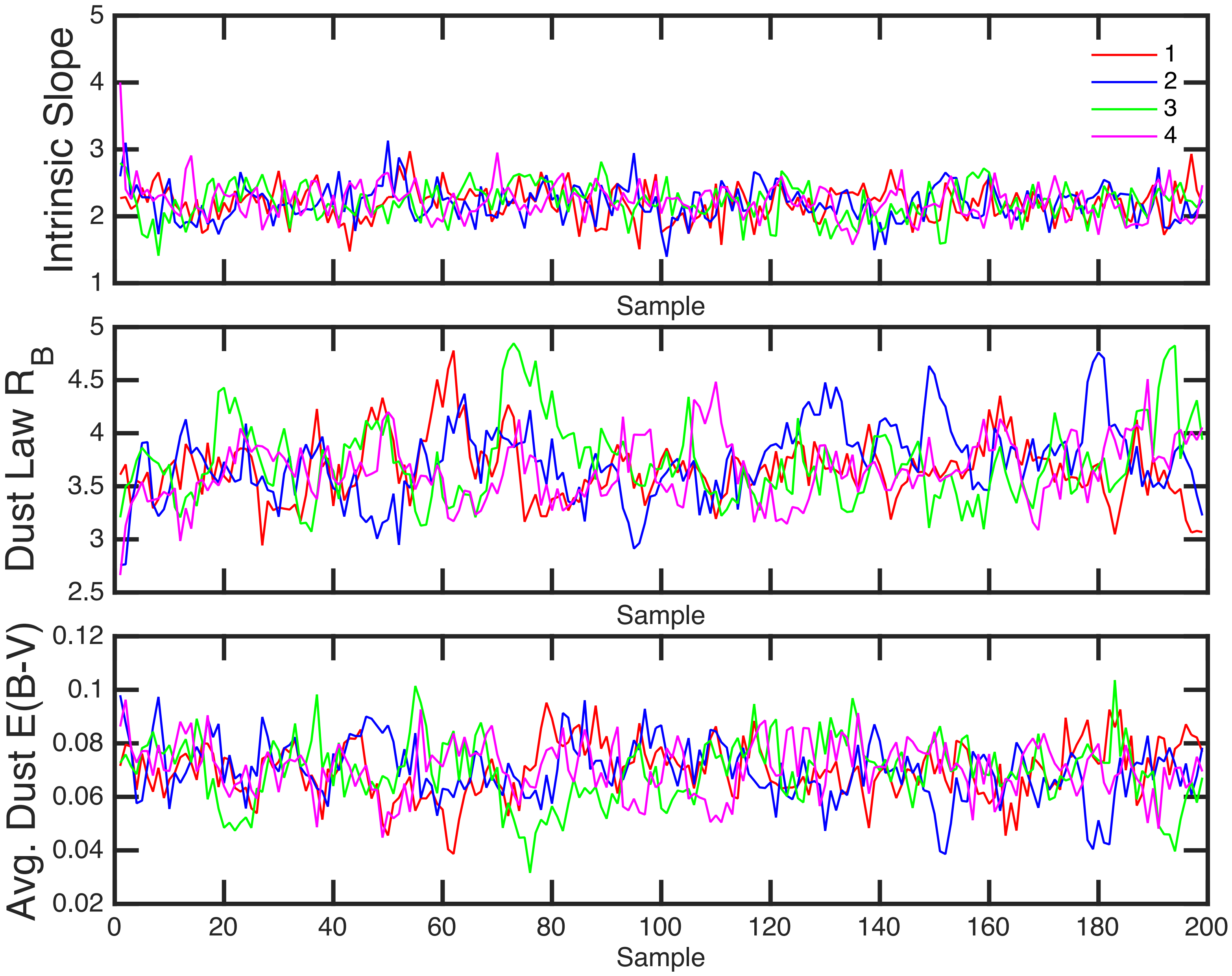
The Data:

Optical LC fits for 277 nearby (low-z < 0.1) SN Ia (CfA, CSP) cross-calibrated with Pan-STARRS [Scolnic+15]



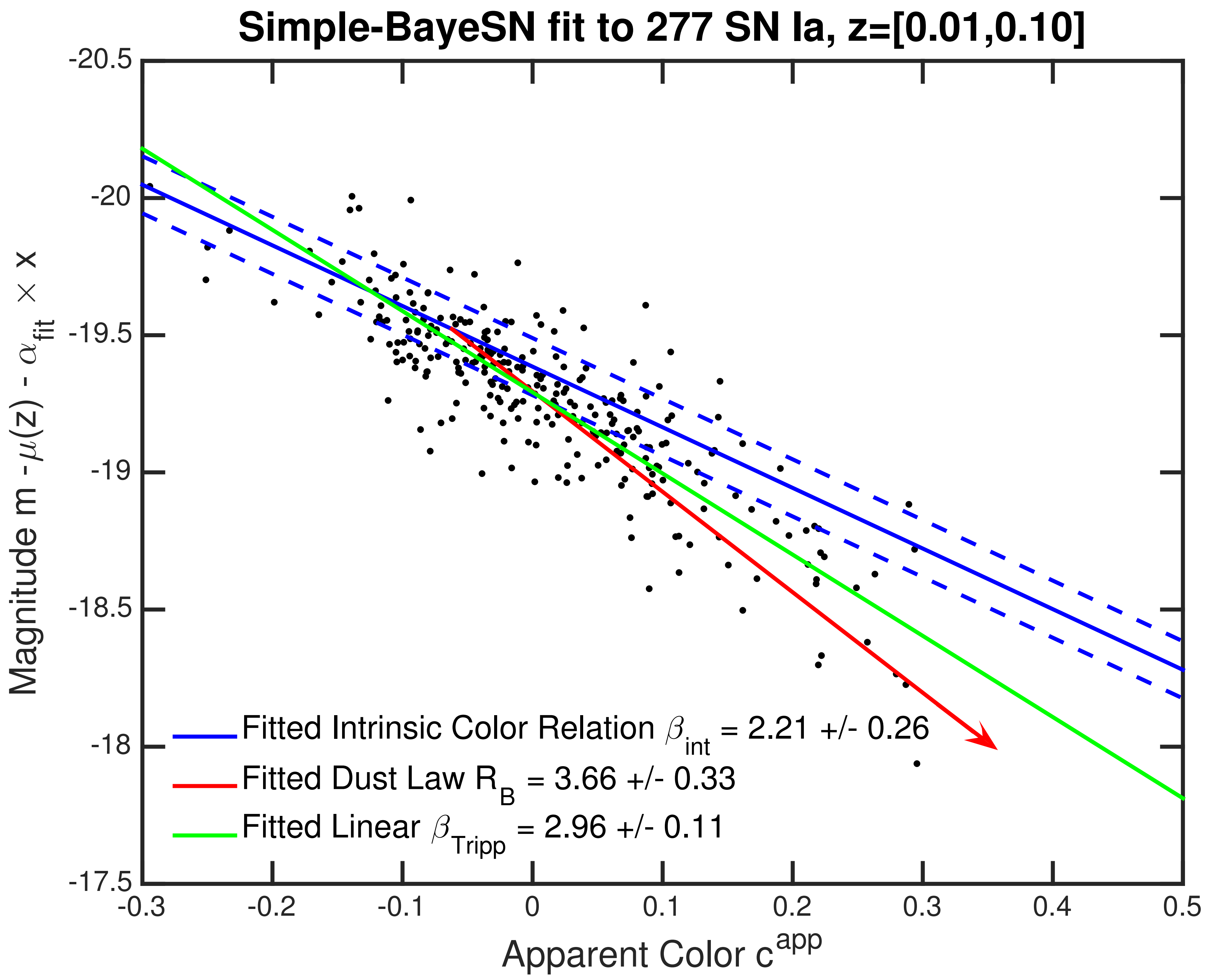
Bayesian Posterior Inference & Statistical Computation

- Estimate Intrinsic Relation, Dust Law, Dust Population, etc.
- Gibbs Sampling utilizes conditionals of full posterior to update MCMC steps
- Explore joint posterior probability of all parameters



Four Parallel MCMC Chains

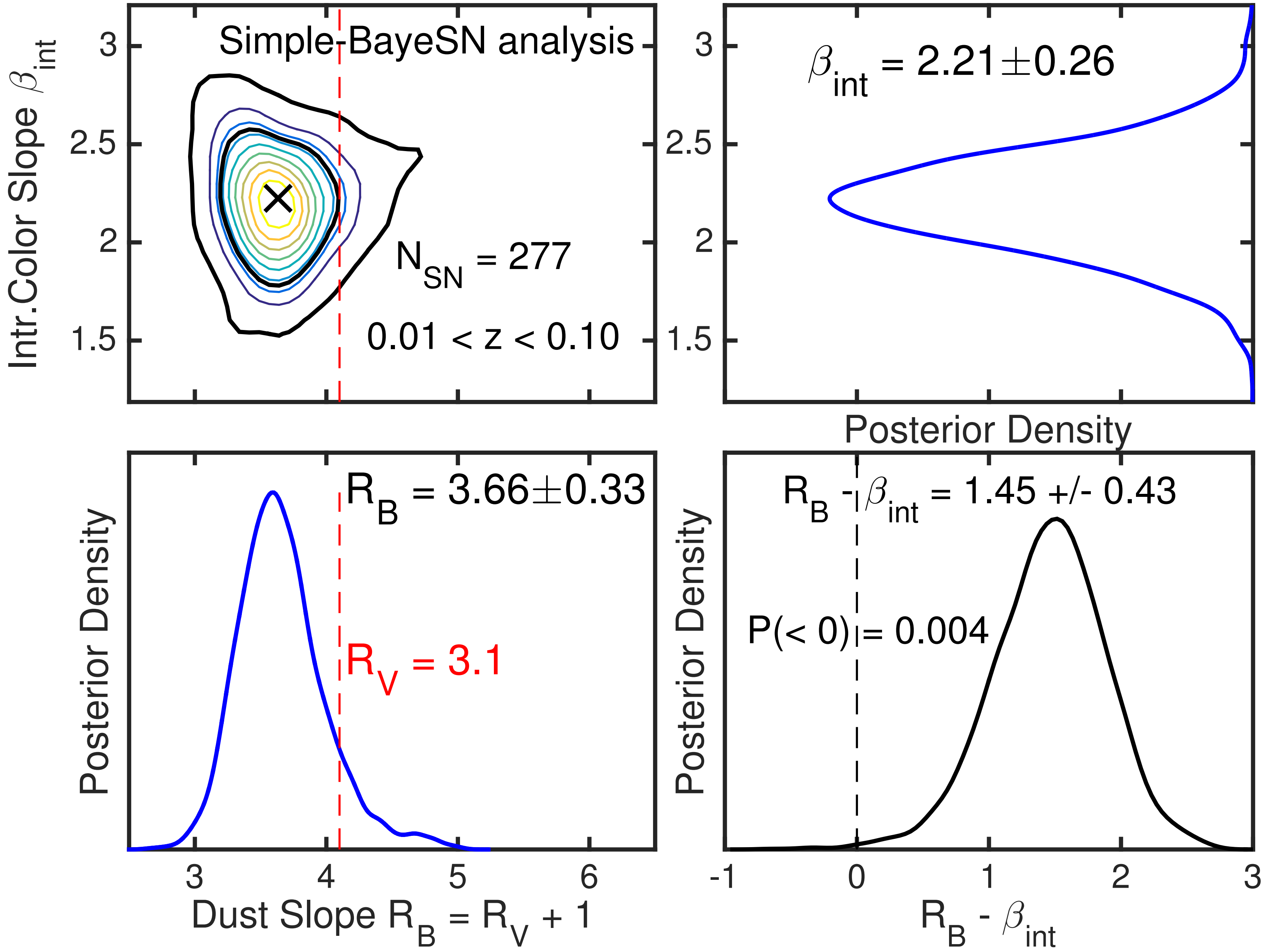
Results: Discerning Dust vs. Intrinsic Variations



Intrinsic Color-Magnitude Slope ≠ Dust Reddening Vector!

(Color-Magnitude Effects NOT described by a single slope β!)

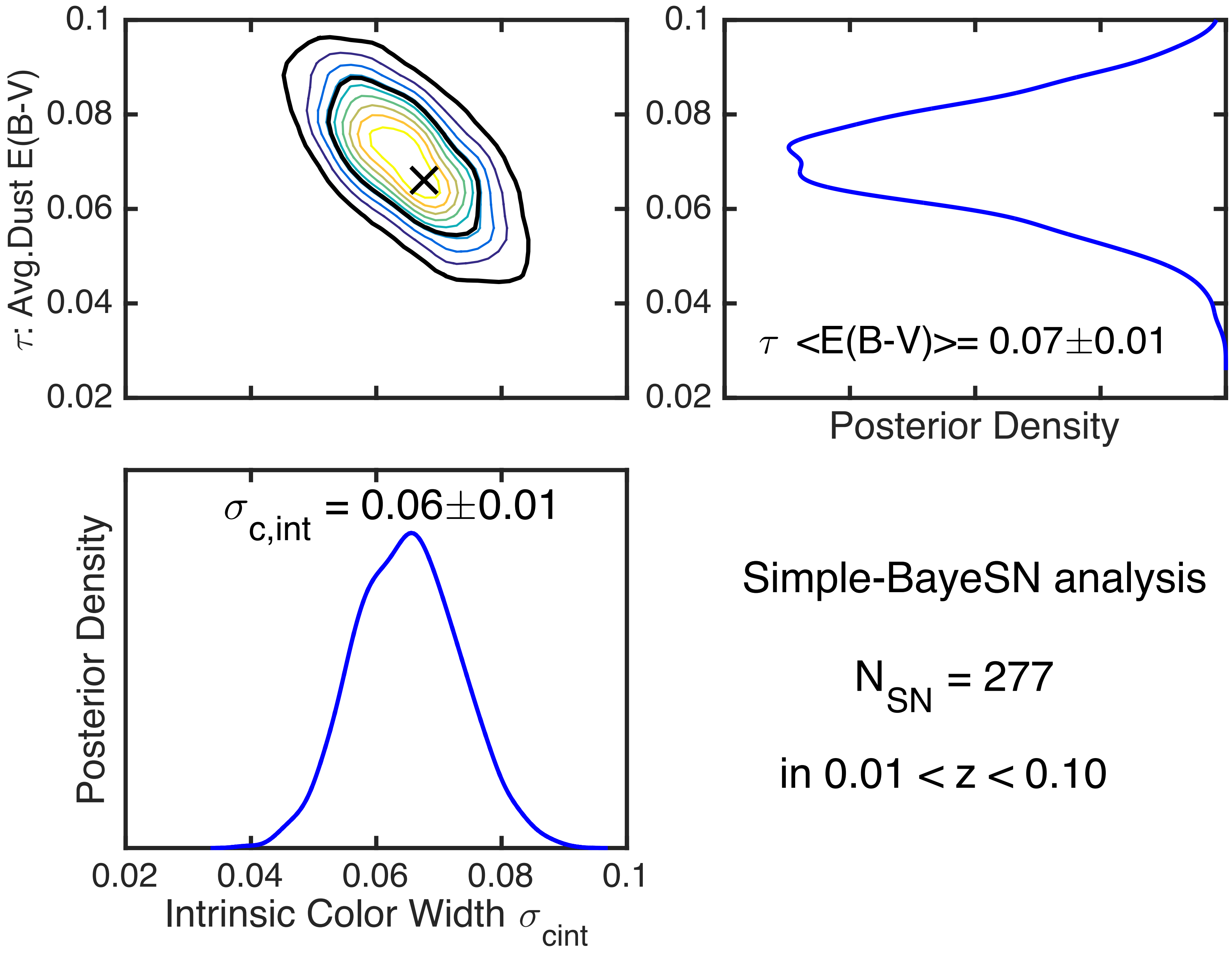
Results: Inferring Dust Extinction/Reddening (R_B) vs. Intrinsic Color-Luminosity Trend (β_{int})



Dust Reddening Vector consistent with Milky Way dust (R_V = 3.1)!

Intrinsic Color-Magnitude Slope ≠ Dust Reddening Vector!

Results: Inferring Population Distributions of SN Ia Intrinsic Color vs Host Galaxy Dust



Simple-BayeSN analysis

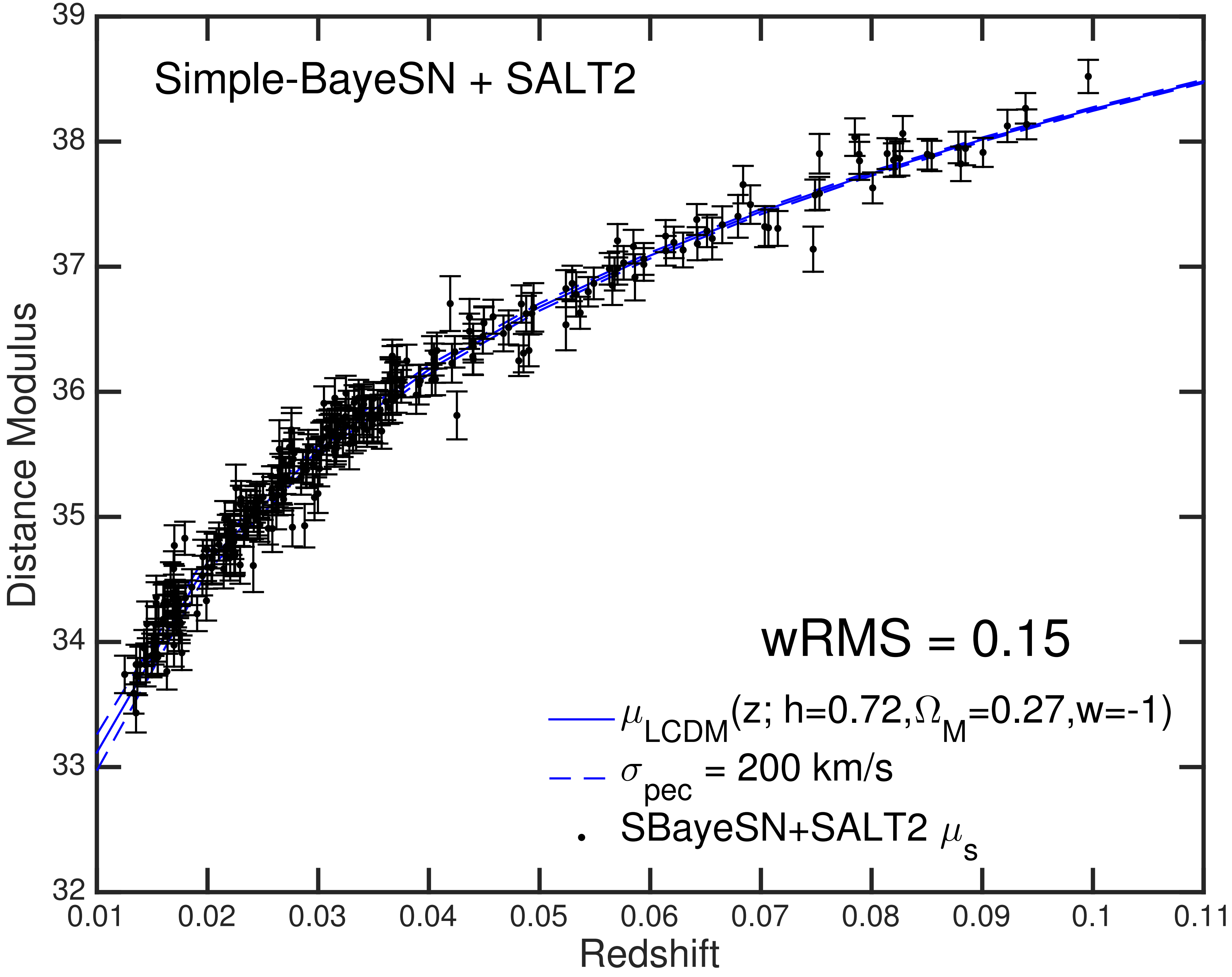
$N_{SN} = 277$

in $0.01 < z < 0.10$

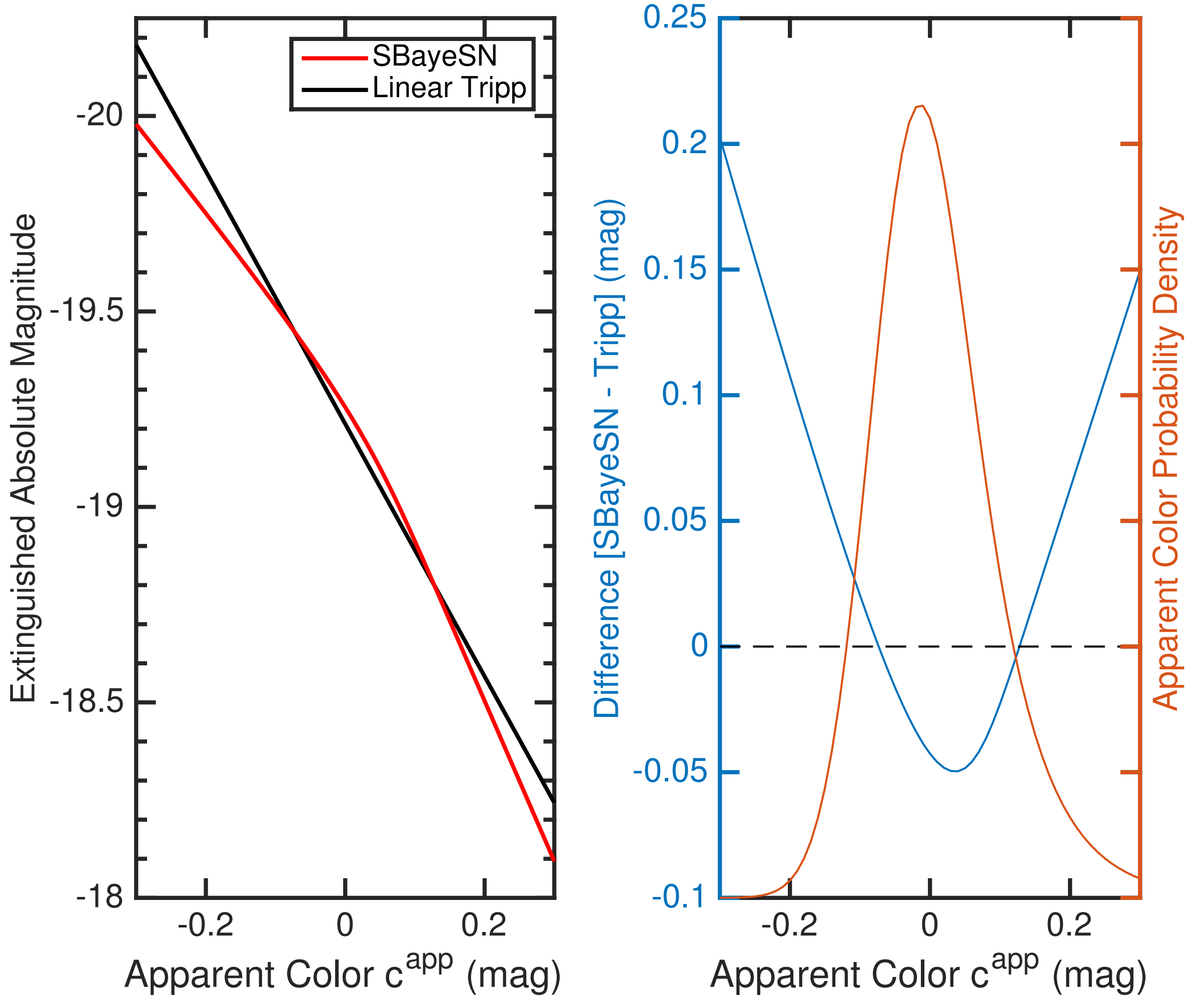
Roughly Equal Contributions to Total Apparent Color Variance

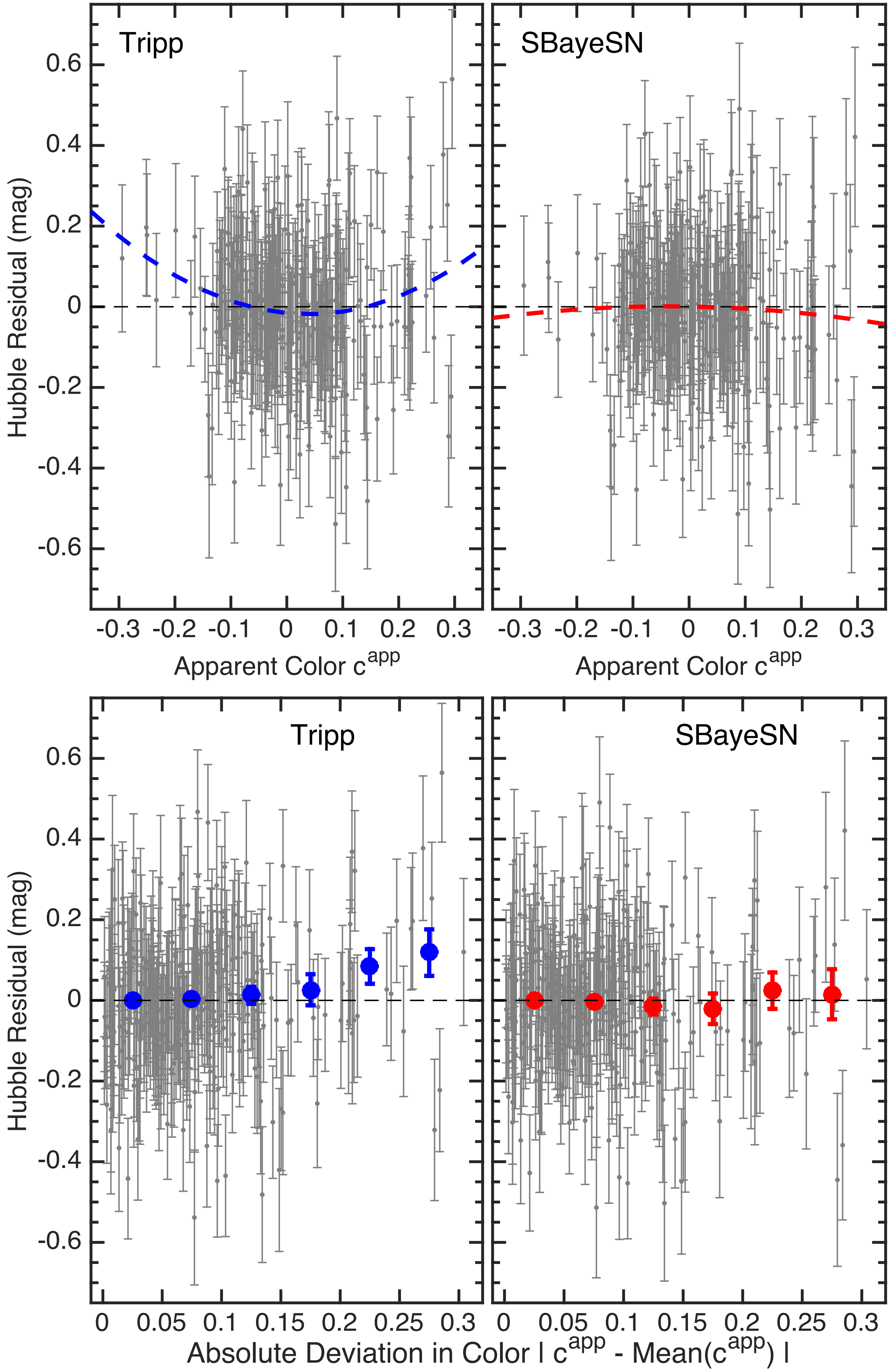
Hubble Diagram: Use Trained Model Hyperparameters to Predict Photometric Distances based on SN Ia Light Curve Data:

$$P(\mu_s | \, d_s, \hat{\Theta}_{\text{SN}}, \hat{\tau}, \hat{R}_B)$$



Hubble Residuals





Simple-BayeSN
Corrects ~ 0.1 mag bias
in tails of SN Ia
color distribution
relative to Linear Tripp fit

Summary

- Current Optical SN Ia Surveys systematically limited by confounding of host galaxy dust effects with intrinsic SN Ia variations
- Conventional analysis (Tripp+SALT2) too simplistic: does not account for physically distinct intrinsic color-magnitude variation vs. dust reddening-extinction
- Simple-BayeSN: Hierarchical Bayesian / Probabilistic Generative Model for intrinsic variation & dust effects applied to current optical data
- Get a sensible dust law, $R_B = 3.7 \pm 0.3$ ($R_V = 2.7 \pm 0.3$) different from intrinsic slope $\beta_{int} = 2.2 \pm 0.3$!
- Future: applications to high-z sample; extensions to rest-frame NIR (Mandel+09,11); spectroscopic indicators of luminosity & color (Mandel+14)