Beyond population synthesis: MCMC models of high mass X-ray binaries

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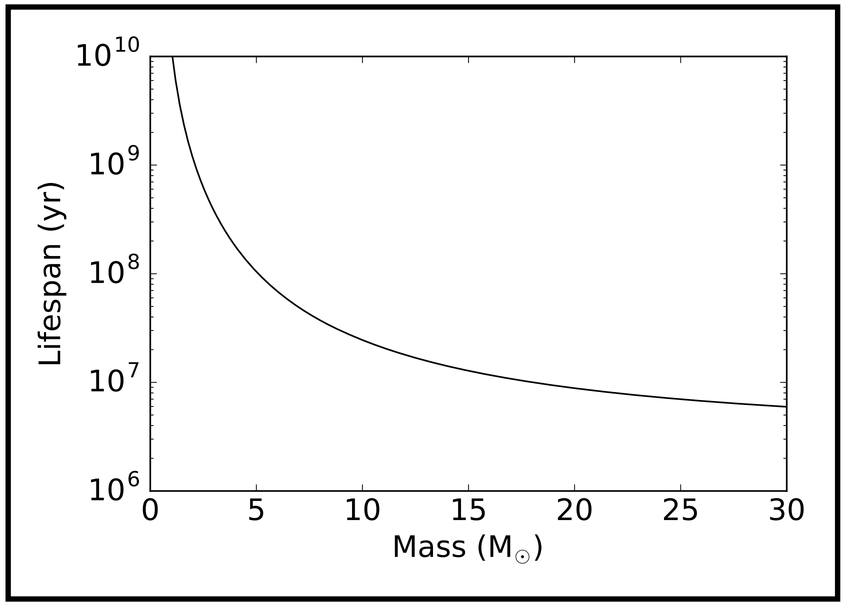
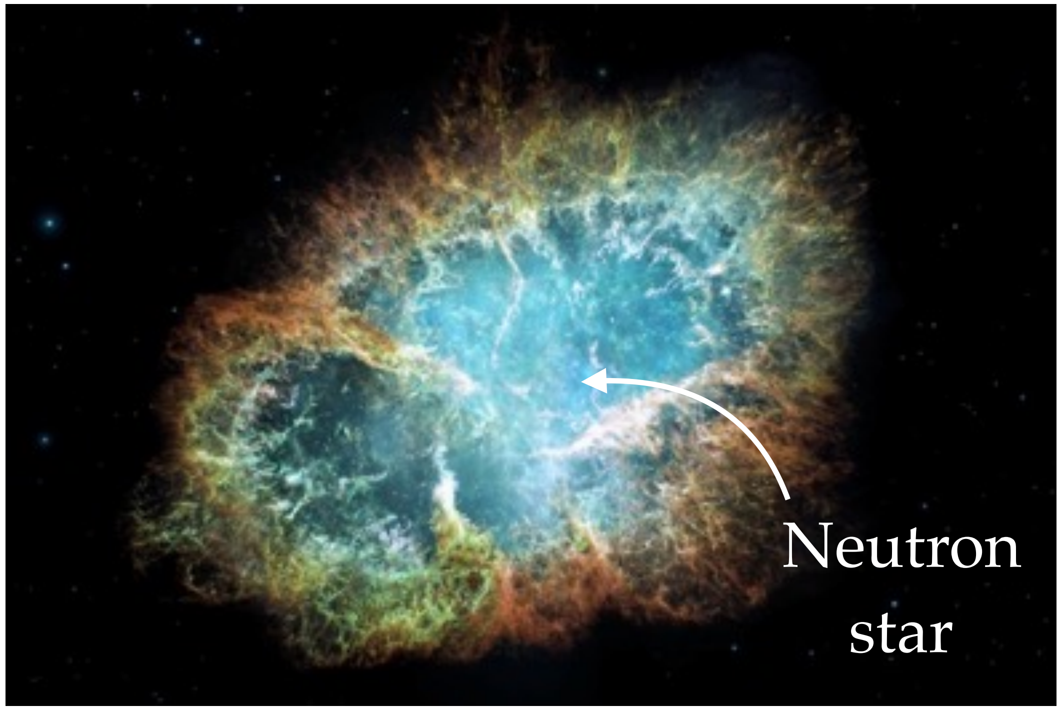
Massive stellar evolution crash course

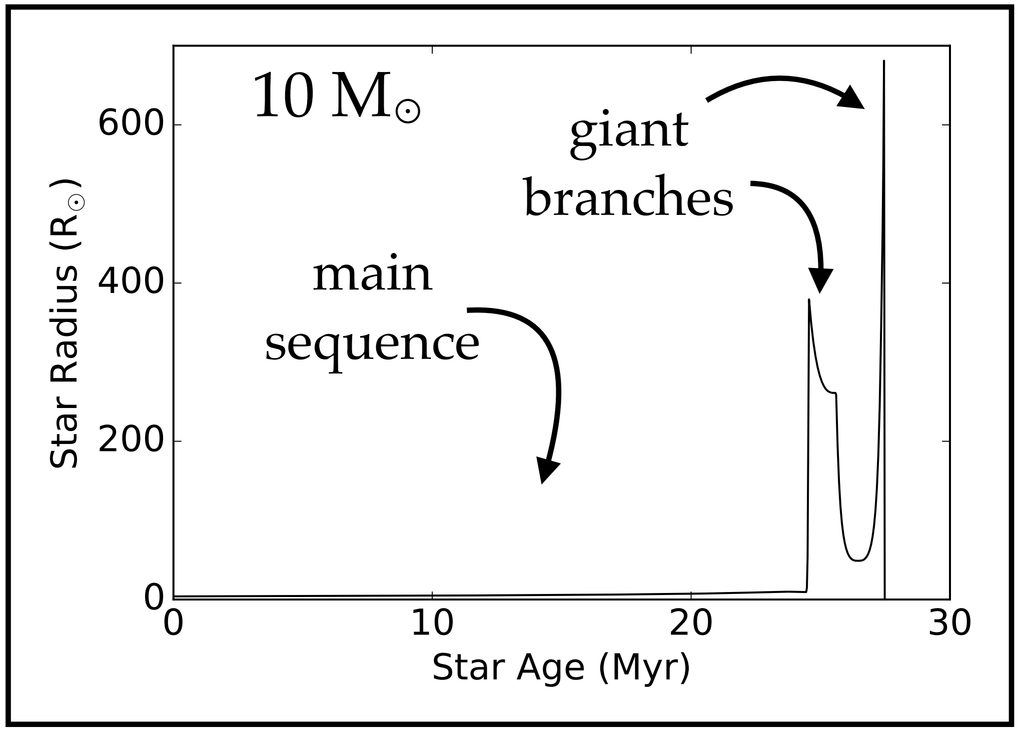
Massive stars die young



end their lives in a supernova (SN)



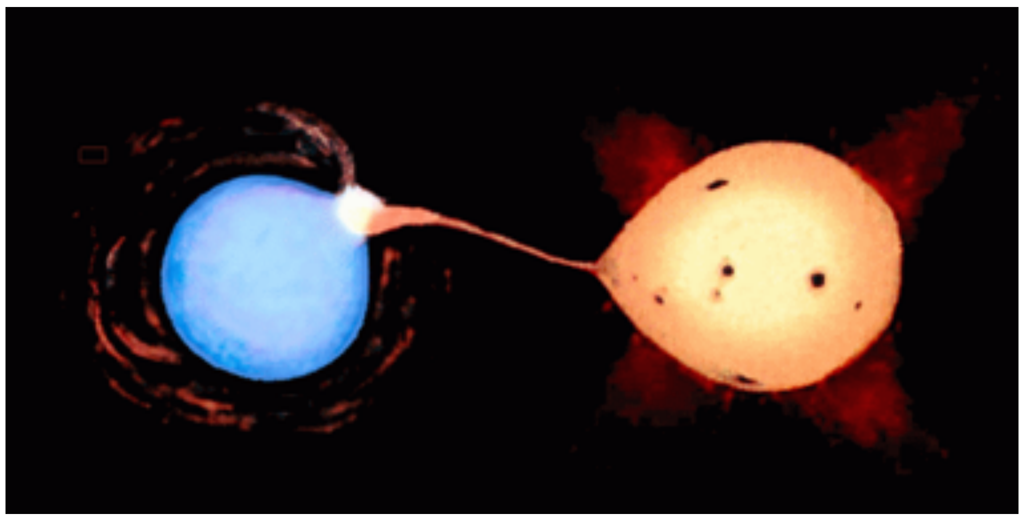
"puff" up



leaving behind a neutron star (NS) or black hole (BH)

Binary stellar evolution crash course

Mass transfer can occur when there is a companion



SN affects the orbit

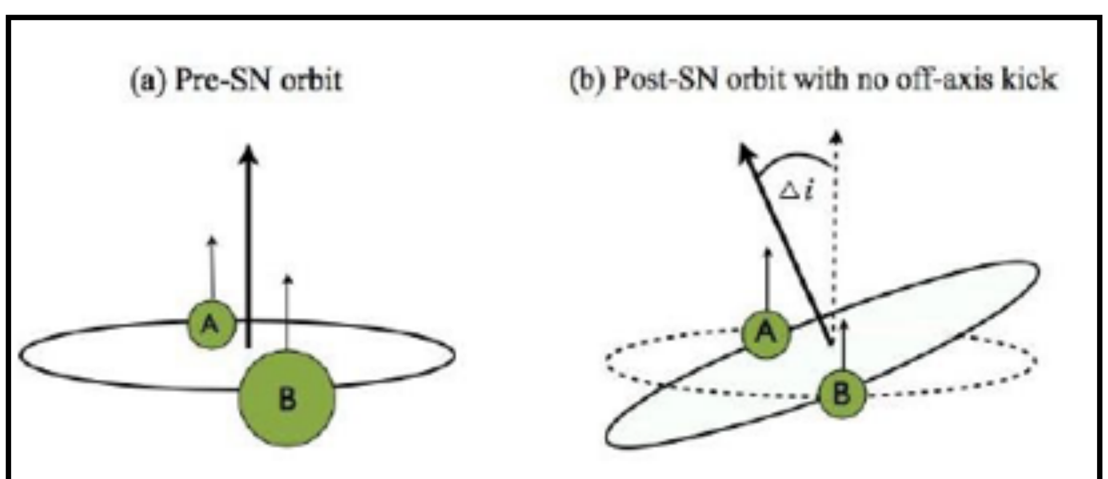


Image: Kyle Kremer

Wind accretion onto the NS or BH

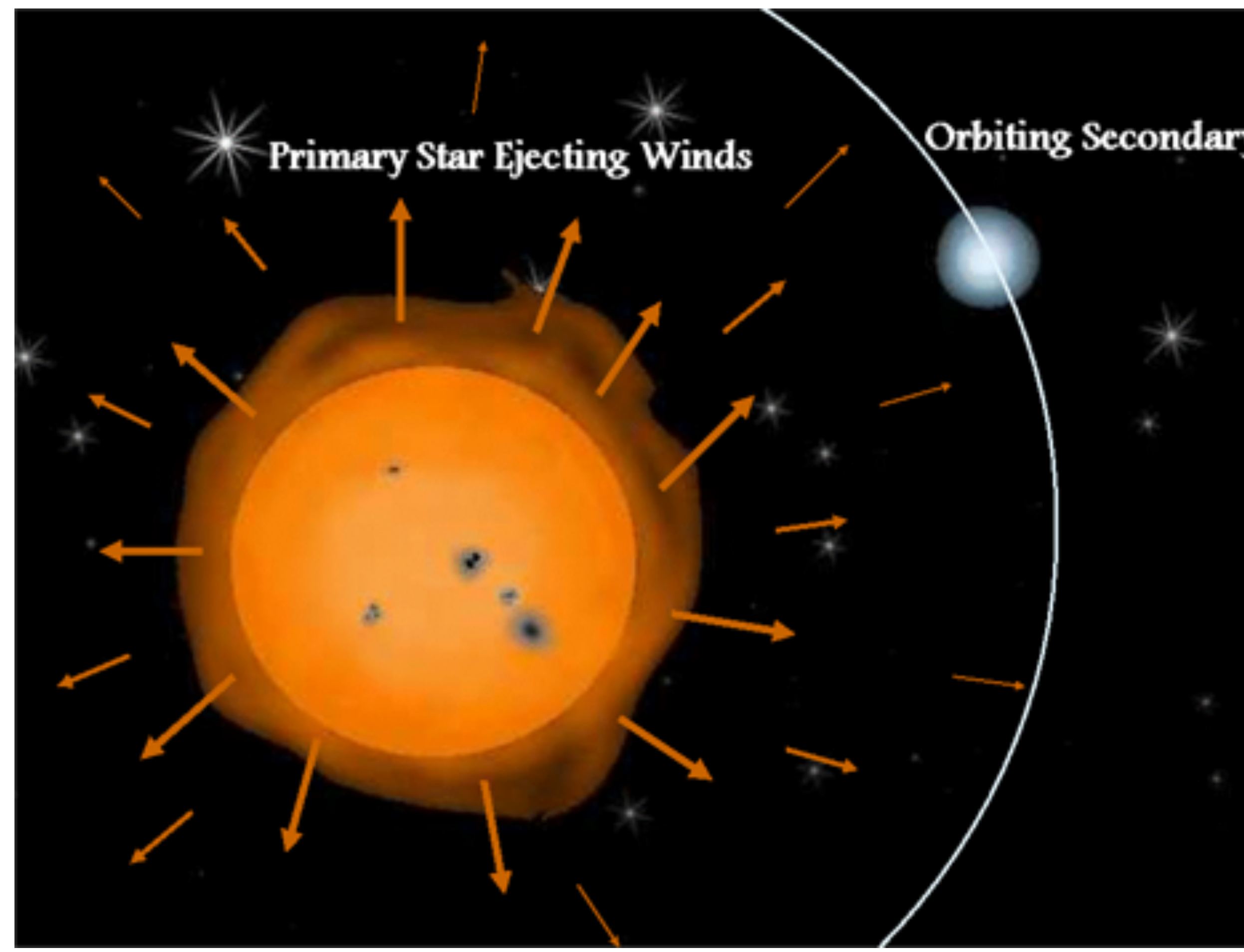
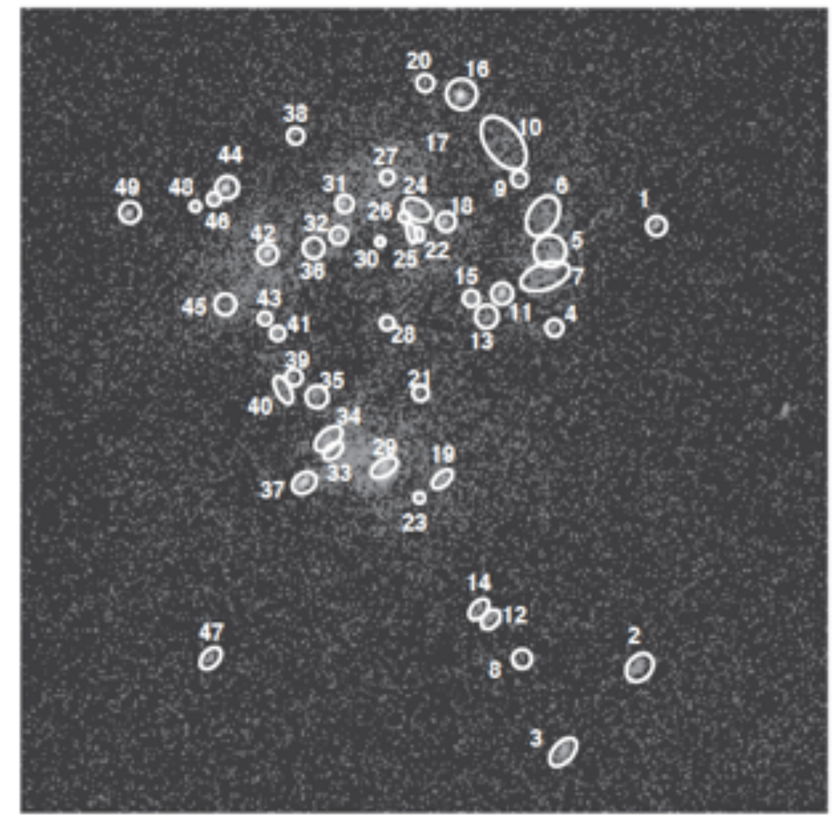


Image: Mathew Bailey

X-ray sources in nearby galaxies

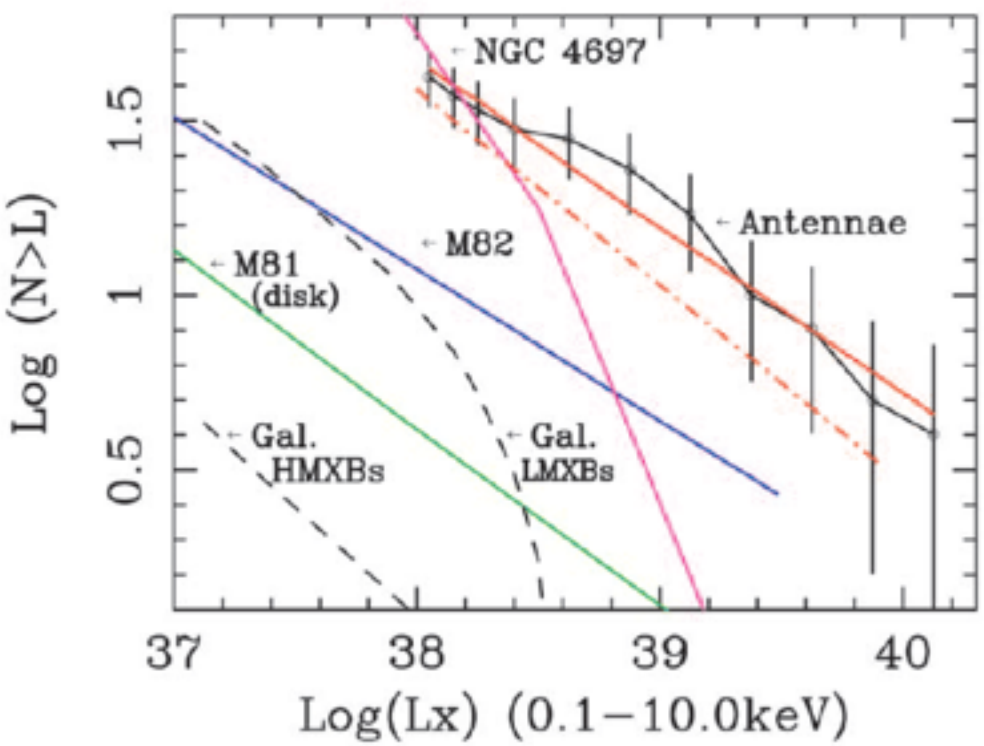
Optical

X-rays



Zezas et al. (2002)

X-ray luminosity functions



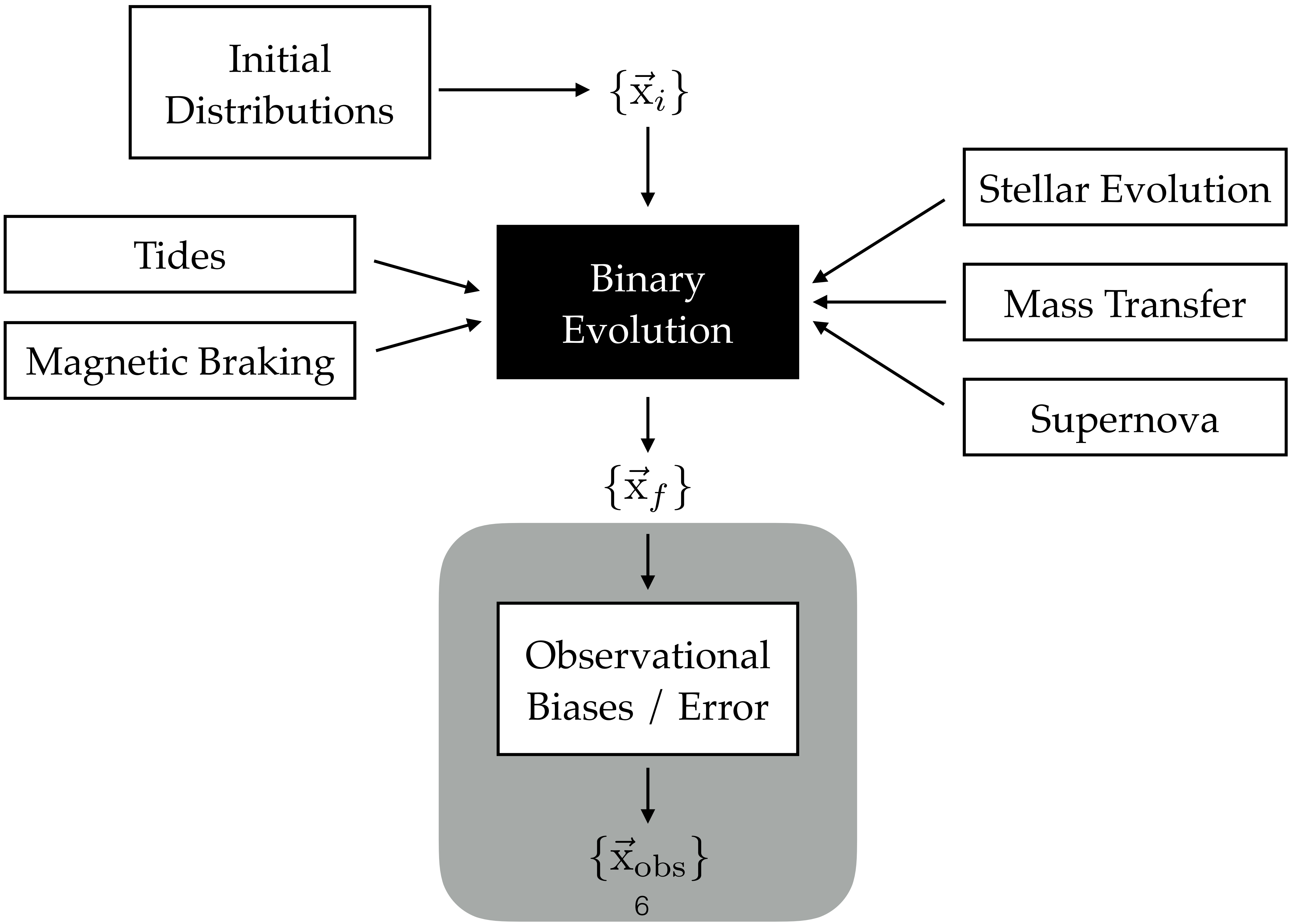
Zezas & Fabbiano (2002)

Notation

Model variables	
M	Model
$\vec{x}_i$	Initial parameters
$\vec{x}_f$	Current parameters
$\{\vec{x}_i\}$	Set of initial parameters, for all systems
$\{\vec{x}_f\}$	Set of current parameters, for all systems

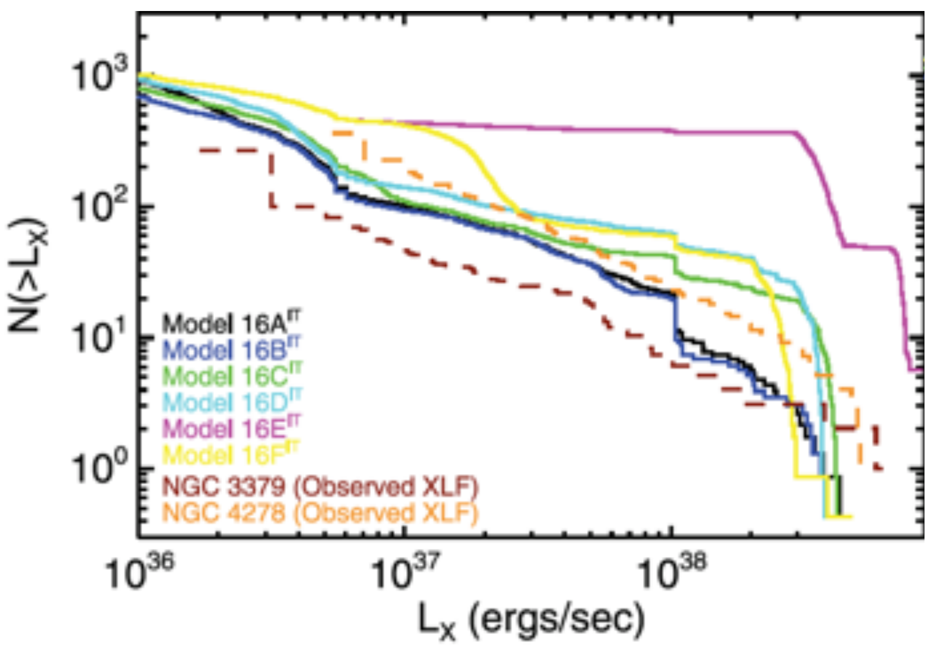
Binary variables	
M_1	Primary mass
M_2	Secondary mass
a	Orbital separation
e	Orbital eccentricity
v_k	Kick velocity
θ	Kick polar angle
ϕ	Kick azimuthal angle
α	Coordinate - right ascension
δ	Coordinate - declination
t	Birth time / age
$P_{\rm orb}$	Orbital period
$v_{\rm sys}$	Velocity of the system

Population synthesis basics

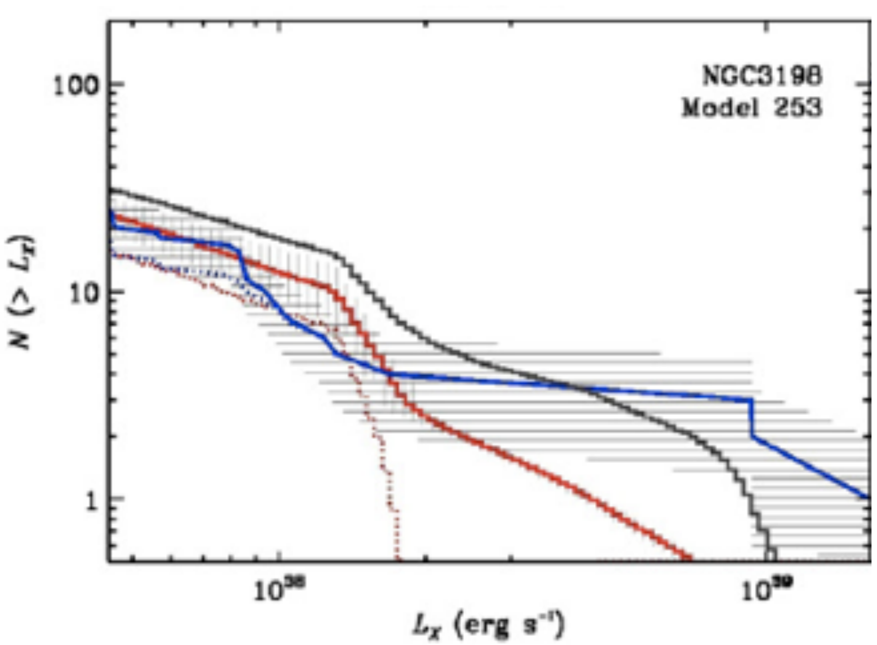


Population synthesis goals

Model selection

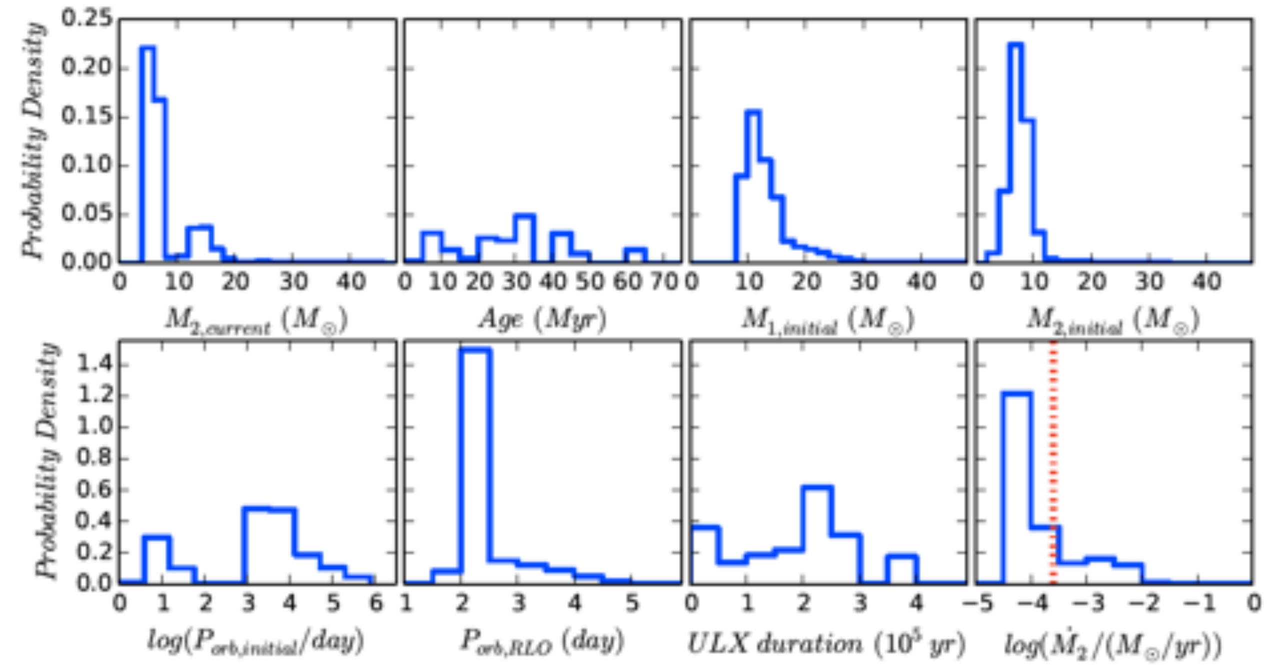


Fragos et al. (2008)



Tzanavaris et al. (2013)

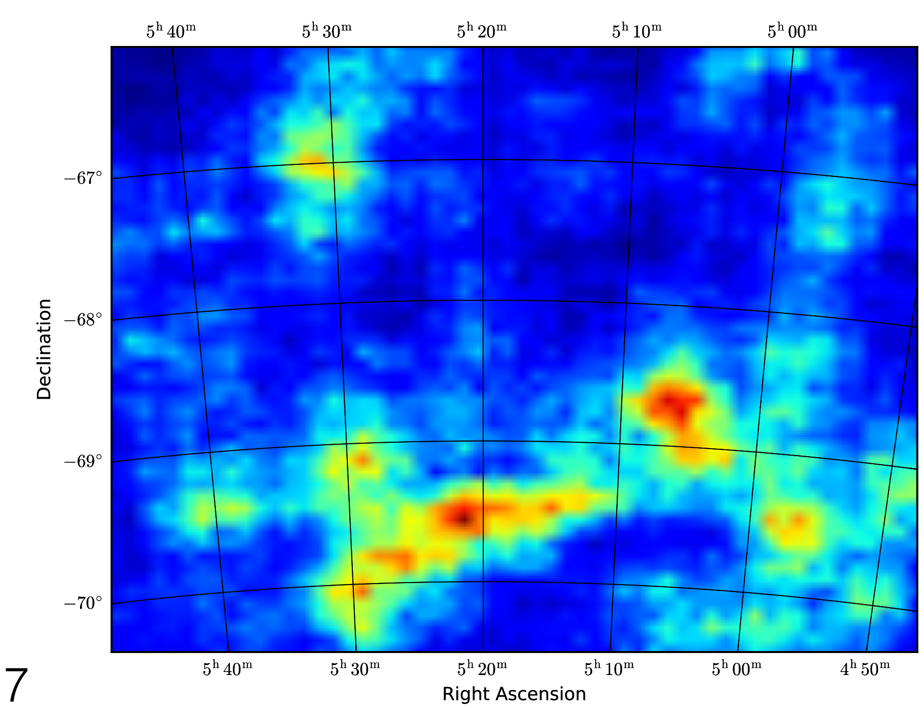
Individual system analysis



M82 X-2

Fragos et al. (2015)

Observational prediction



Expected HMXB population in the Large Magellanic Cloud

Population synthesis math: model selection

Goal is to compare our physics with observed population

$$P(M|\{\vec{x}_f\}) = \frac{P(\{\vec{x}_f\}|M)P(M)}{P(\{\vec{x}_f\})}$$

Observed systems are independent

$$P(\{\vec{x}_f\}|M) = \prod P(\vec{x}_f|M)$$

Model doesn't directly provide us with a population

$$P(\vec{x}_f|M) = \int \mathrm{d}\vec{x}_i \; P(\vec{x}_f|\vec{x}_i, M) \; P(\vec{x}_i|M)$$

Population synthesis uses importance sampling

$$P(\vec{x}_f|M) \approx \frac{1}{N} \sum_j P(\vec{x}_f|\vec{x}_{i,j}, M)$$

$$\vec{x}_{i,j} \sim P(\vec{x}_i|M)$$

Only select binaries

$$P(\vec{x}_f|\vec{x}_{i,j}, M) = \begin{cases} 1 & \vec{x}_f \in \text{binary} \\ 0 & \vec{x}_f \text{ else} \end{cases}$$

Population synthesis

Model selection $$P(M|\{\vec{x}_f\}) \propto \; P(M) \prod_{\mathrm{all}\ \vec{x}_f} \frac{1}{N} \sum_j P(\vec{x}_f|\vec{x}_{i,j}, M) \; P(\vec{x}_i|M)$$

Observational prediction $$P(\vec{x}_f) \approx \frac{1}{N} \sum_j P(\vec{x}_f|\vec{x}_{i,j}, M) \; P(\vec{x}_i|M)$$

Individual system analysis $$P(\vec{x}_i|\vec{x}_f, M) \propto \; P(\vec{x}_f|\vec{x}_i, M) \; P(\vec{x}_i|M)$$

Essentially all the same calculation:
identify the binary initial conditions of relevance

Demonstrative example: double neutron stars

Eccentricity Orbital period

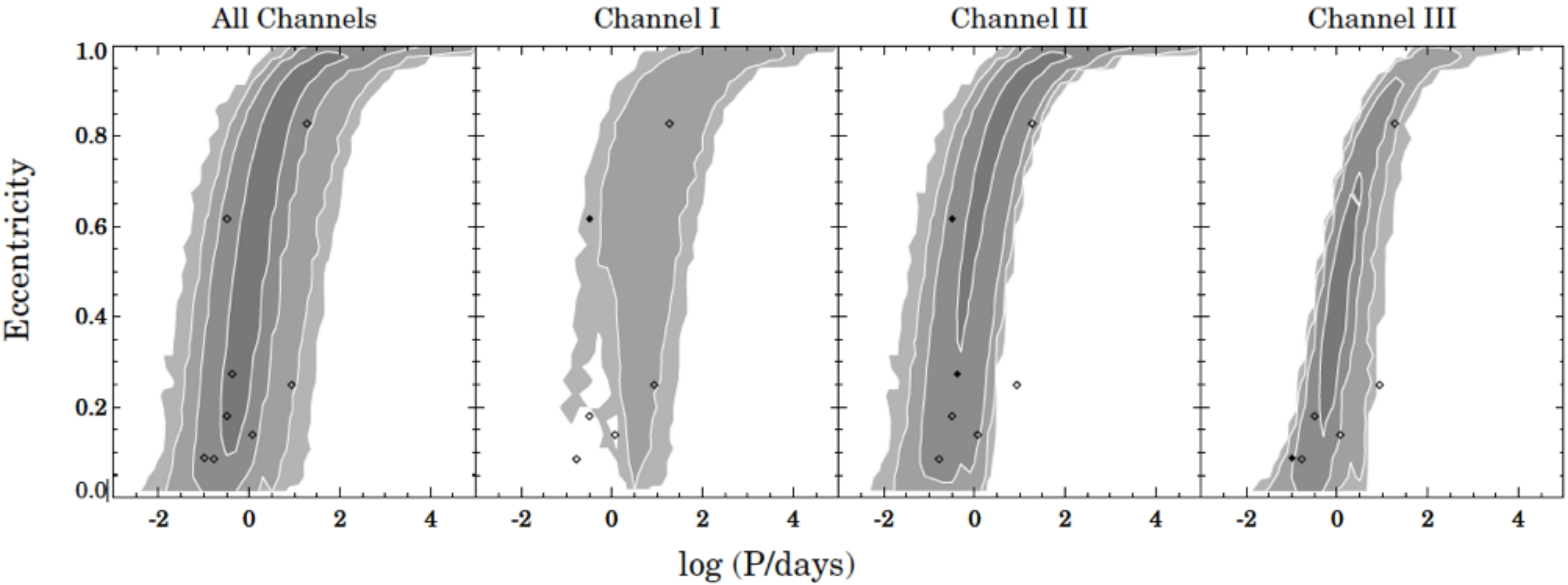
8 high quality systems

DNS	e^d	P^d (days)	P_s (ms)]	Pulsar Mass ($M_\odot$)	Companion Mass ($M_\odot$)
B1534+12	0.274	0.421	37.9	1.3332(10)	1.3452(10)
B1913+16	0.617	0.323	59.0	1.4408(3)	1.3873(3)
J0737−3039	0.088	0.102	22.7	1.337(5)	1.250(5)
J1518+4904	0.249	8.634	40.9	$1.56^{+0.13}_{-0.45}$	$1.05^{+0.45}_{-0.11}$
J1756−2251	0.181	0.320	28.5	1.341(7)	1.230(7)
J1811−1736	0.828	18.779	104.2	$1.62^{+0.22}_{-0.55}$	$1.11^{+0.53}_{-0.15}$
J1829+2456	0.139	1.176	41.0	$1.14^{+0.28}_{-0.48}$	$1.36^{+0.50}_{-0.17}$
J1906+0746[a]	0.085	0.166	144.1	1.248(18)	1.365(18)
J1753−2240[b]	0.304	13.638	95.1		
B2127+11C[c]	0.680	0.335	30.5	1.35(4)	1.36(4)

Can ignore observational uncertainties and biases

Andrews et al. (2015)

Double neutron star orbit distribution

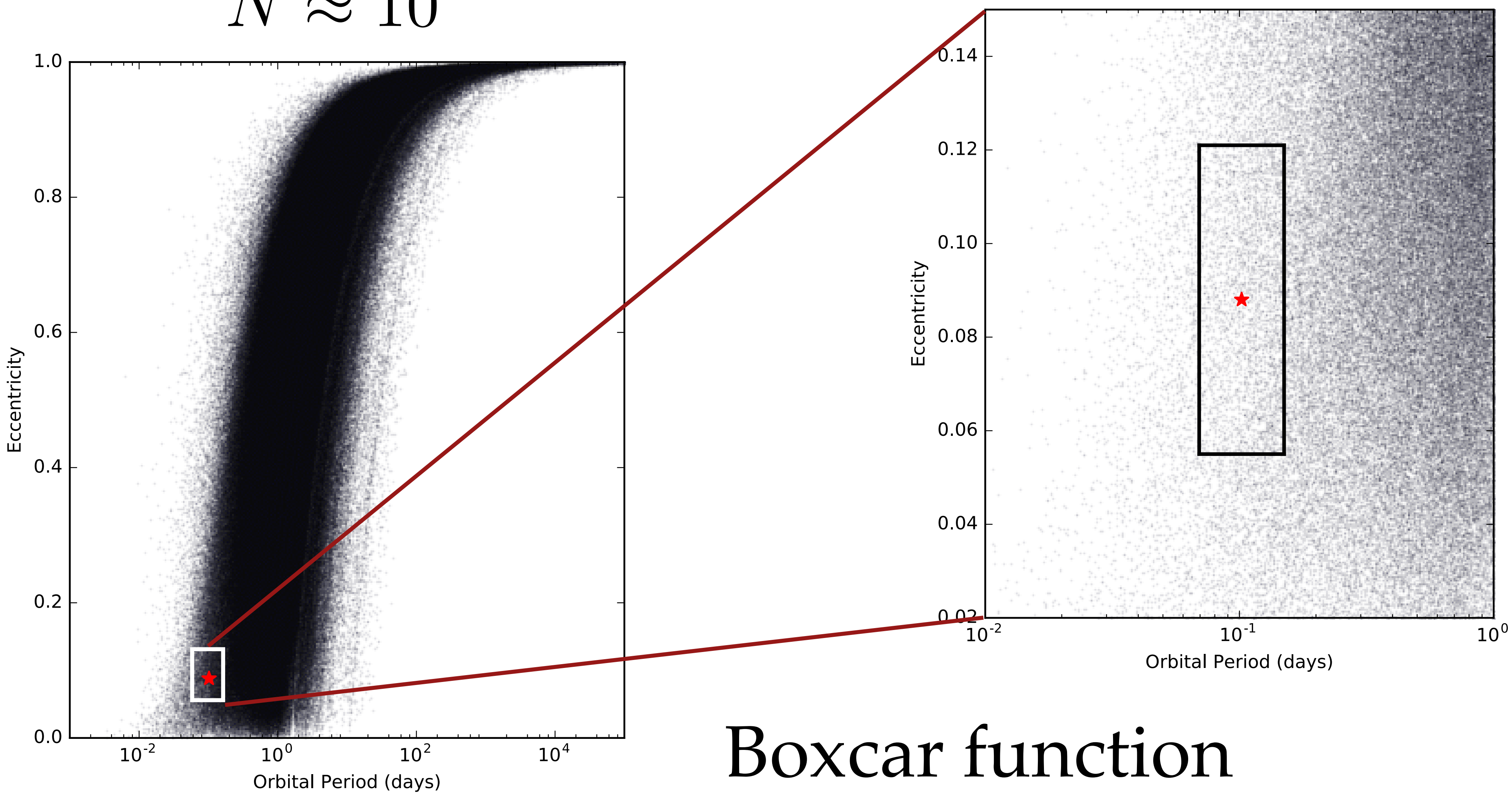


Data consist of:
1. Eccentricity
2. Orbital period

Andrews et al. (2015)

But, how to determine the likelihood?

$N \approx 10^6$



Basic prescription: Box size dependent on number of data points (shot noise)

$$N_{\rm box} \leq \sqrt{N}$$

Boxcar function

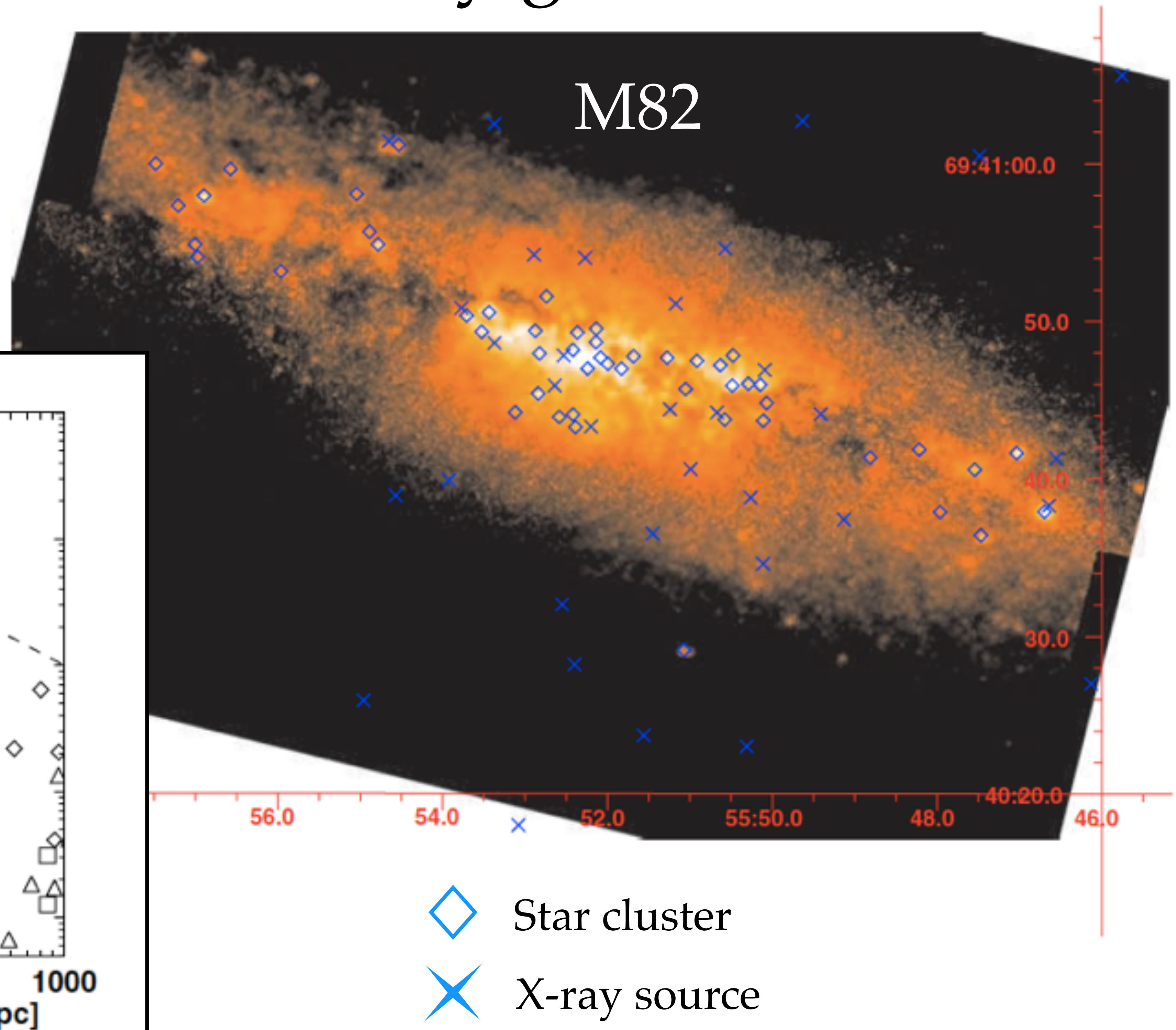
$$P(\vec{x}_f|M) = \frac{1}{N}\sum_{j\in N} P(\vec{x}_f|\vec{x}_{i,j}, M)$$

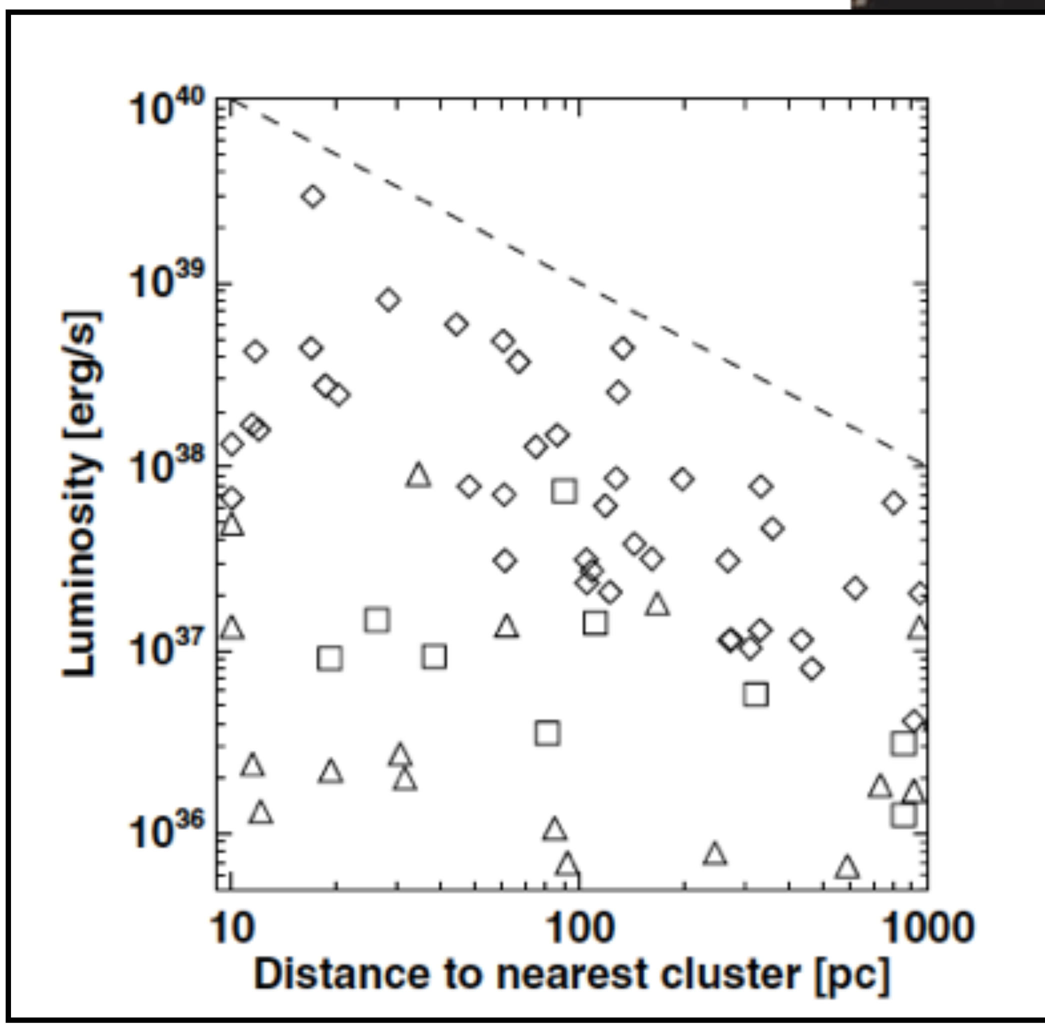
$$P(\vec{x}_f|\vec{x}_{i,j}, M) = \begin{cases} 1 & \vec{x}_f \in \text{box} \\ 0 & \vec{x}_f \ \text{else} \end{cases}$$

PROBLEM: most of the parameter space is in "else"

HMXBs in nearby galaxies

SN kick causes offset



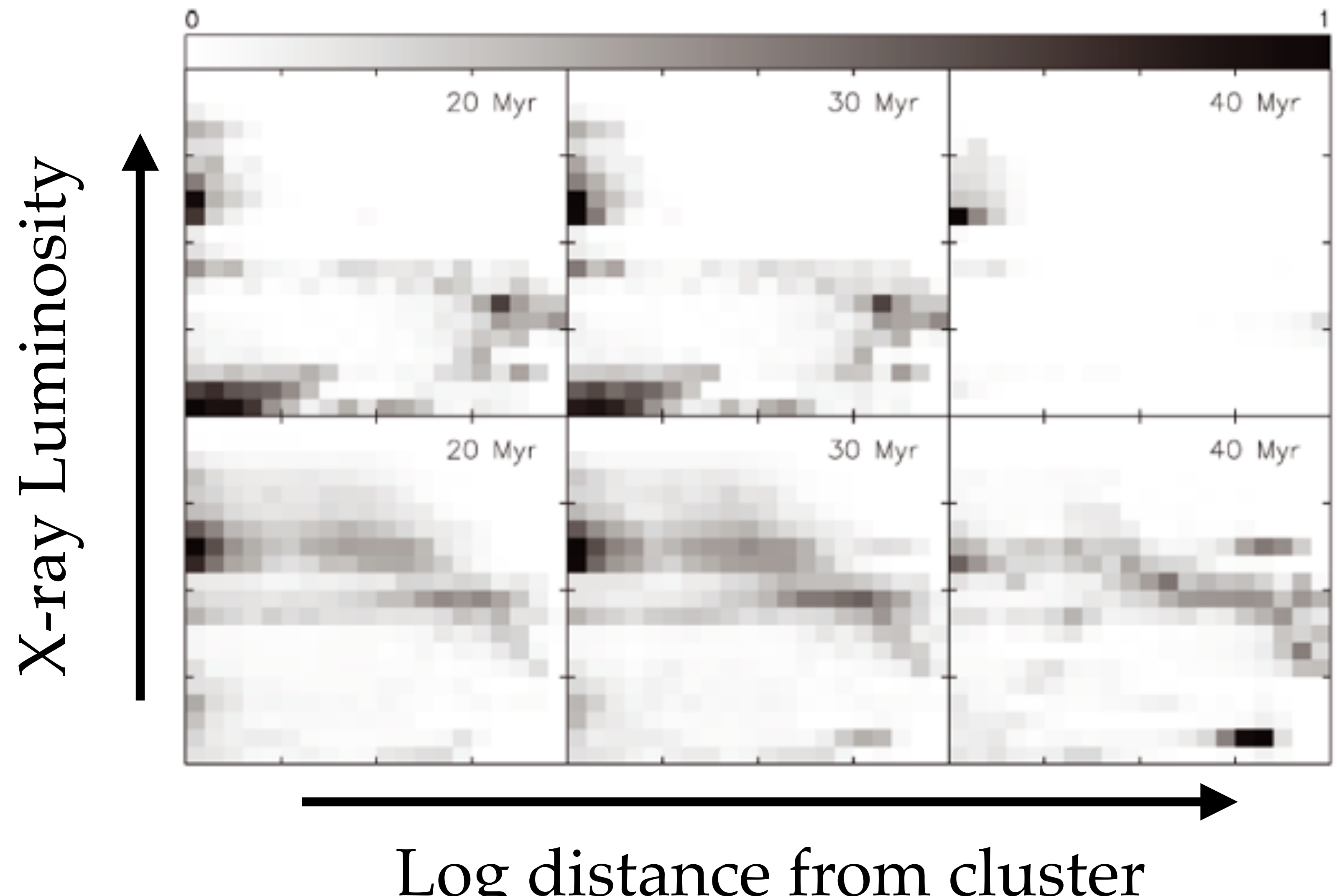


M82, NGC 1569, NGC 5253

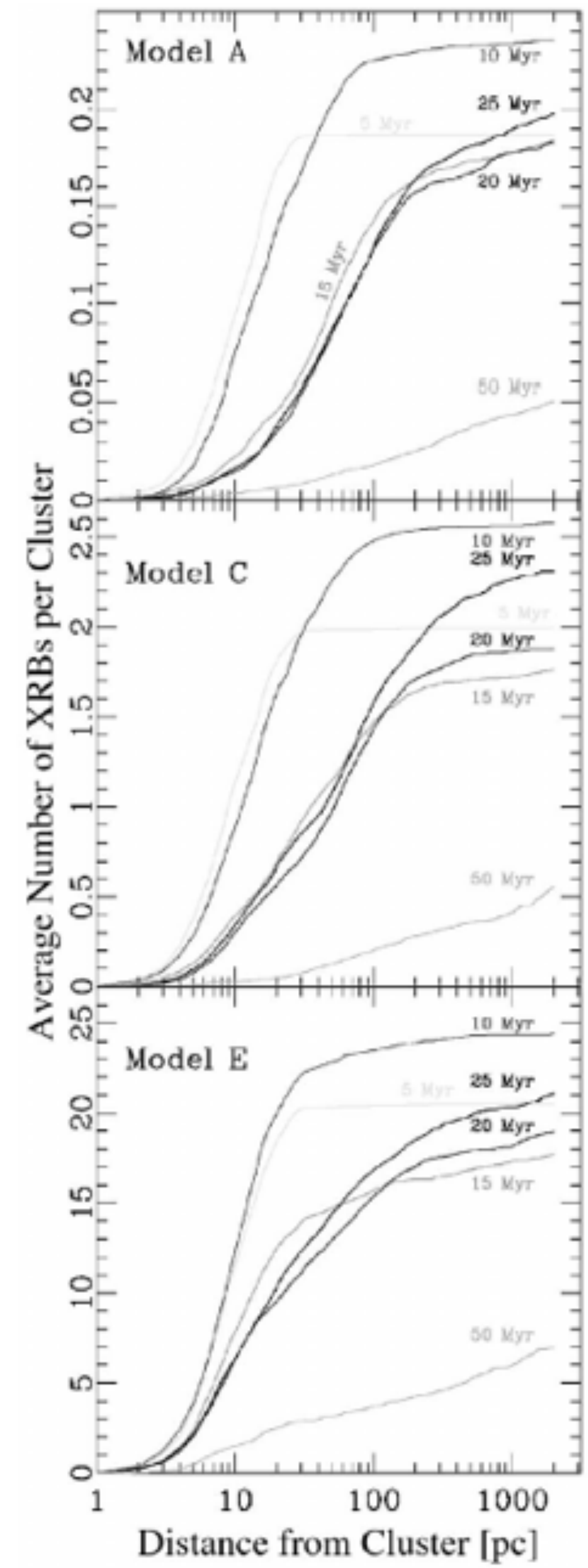
Kaaret et al. (2004)

HMXB population synthesis

Can reproduce general trends



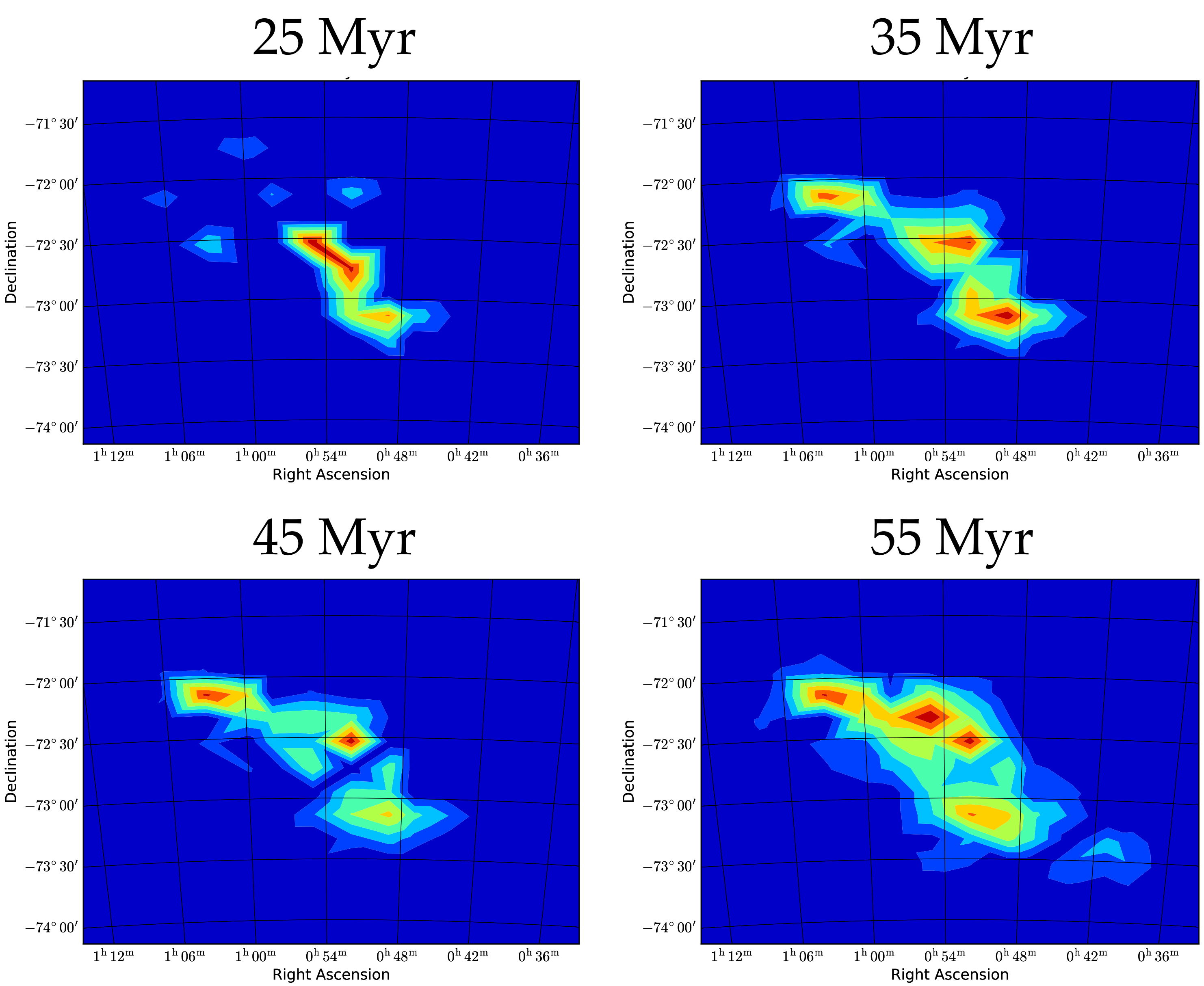
Zuo & Li (2010)



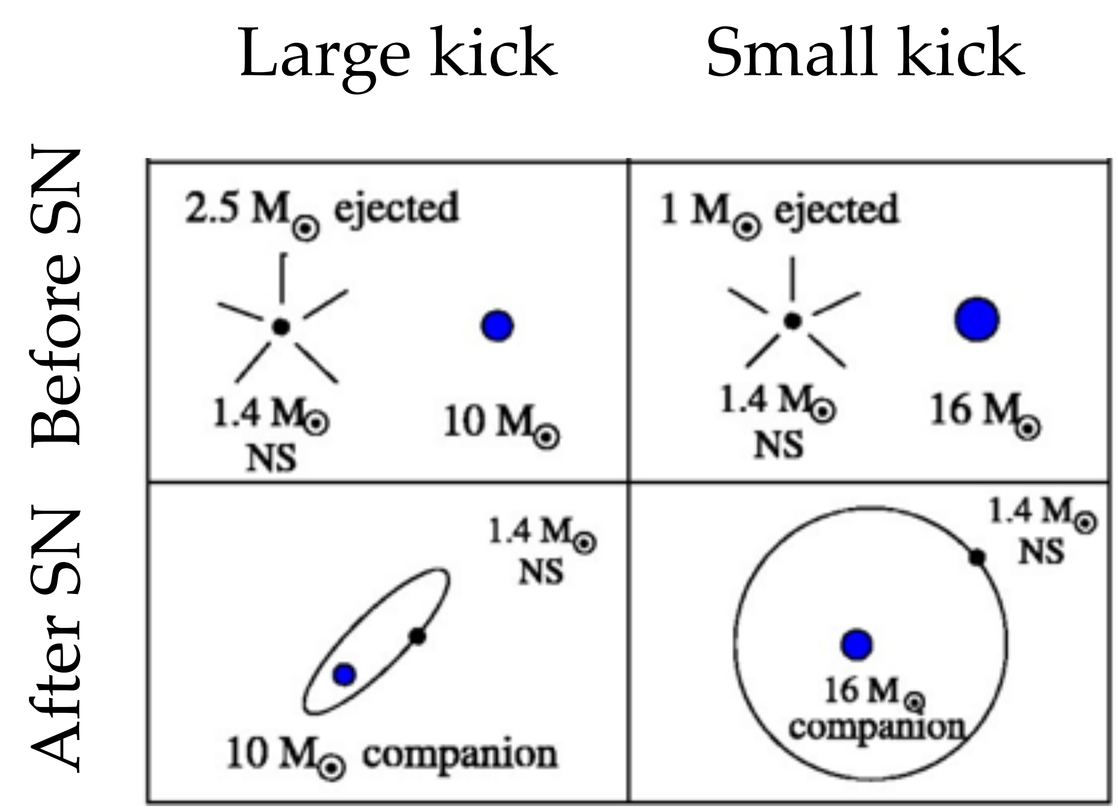
Sepinsky et al. (2005)

Core project idea

Star formation history



Individual systems' orbits



Podsiadlowski et al. (2004)

Back to the math: Our model

$$P(\vec{x}_f|M) = \int \mathrm{d}\vec{x}_i \; P(\vec{x}_f|\vec{x}_i, M) \; P(\vec{x}_i|M)$$ Marginalize

$$\vec{x}_i = \{M_{1,i}, M_{2,i}, a_i, e_i, \vec{v}_k, \alpha_i, \delta_i, t_i\}$$ Initial binary parameters

$$\vec{x}_f = \{\alpha, \delta, P'_{\mathrm{orb}}, e', M'_2\}$$ Observations

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Marginalize again to account for observational uncertainties

$$P(\vec{x}_f|M) = \int \mathrm{d}v_{\mathrm{sys}} \; \mathrm{d}P_{\mathrm{orb}} \; \mathrm{d}e \; \mathrm{d}M_2 \; \mathrm{d}\vec{x}_i \; P(\vec{x}_f, v_{\mathrm{sys}}, P_{\mathrm{orb}}, e, M_2|\vec{x}_i, M) \; P(\vec{x}_i|M)$$

$$P(\vec{x}_f|M) = \int \mathrm{d}\vec{x}_i \; \mathrm{d}v_{\mathrm{sys}} \; \mathrm{d}P_{\mathrm{orb}} \; \mathrm{d}e \; \mathrm{d}M_2$$

$$\times P(P'_{\mathrm{obs}}|P_{\mathrm{orb}}) \; P(e'|e) \; P(M'_2|M_2)$$ Observational uncertainties

$$\times P(\alpha, \delta|v_{\mathrm{sys}}, \vec{x}_i, M)$$ Distance traveled

$$\times P(v_{\mathrm{sys}}, P_{\mathrm{orb}}, e, M_2|\vec{x}_i, M)$$ Binary evolution

$$\times P(\vec{x}_i|M)$$ Initial binary probabilities

Andrews et al. (in prep.)

Binary evolution

$$P(\vec{x}_f|M) = \int \mathrm{d}\vec{x}_i \ \mathrm{d}v_{\rm sys} \ \mathrm{d}P_{\rm orb} \ \mathrm{d}e \ \mathrm{d}M_2$$
$$\times P(P'_{\rm obs}|P_{\rm orb}) \ P(e'|e) \ P(M'_2|M_2)$$
$$\times P(\alpha, \delta|v_{\rm sys}, \vec{x}_i, M)$$
$$\times P(v_{\rm sys}, P_{\rm orb}, e, M_2|\vec{x}_i, M)$$
$$\times P(\vec{x}_i|M)$$

$$P(v_{\rm sys}, P_{\rm orb}, e, M_2|\vec{x}_i, M) = \delta\left[v_{\rm sys} - f_1(\vec{x}_i)\right] \times \delta\left[P_{\rm orb} - f_2(\vec{x}_i)\right] \times \delta\left[e - f_3(\vec{x}_i)\right] \times \delta\left[M_2 - f_4(\vec{x}_i)\right]$$

Starred quantities are solutions to delta functions

Integral reduces:

$$P(\vec{x}_f|M) = \int \mathrm{d}\vec{x}_i \ P(P'_{\rm obs}|P^*_{\rm orb}) \ P(e'|e^*) \ P(M'_2|M^*_2)$$
$$\times P(\alpha, \delta|v^*_{\rm sys}, \vec{x}_i, M) \ P(\vec{x}_i|M)$$

MCMC Approach:

$\vec{x}_i$	Model parameters
$P(\vec{x}_i\|M)$	Prior probabilities
$P(P'_{\rm obs}\|P^*_{\rm orb}) \ P(e'\|e^*) \ P(M'_2\|M^*_2) \times P(\alpha, \delta\|v^*_{\rm sys}, \vec{x}_i, M)$	Likelihood

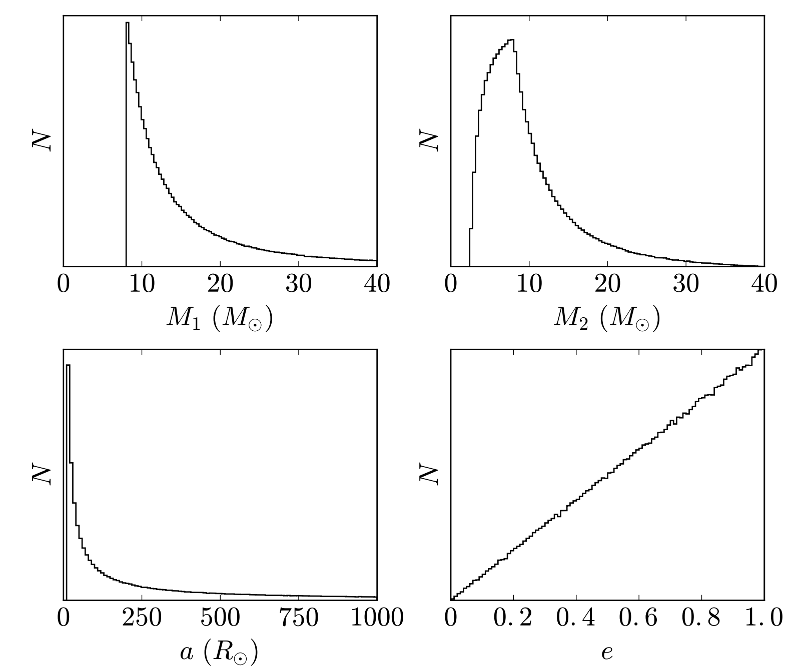
Priors: binary/kick parameters

$\vec{x}_i = \{M_{1,i}, M_{2,i}, a_i, e_i, \vec{v}_k, \alpha_i, \delta_i, t_i\}$

$$P(\vec{x}_i|M) = P(M_{1,i})\ P(M_{2,i}|M_{1,i})\ P(a_i) \times P(e_i)\ P(v_k)\ P(\theta_k)\ P(\phi_k) \times P(\alpha_i, \delta_i, t_i | M_{1,i}, M_{2,i}, a_i, e_i, \vec{v}_k)$$

binary priors

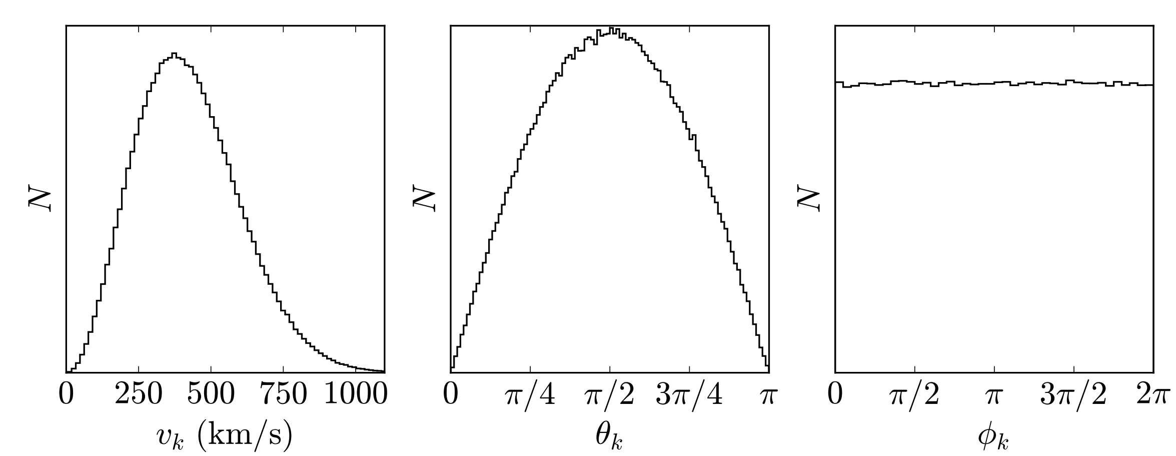
- $P(M_{1,i}) \propto M^{-2.35}$ ← Initial Mass Function
- $P(M_{2,i}|M_{1,i}) \propto 1/M_1$ ← Mass Ratio
- $P(a_i) \propto 1/a$ ← Binary Separation
- $P(e_i) \propto e$ ← Eccentricity



SN kick priors

$P(v_k) \propto v_k^2 \exp\left[-v_k^2/2\sigma^2\right]$
$P(\theta_k) \propto \sin\theta$
$P(\phi_k) \propto 1$

} Maxwellian, isotropic kicks



Priors: birth position and time

$$P(\alpha_i, \delta_i, t_i | M_{1,i}, M_{2,i}, a_i, e_i, \vec{v}_k) = C_{\rm SFH}\ {\rm SFR}(\theta, \phi, t_i)$$

Normalization constant

Star formation rate

Calculating the normalization constant

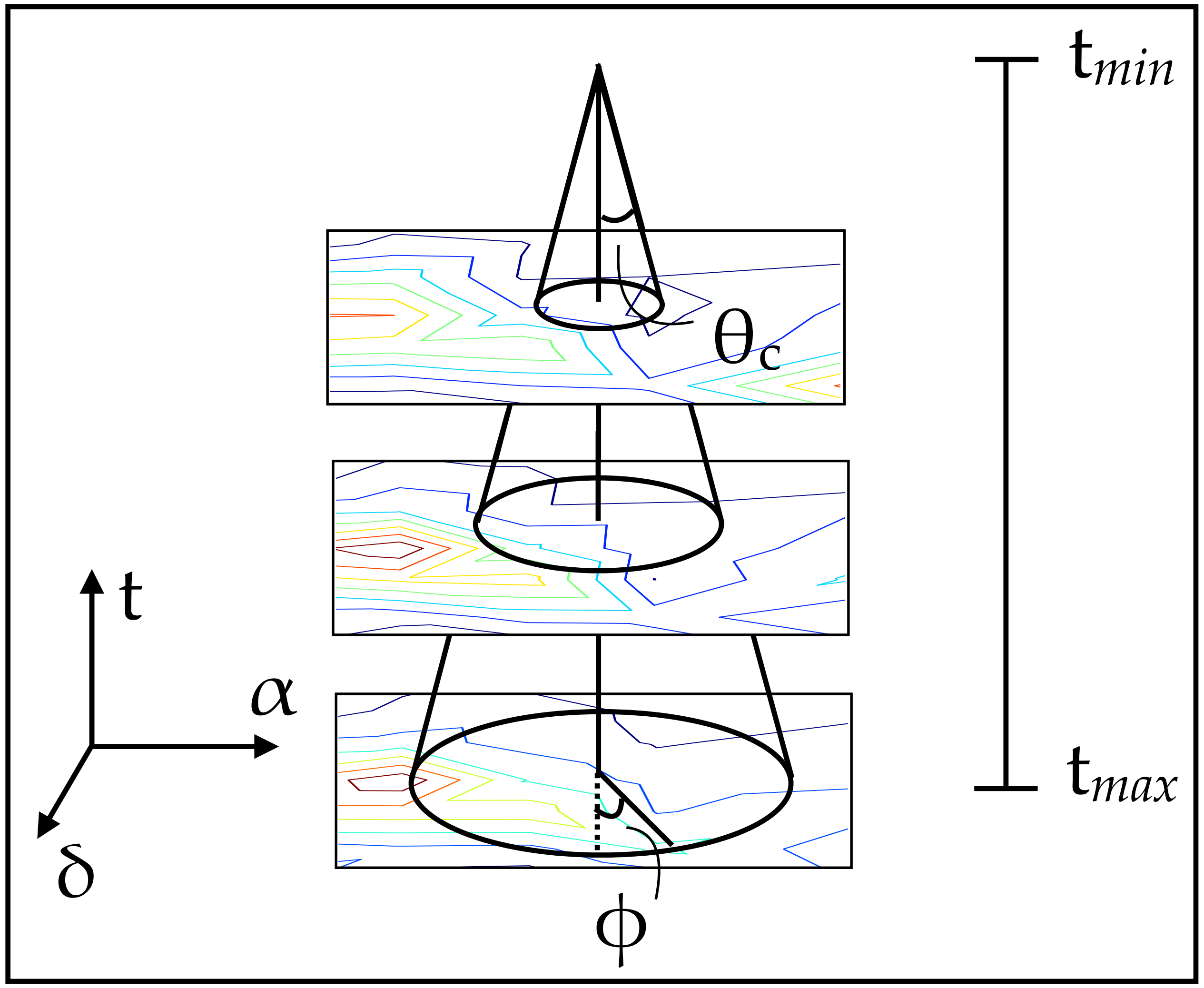
$$1 = C_{\rm SFH} \int_{t_{\rm min}}^{t_{\rm max}} \int_0^{2\pi} \int_0^{\theta_c} {\rm d}t_i\ {\rm d}\phi\ {\rm d}\theta\ {\rm SFR}(\theta, \phi, t_i)$$

Monte Carlo integrate

$$\frac{1}{C_{\rm SFH}} \approx \frac{\pi \theta_C^2 (t_{\rm max} - t_{\rm min})}{3N} \sum_j {\rm SFR}(\theta_j, \phi_j, t_{i,j})$$

$$\phi_j \sim U(0, 2\pi)$$
$$\theta_j = \theta_C\ \sqrt{y_1};\ \ y_1 \sim U(0,1)$$
$$t_{i,j} = \sqrt[3]{y_2}\,(t_{\rm max} - t_{\rm min}) + t_{\rm min};\ \ y_2 \sim U(0,1)$$

Inverse transform sampling



Andrews et al. (in prep.)

Likelihood 1: Orbital parameters

Gaussian uncertainties

$$P(P'_{\rm obs}|P^*_{\rm orb}) = \mathcal{N}(P'_{\rm obs}|P^*_{\rm orb}, \sigma^2)$$

$$P(e'|e^*) = \mathcal{N}(e'|e^*, \sigma^2)$$

$$P(M'_2|M^*_2) = \mathcal{N}(M'_2|M^*_2, \sigma^2)$$

Measurable with work

Measurable with lots of work

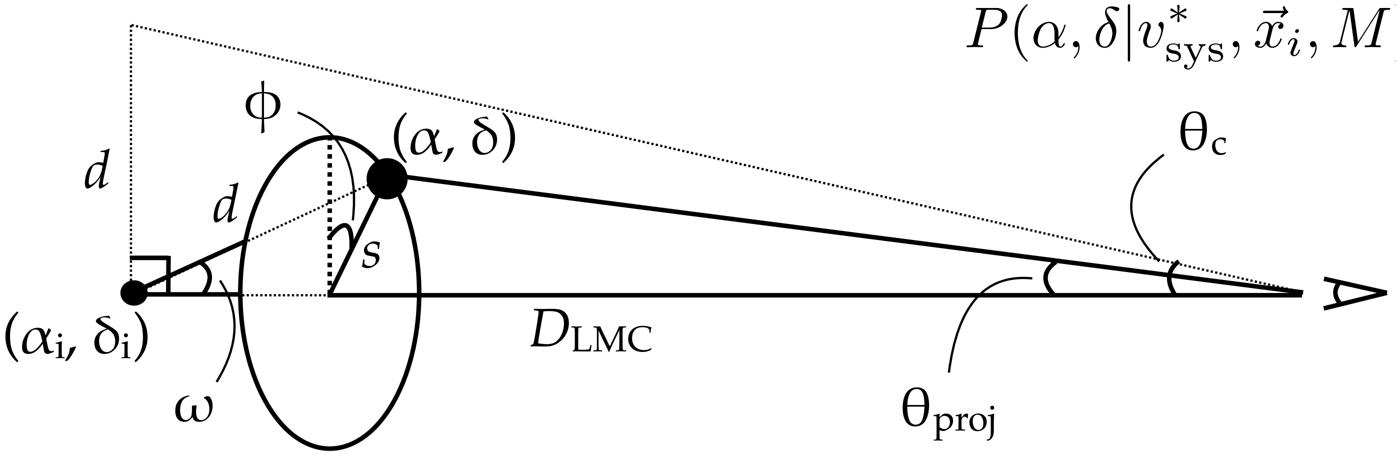
Derived from photometry

Future direction: use photometry directly

How to deal with uncertainties here?

Likelihood 2: Position

$$J_{\rm coor} = \left| \frac{{\rm d}\theta_{\rm proj}}{{\rm d}\alpha}\frac{{\rm d}\phi}{{\rm d}\delta} - \frac{{\rm d}\phi}{{\rm d}\alpha}\frac{{\rm d}\theta_{\rm proj}}{{\rm d}\delta} \right|$$



$$\begin{aligned} P(\alpha,\delta|v^*_{\rm sys},\vec{x}_i,M) &= \int {\rm d}\omega\ P(\alpha,\delta,\omega|v^*_{\rm sys},\vec{x}_i,M) \\ &= \int {\rm d}\omega\ P(\theta_{\rm proj},\phi,\omega|v^*_{\rm sys},\vec{x}_i,M)\ J_{\rm coor} \\ &= \int {\rm d}\omega\ P(\theta_{\rm proj}|\omega,v^*_{\rm sys},\vec{x}_i,M)\ P(\phi)\ P(\omega)\ J_{\rm coor} \end{aligned}$$

Projected separation — Azimuthal angle — Polar angle

$P(\omega) = \sin\omega;\ \omega \in [0,\pi]$

$P(\phi) = \frac{1}{2\pi};\ \phi \in [0,2\pi]$

$P(\theta_{\rm proj}|\omega,v^*_{\rm sys},\vec{x}_i,M) = \delta\left[G(\omega)\right]$

$G(\omega) = \theta_{\rm proj} - \theta_C \sin\omega$

$$\int {\rm d}\omega\ P(\phi)P(\omega)\delta\left[G(\omega)\right] J_{\rm coor} = \sum_i \frac{P(\omega_i^\star)P(\phi)J_{\rm coor}}{\left|\frac{{\rm d}G(\omega)}{{\rm d}\omega}\right|_{\omega_i^*}}$$

$$P(\alpha,\delta|v^*_{\rm sys},\vec{x}_i,M) = \begin{cases} 0, & \theta_{\rm proj} \geq \theta_C \\ \frac{\tan\omega^*}{2\pi\theta_C} J_{\rm coor}, & \theta_{\rm proj} < \theta_C \end{cases}$$

$$\sin\omega^\star = \frac{\theta_{\rm proj}}{\theta_C}$$

Andrews et al. (in prep.)

Model summary

10 model parameters

- 4 initial binary
- 3 supernova kick
- 2 birth coordinate
- 1 birth time

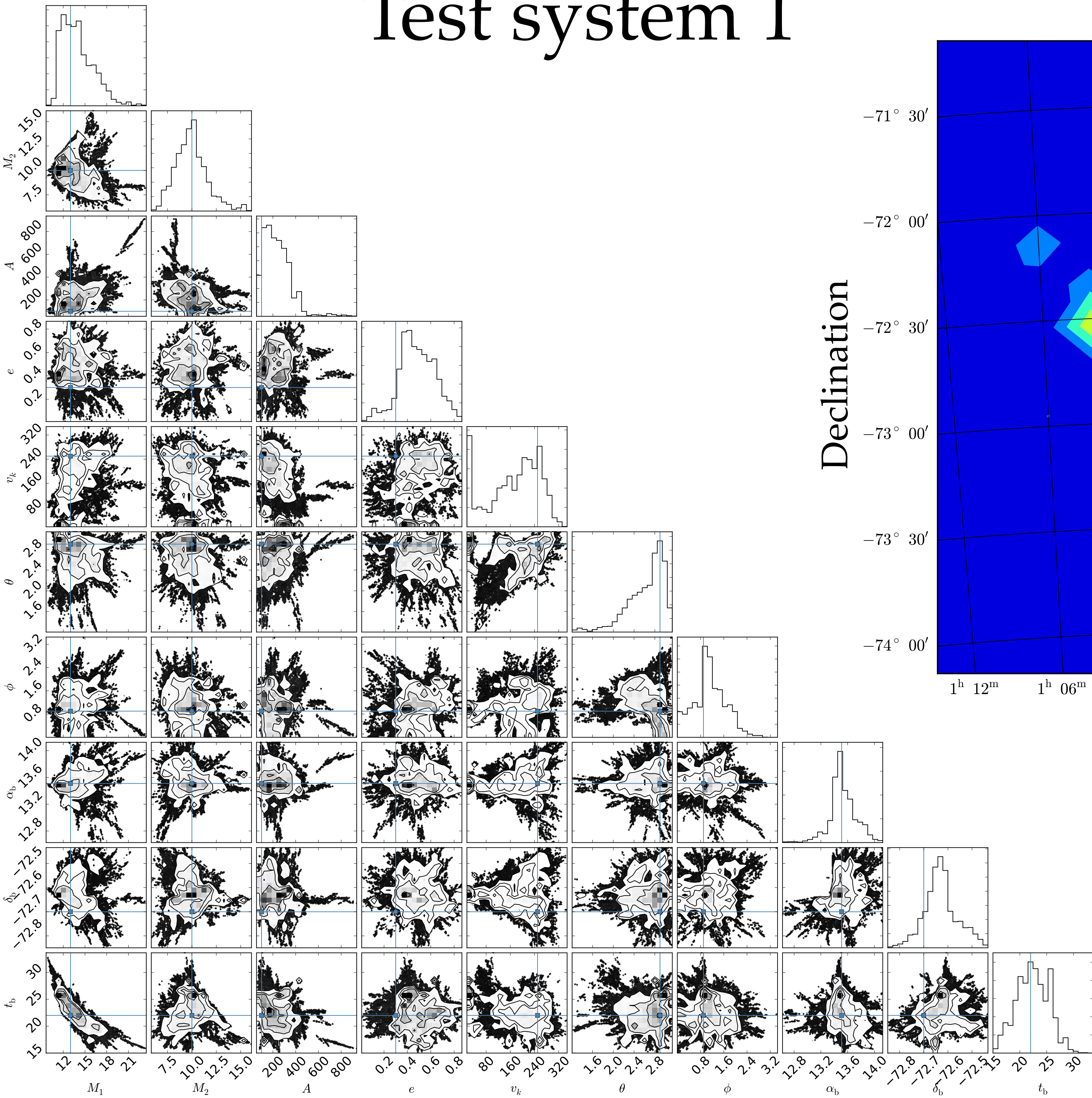
$$\vec{x}_i = \{M_{1,i}, M_{2,i}, a_i, e_i, \vec{v}_k, \alpha_i, \delta_i, t_i\}$$

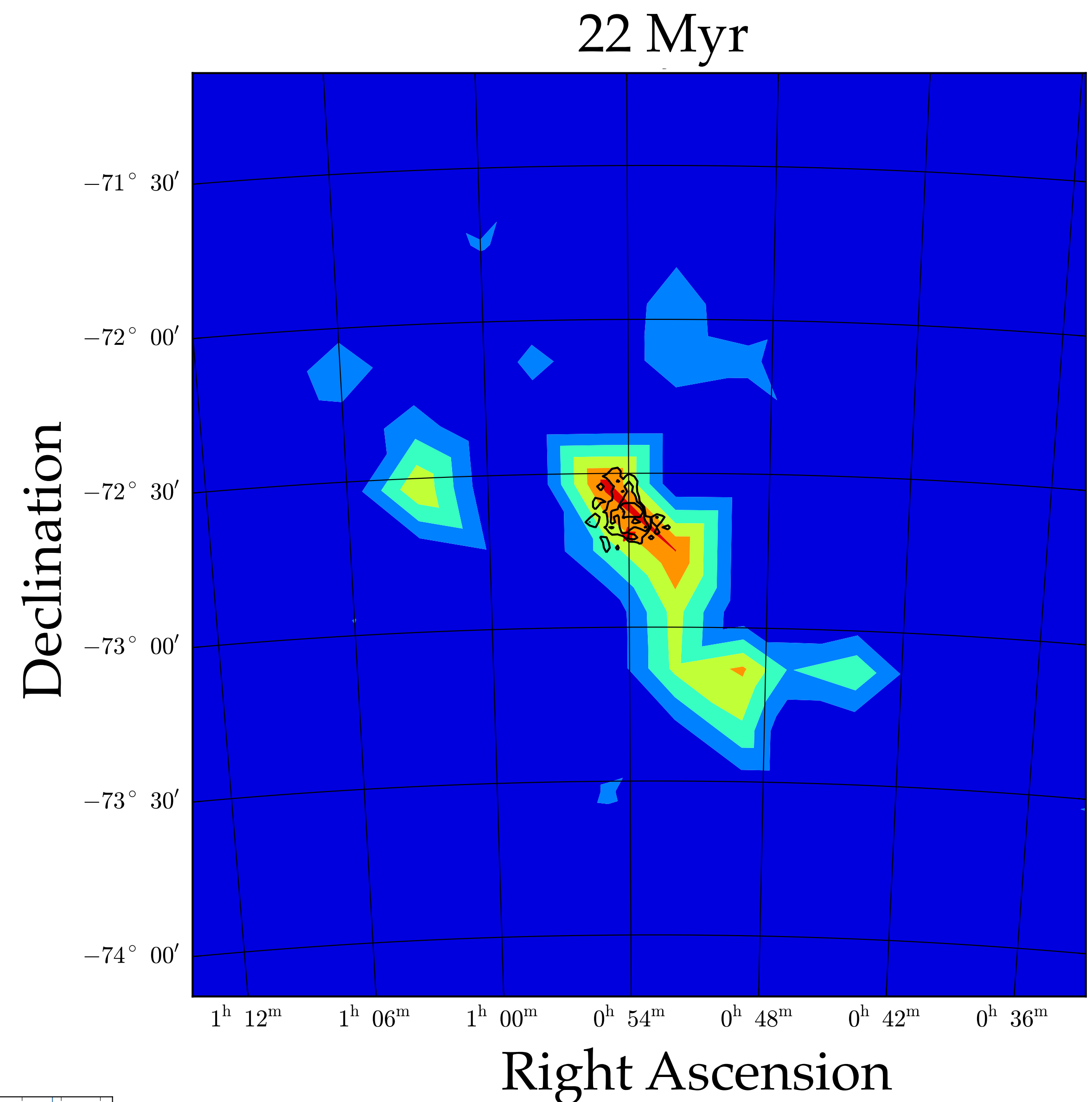
Likelihood:	binary evolution, star formation history
Priors:	orbital parameters, present day position

Numerical Tool: emcee Affine invariant MCMC ensemble sampler (Foreman-Mackey et al. 2012) http://dan.iel.fm/emcee/current/

Model Test: Generate mock data and "observe."
Can we recover input parameters?

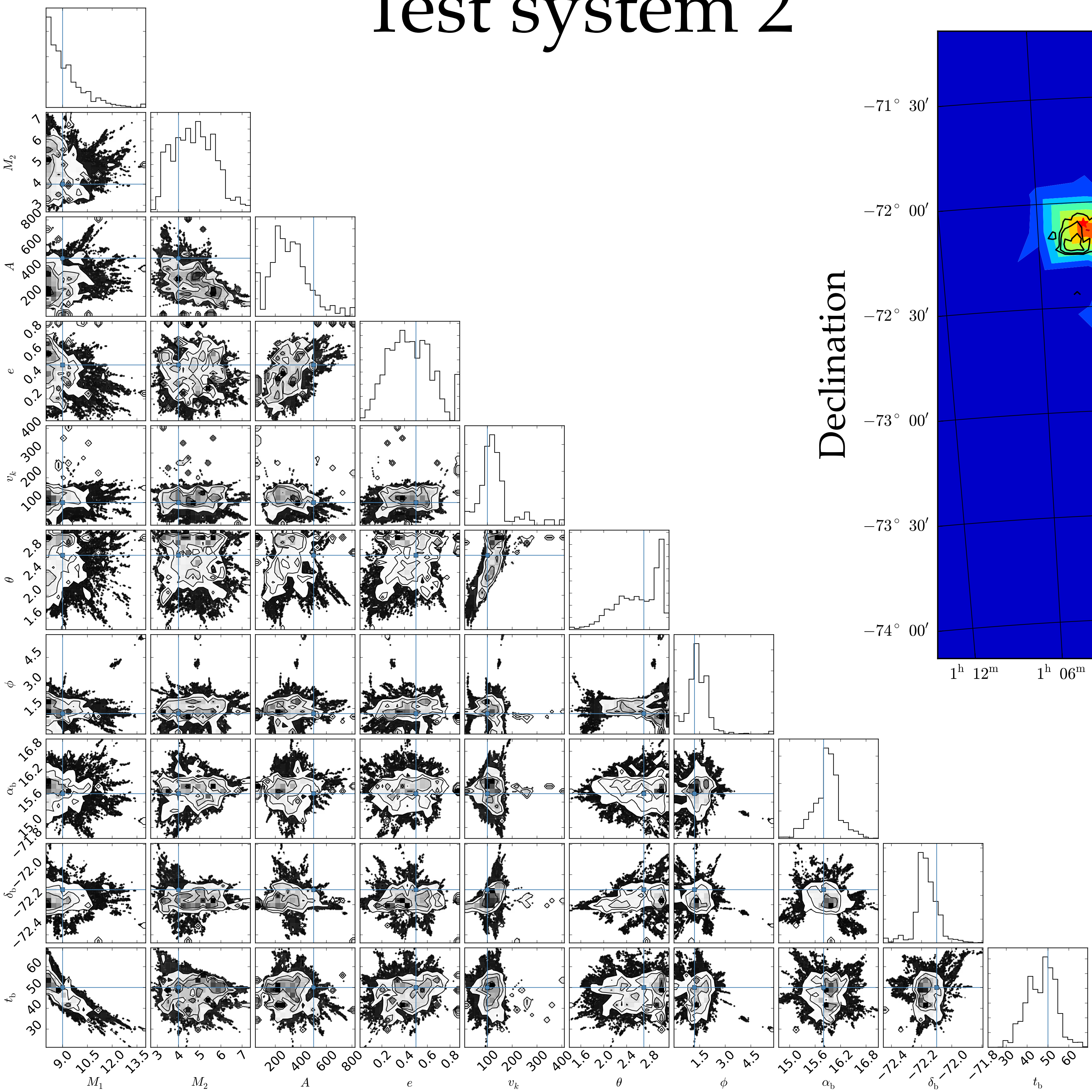
Test system 1

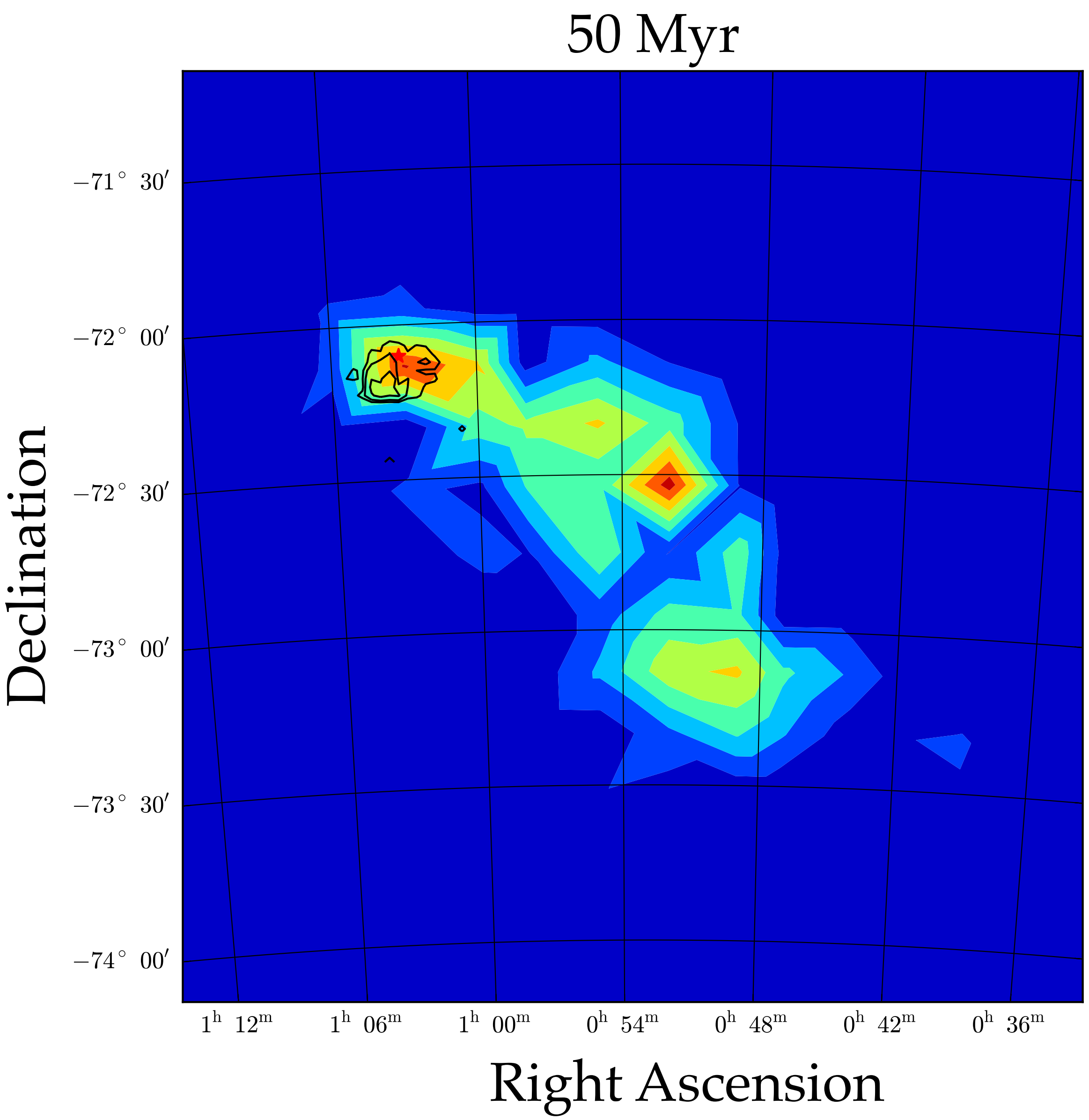




Andrews et al. (in prep.)

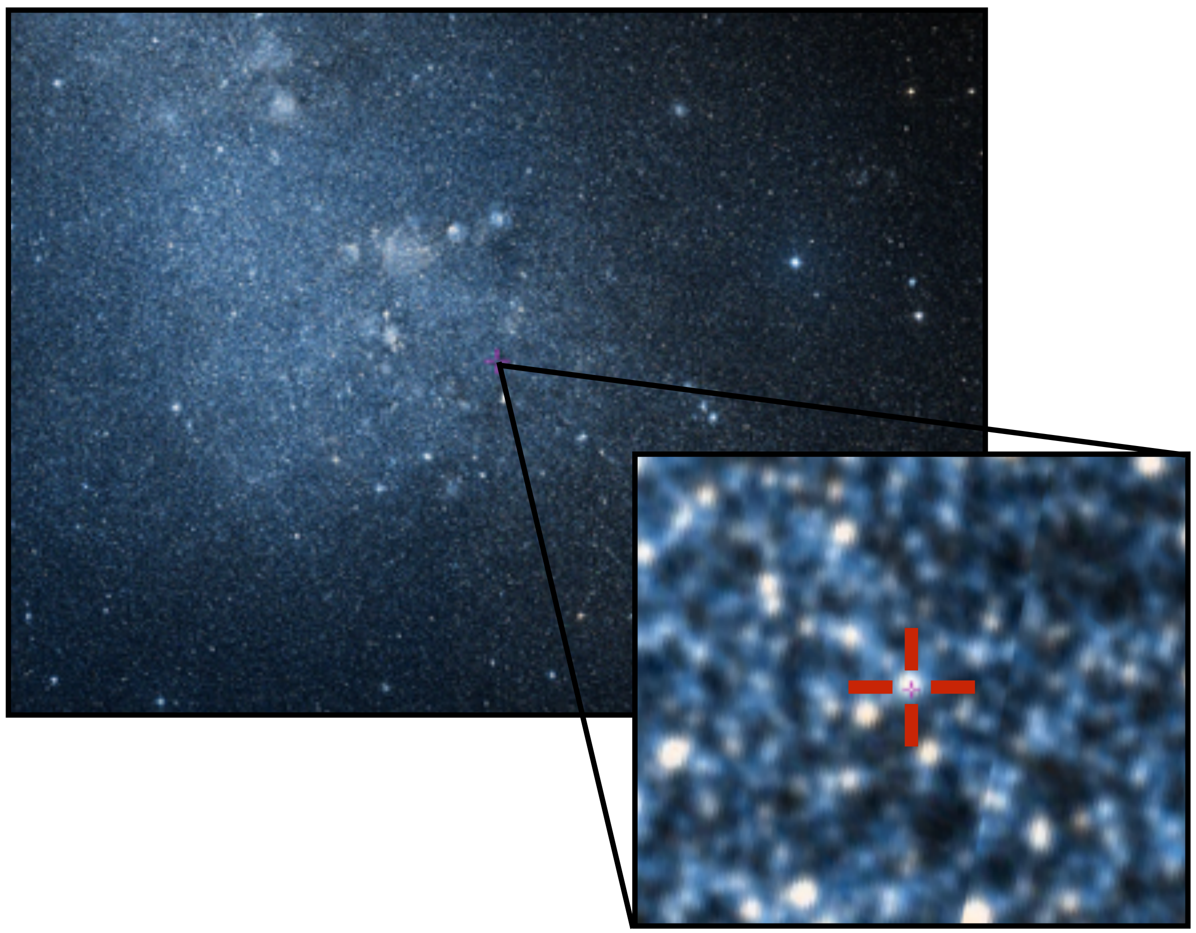
Test system 2





Andrews et al. (in prep.)

SMC J0045-7319



Observed Parameters

(Bell et al. 1995)

$\alpha = 00:45:35.26$

$\delta = -73:19:03.32$

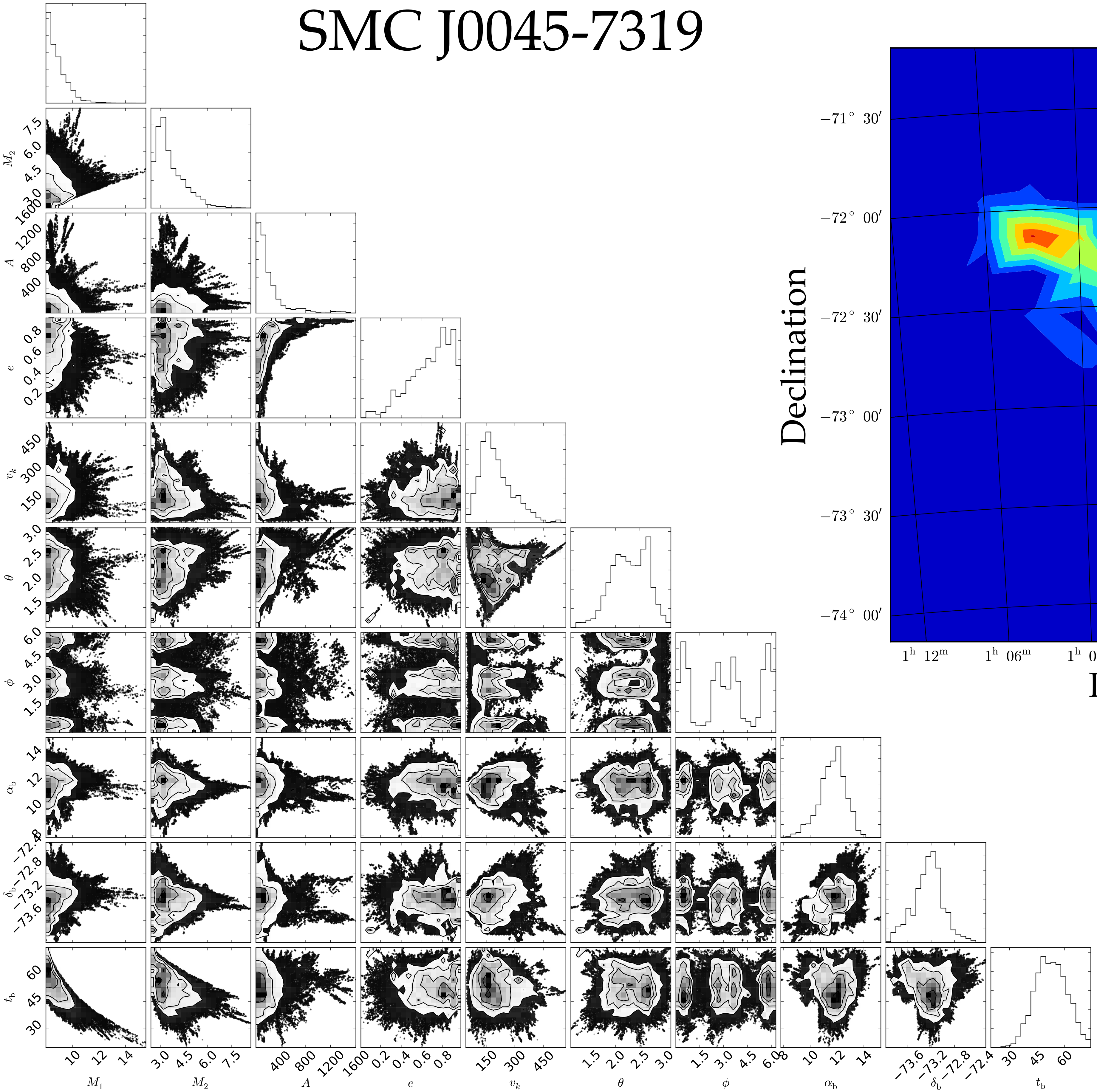
$P_{\rm orb} = 51.169$ days

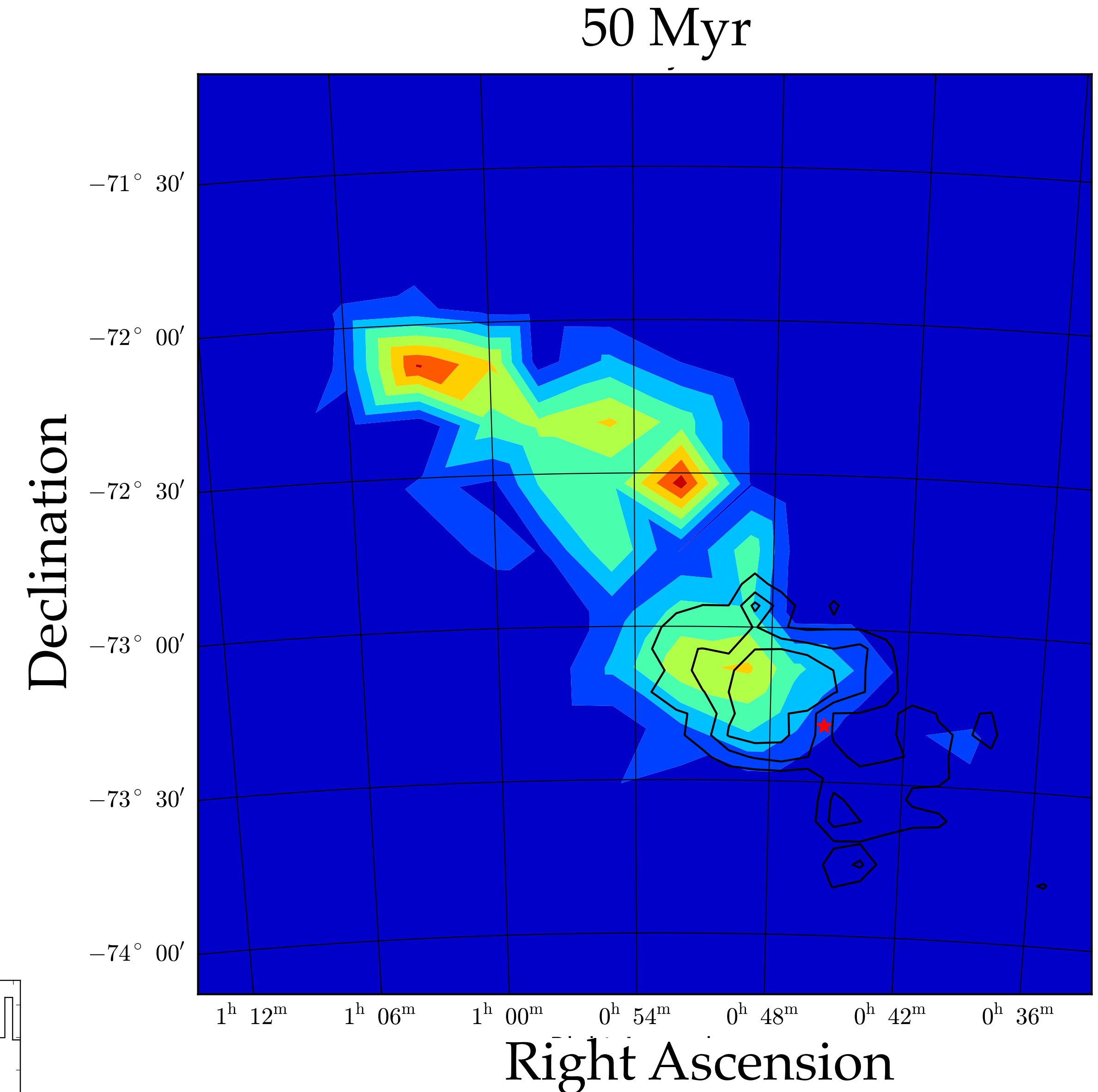
$e = 0.808$

$M_2 = 8.8\ M_\odot$

Andrews et al. (in prep.)

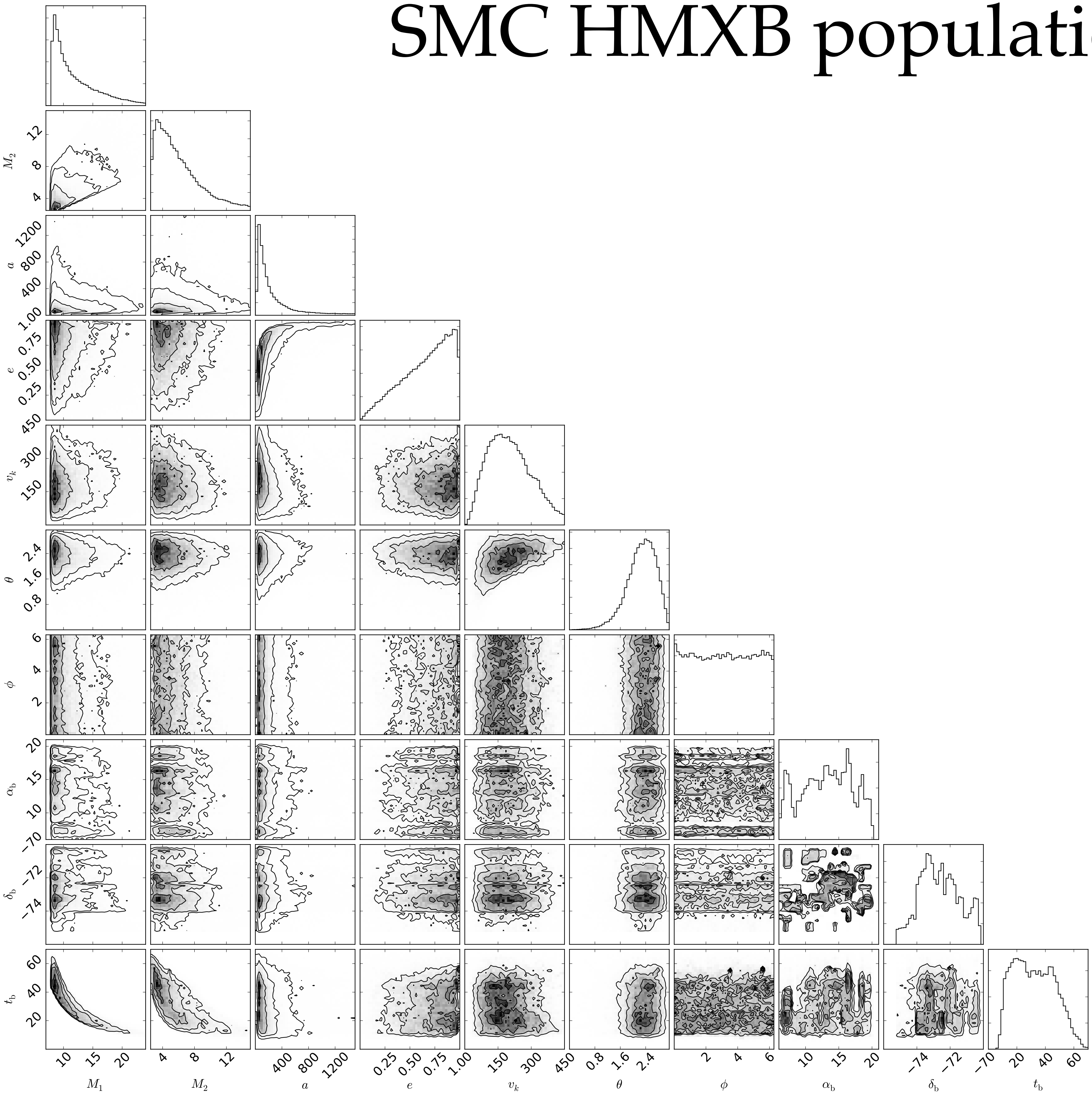
SMC J0045-7319





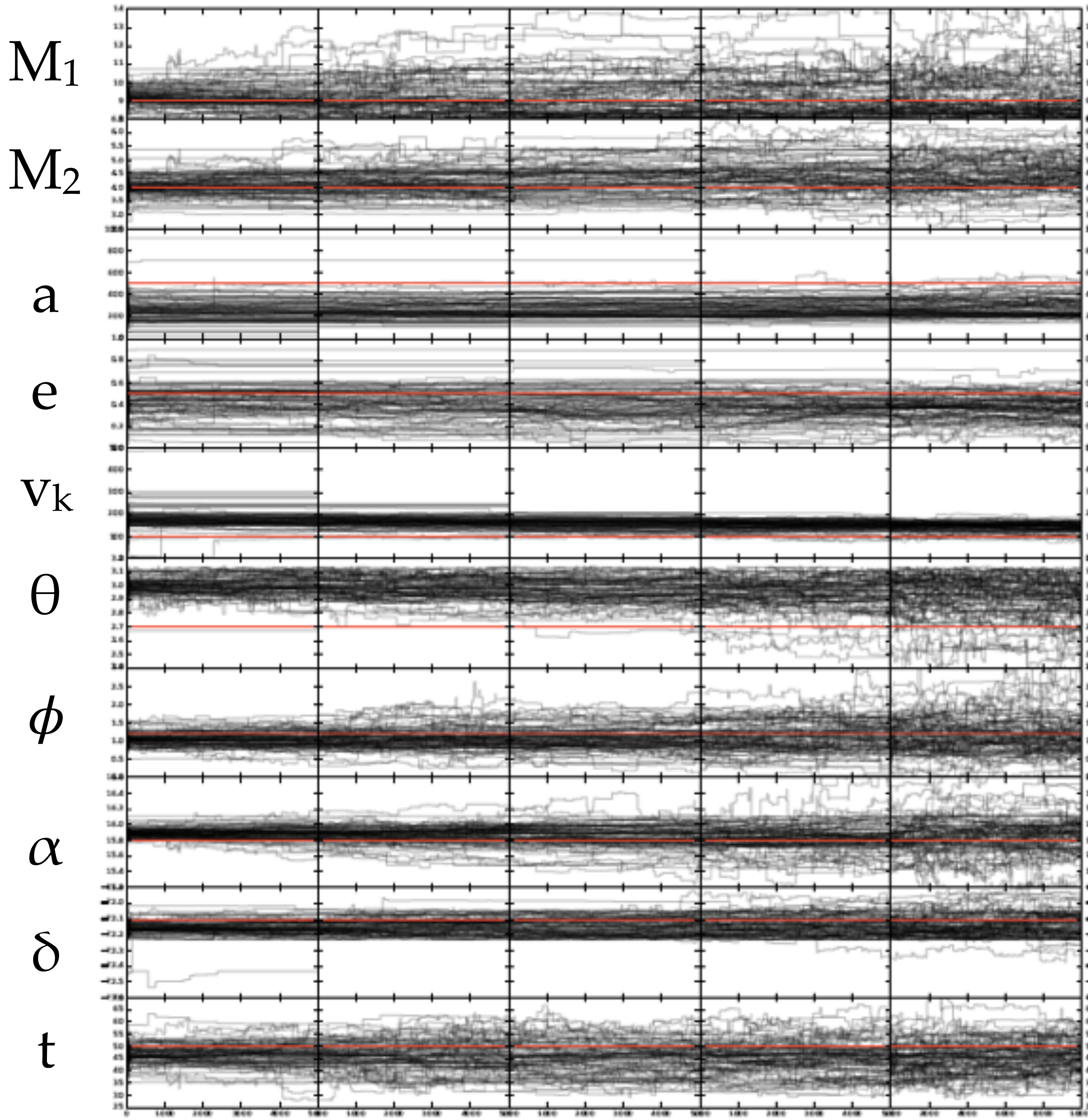
Andrews et al. (in prep.)

SMC HMXB population



Andrews et al. (in prep.)

Test system 2 - Multiple burn-ins



Run with 4 separate burn-ins - 5000 steps each

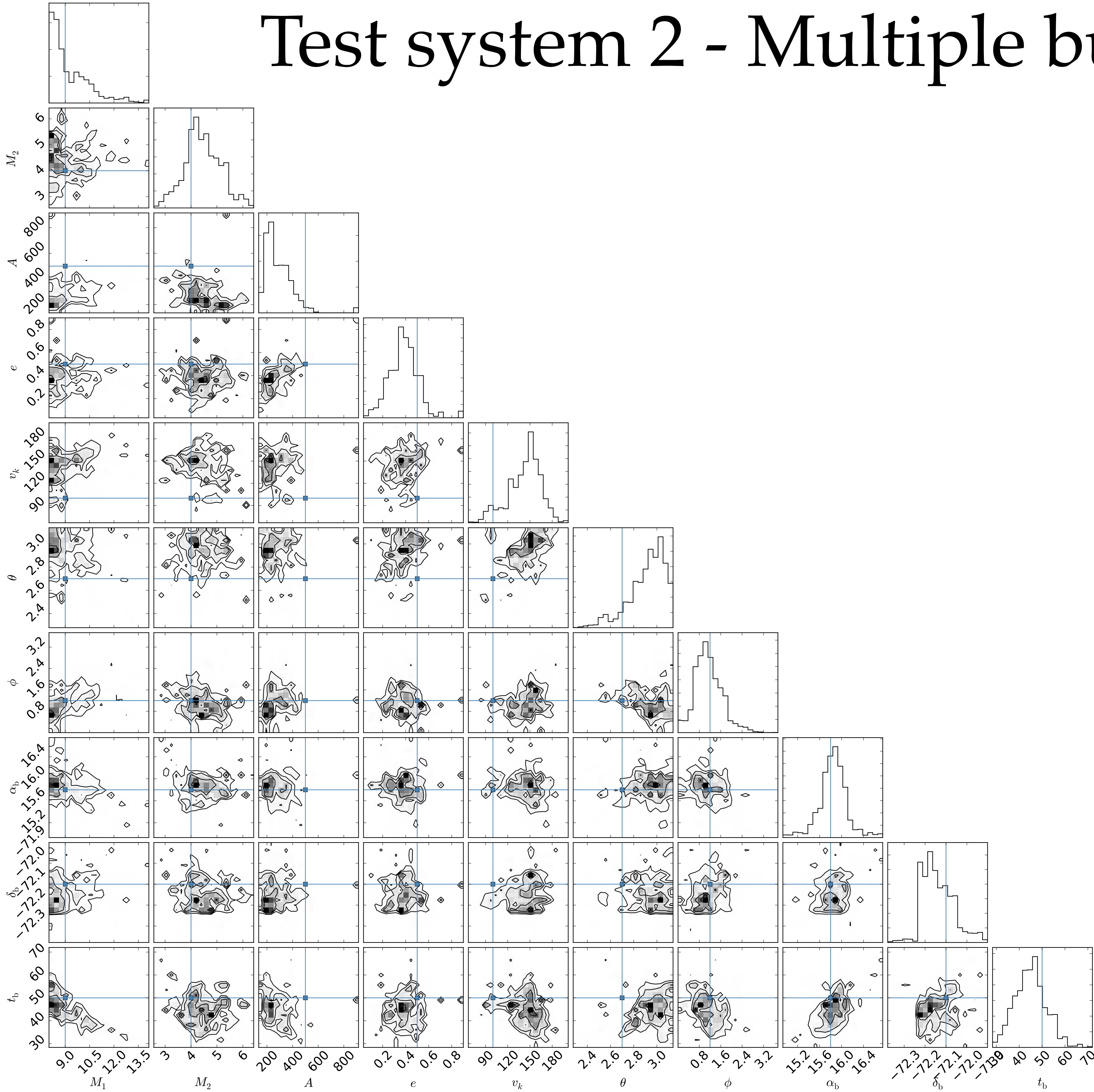
After first 3 burn-ins, move lowest posterior probability chains

Last burn-in should spread distribution around parameter space

Run chains for 10,000 steps

Andrews et al. (in prep.)

Test system 2 - Multiple burn-ins



Discussion questions / future directions

Question 1: How do I add multiple evolutionary channels?

Question 2: How do I simulate multiple systems simultaneously?

Question 3: Evaluate goodness of model?

Making the model hierarchical - if time

Currently, I can calculate individual posterior probabilities separately and combine after

$$P(\vec{x}_f|M) = \int d\vec{x}_i \; P(\vec{x}_f|\vec{x}_i, M) \; P(\vec{x}_i|M)$$

$$P(\{\vec{x}_f\}|M) = \prod P(\vec{x}_f|M)$$

hierarchical parameter priors

Test binary evolution parameterizations

Test binary priors

$$P(M|\{\vec{x}_f\}) \propto \int d\alpha_i \; P(\alpha_i|M) \prod_{\mathrm{all}\ \vec{x}_f} \frac{1}{N} \sum_j P(\vec{x}_f|\vec{x}_{i,j}, \alpha_i, M) \; P(\vec{x}_i|\alpha_i, M)$$

Number of parameters: $10 \times n + \mathrm{len}(\alpha_i)$

Need a new numerical algorithm to handle high dimensionality