Solar DEM Model

$$Y \approx Poisson(\lambda) + N(0, \sigma_b^2)$$

$$Y = I_{ijb} \cdot \tau_b \qquad \lambda_{ijb} = \sum \beta_t \cdot M_{ijbt} \cdot \tau_b$$

I_{ijb} : A solar image in color band *b*, m×n pixels, containing a particular solar feature, for b= 1, 2, … B

$M_{bt} = M_{ijbt}$: The expected *Emission Measure in color band b originating at temperature t, for t = 1, 2, . . . , T.*

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β_t : the proportion of the total volume at temperature *t*.

σ_b : computed from the magnitude of the negative values of I_b in the data.

New Solar DEM Model

$$Y \approx Y_1 + Y_2 + 1/4$$

$$Y = I_{ijb} \cdot \tau_b \qquad Y_1 = Z^2 \qquad Y_2 \sim N(0, \sigma_b^2)$$

$$Z \sim N(\sqrt{\lambda_{ijb}}, \varepsilon^2) \qquad \lambda_{ijb} = \sum \beta_t \cdot M_{ijbt} \cdot \tau_b$$

This will ensure that will never be negative, and using the Normal distribution will help with implementing the Gaussian Random Field model on the spatial correlations.

EM for This Model

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$$Z \sim N(\sqrt{\lambda_{ijb}}, \varepsilon^2) \qquad \lambda_{ijb} = \sum \beta_t \cdot M_{ijbt} \cdot \tau_b$$

E-Step: E(Z|Y….)

M-Step 1: Given σ_b^2 to maximize $\mu = \sqrt{\lambda_{ijb}}$
M-Step 2: Given μ to maximize β_t

EM for This Model

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$$Y = I_{ijb} \cdot \tau_b \qquad Y_1 = Z^2 \qquad Y_2 \sim N(0, \sigma_b^2)$$

$$Z \sim N(\sqrt{\lambda_{ijb}(t)}, \varepsilon^2) \qquad \lambda_{ijb}(t) = \sum \mu_t \cdot M_{ijbt} \cdot \tau_b$$

Problem:
The M-step can't go well.
M-Step1: goes slow because of too many pixels;
M-Step2: We use Fisher Scoring, and it converges poorly because of positive constraints of $\lambda_{ijb}(t)$

MCMC for This Model

$$Y \approx Y_1 + Y_2 + 1/4$$

$$Y = I_{ijb} \cdot \tau_b \qquad Y_1 = Z^2 \qquad Y_2 \sim N(0, \sigma_b^2)$$

$$Z \sim N(\sqrt{\lambda_{ijb}(t)}, \varepsilon^2) \qquad \lambda_{ijb}(t) = \sum \mu_t \cdot M_{ijbt} \cdot \tau_b$$

Step 1: $f(Z \mid Y, \beta, M, \varepsilon^2, \sigma_b^2)$

Step 2: $Y_1 = Z^2$ Step 2': $f(\beta \mid Z, M, \varepsilon^2)$

Step 3: $f(\sigma_b^2 \mid Y, Y_1)$ Step 3': $f(\varepsilon^2 \mid Z, M, \beta)$

MCMC for This Model

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$$Y = I_{ijb} \cdot \tau_b \qquad Y_1 = Z^2 \qquad Y_2 \sim N(0, \sigma_b^2)$$

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I use both Gibbs Sampling and Metropolis-Hastings Algorithm.

Problem:

1. Each iteration costs too much time.
2. Acceptance Rate is too low.