Systematic Errors

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Alanna Connors, David van Dyk, Jin Xu,
Aneta Siemiginowska, Xiao-Li Meng, Shandong Min,
Andreas Zezas, Taeyoung Park, Hyunsook Lee, Rima Izem
Jeremy Drake, Pete Ratzlaff

Pretty good. Too many gross errors and mistakes [illegible]

The line between good humored exuberance and presumptuous cuteness is not always clear. Unless you are pretty sure of your mastery of the subject it is usually better to err on the side of conservatism.

Hydrogenic Optical Emission Spectra

Experiment Number 17

ὁρῶμεν τούτοις . πάντες λάμπουσι καὶ θαυμάζομεν τὸ τοῦ πάντα καλὸν μόνον.

MIT 8.13 Physics Junior Lab

Elizabeth Cavicchi
Alanna Connors
8.13 Junior Physics Lab
October 21, 1976

Error Analysis

Photographs of discrete spectral lines lend themselves to systematic error rather than random statistical fluctuations. For example, the lines on the spectrograms were not well resolved, so our measurements were only to within ± .05 cm. A dispersion of about 15 Å/mm and a slit width of about .004 cm. gives a line width of ~.06 cm, which was roughly the same as our uncertainty in measurement. Also, each time we put a new exposure on one film, (as will for example the film that has Hg, H, and D on it) we had to realign the light source to make sure it reached the mirror. This involved closing the shutter slides and taking the top off the Cenco spectrograph. All that banging around could have, say, jolted the mirror slightly, causing our reference Mercury green line to be off somewhat from the Hydrogen, Deuterium, and Rubidium.

We also had trouble with spurious lines, such as those yellow ones in the Hydrogen spectrum. Presumably they were due to stray light from nearby sources used in other experiment. In the Rubidium, the exposure was so long that even faint 'outside' lines might show up. Indeed, it was hard to tell which lines corresponded to different [illegible] series. (i.e. n, different) and which were due to just spin-orbit splitting.

Calculating uncertainty:

For each measurement we had ±.05 cm

a] Hydrogen Balmer series:

Calculating dispersion from Hg lines gives 3.75 ± .05 ⇒ ± 1.5%

∴ the uncertainty in the dispersion = ± 1.5%

Since the uncertainty in the H measurements was ± 1.5%, the total uncertainty for each wavelength was ± 3%

Calculating the slope involved adding, squaring and subtracting 4 terms. so for the slope, total uncertainty was ± 12%

b] [illegible] For this, since our uncertainty (± .05 cm.) was roughly equal to our measurement, .06 ± .01 cm., we can only expect an order of magnitude correlation

c] Rubidium: For this we had a ± .5 cm. difference in the Hg since it was so hard to read, so our uncertainty is ten times that in a] and b], or ± ~20%. for the lines, and ~± 60% for the spin-orbit splitting.

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ACCOUNTING FOR CALIBRATION UNCERTAINTIES IN X-RAY ANALYSIS: EFFECTIVE AREAS IN SPECTRAL FITTING

Hyunsook Lee[1], Vinay L. Kashyap[1], David A. van Dyk[2], Alanna Connors[3], Jeremy J. Drake[1], Rima Izem[4], Xiao-Li Meng[5], Shandong Min[2], Taeyoung Park[6], Pete Ratzlaff[1], Aneta Siemiginowska[1], and Andreas Zezas[7,8]
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ABSTRACT

While considerable advance has been made to account for statistical uncertainties in astronomical analyses, systematic instrumental uncertainties have been generally ignored. This can be crucial to a proper interpretation of analysis results because instrumental calibration uncertainty is a form of systematic uncertainty. Ignoring it can underestimate error bars and introduce bias into the fitted values of model parameters. Accounting for such uncertainties currently requires extensive case-specific simulations if using existing analysis packages. Here, we present general statistical methods that incorporate calibration uncertainties into spectral analysis of high-energy data. We first present a method based on multiple imputation that can be applied with any fitting method, but is necessarily approximate. We then describe a more exact Bayesian approach that works in conjunction with a Markov chain Monte Carlo based fitting. We explore methods for improving computational efficiency, and in particular detail a method of summarizing calibration uncertainties with a principal component analysis of samples of plausible calibration files. This method is implemented using recently codified *Chandra* effective area uncertainties for low-resolution spectral analysis and is verified using both simulated and actual *Chandra* data. Our procedure for incorporating effective area uncertainty is easily generalized to other types of calibration uncertainties.

Key words: methods: data analysis – methods: statistical – techniques: miscellaneous – X-rays: general

Online-only material: color figures

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many cases. That is, we assume that the distribution in the calibration samples differ only in their (loosely defined) average and that differences in their variances can be ignored. Under this assumption, we can easily generate calibration replicates based on the first J principal components as

$$A^{\rm rep} = \bar{A} + (A_0^* - A_0) + \sum_{j=1}^{J} e_j r_j v_j + \xi e_{J+1}, \qquad (12)$$

$$= A_0^* + \delta\bar{A} + \sum_{j=1}^{J} e_j r_j v_j + \xi e_{J+1}, \qquad (13)$$

where A_0^* is the observation-specific effective area that would currently be created by users, A_0 is the nominal default effective area from calibration, $\delta\bar{A} = \bar{A} - A_0$, $\xi = \sum_{j=J+1}^{L} r_j v_j$, and $(e_1, \ldots, e_{J+1})$ are independent standard normal random variables. In addition to the first J principal components, this representation aims to improve the replicates by including the residual sum of the remaining $L-J$ components. Equation (12) shows how we account for A_0^*. If A_0^* were equal to A_0, Equation (12) would reduce to the standard PCA representation. To account for the observation-specific effective area, we add the offset $A_0^* - A_0$. Equation (13) rearranges the terms to express $A^{\rm rep}$ as the sum of calibration quantities that we propose to provide in place of $\mathcal{A}$. In particular, using Equation (13), we can generate any number of Monte Carlo replicates from $\mathcal{A}$, using only $\delta\bar{A}$, A_0^*, $(r_1 v_1, \ldots, r_L v_L)$, and ξ. In this way, we need only

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e.g.,
sample arrays
additive PCA
multiplicative PCA
polynomials
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2D arrays/functions
multiscale components
parametric draws from pdfs

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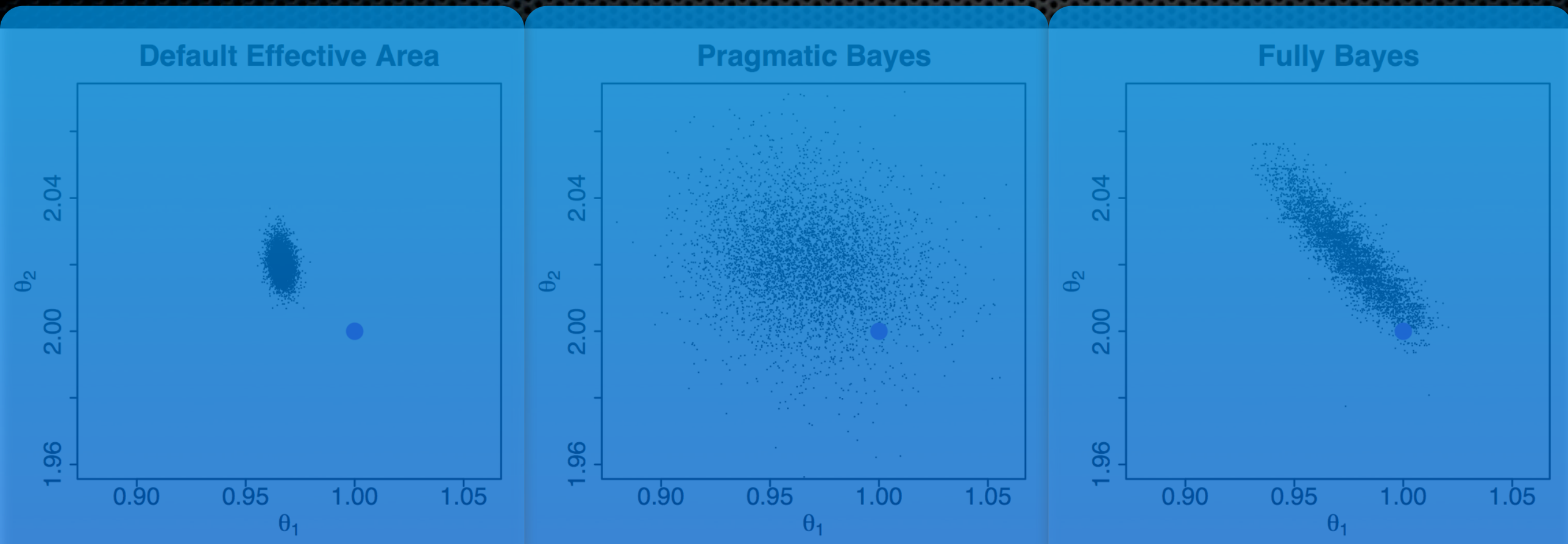
any definable distribution p(A)

fitting to simulated data

$$f(\varepsilon;\theta) = \theta_3\, \varepsilon^{-\theta_1}\, e^{-\theta_2\, \sigma(\varepsilon)}$$



(David van Dyk & Jin Xu)

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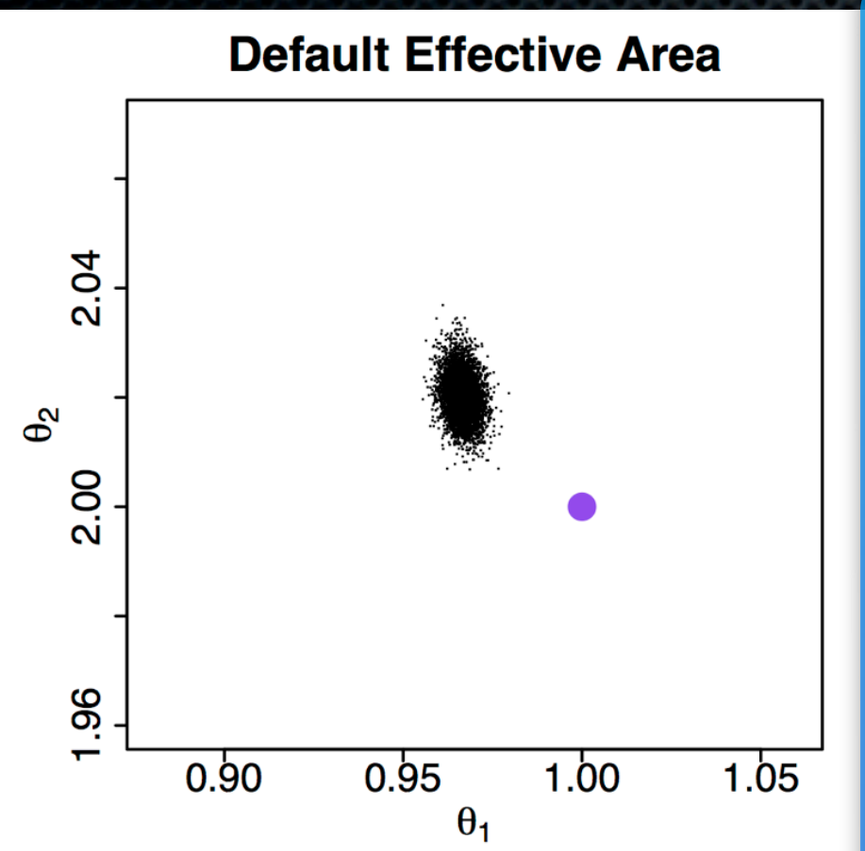

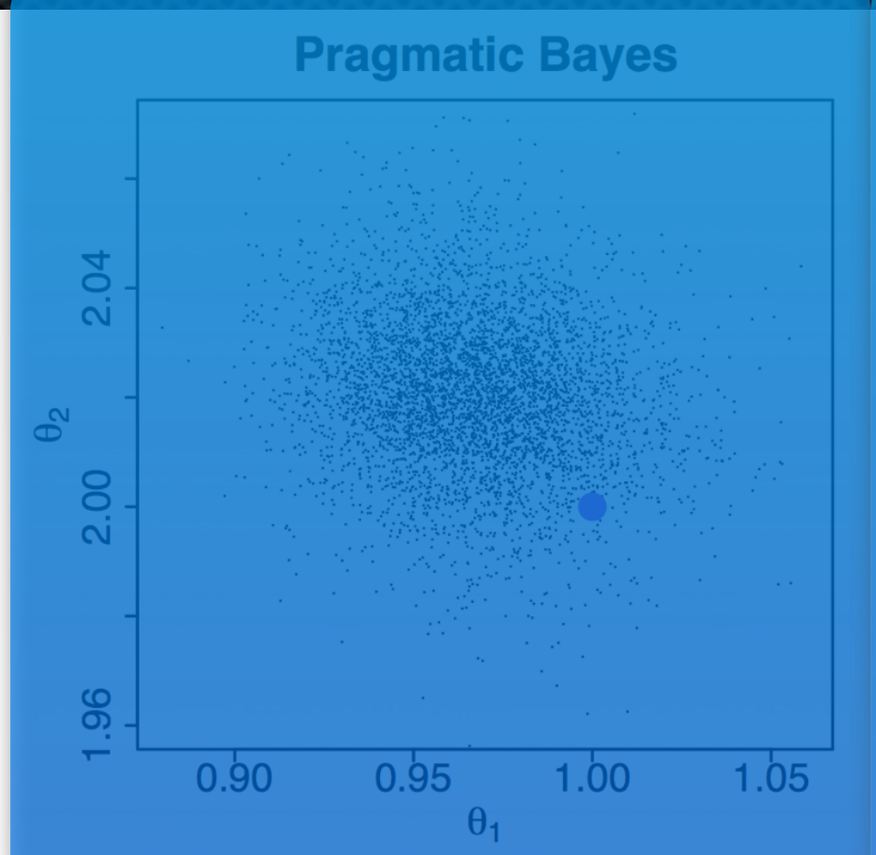

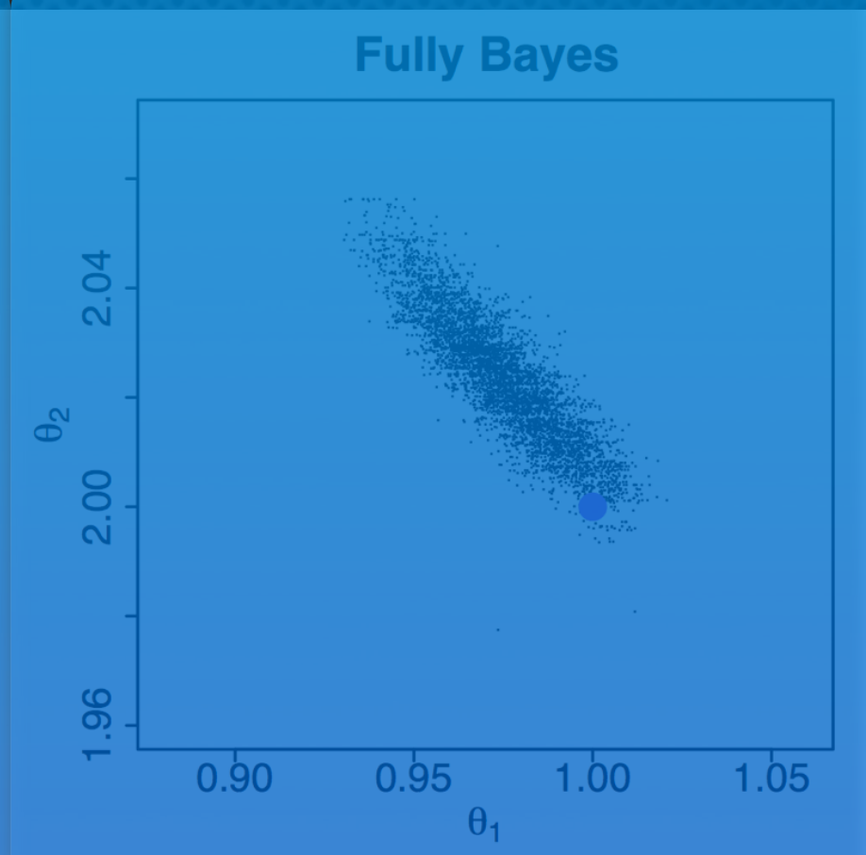

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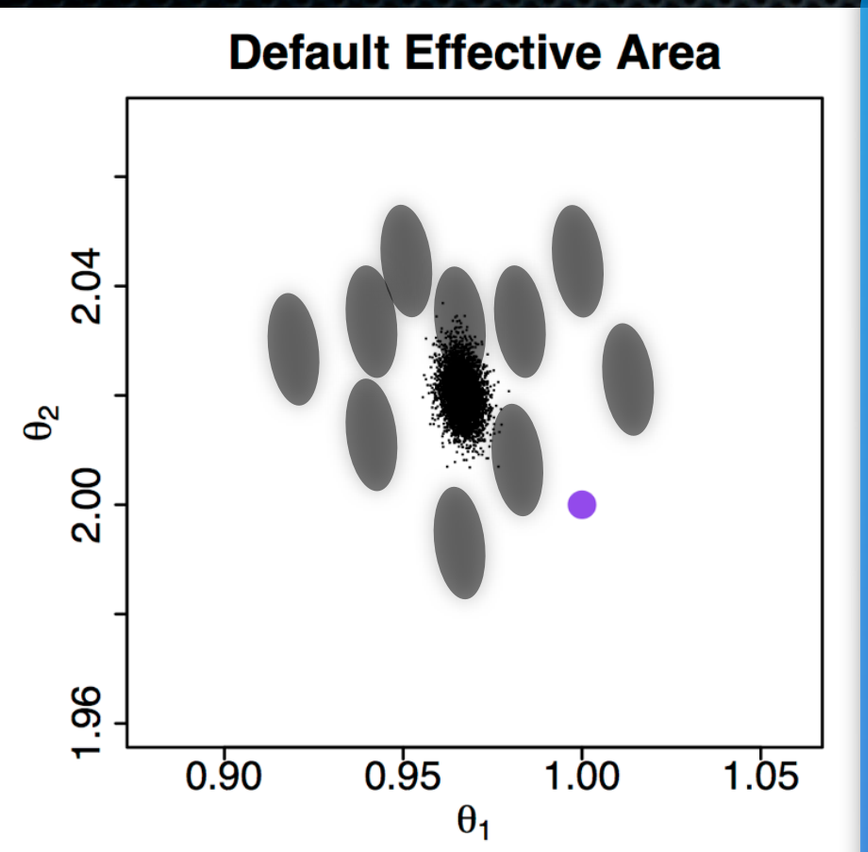

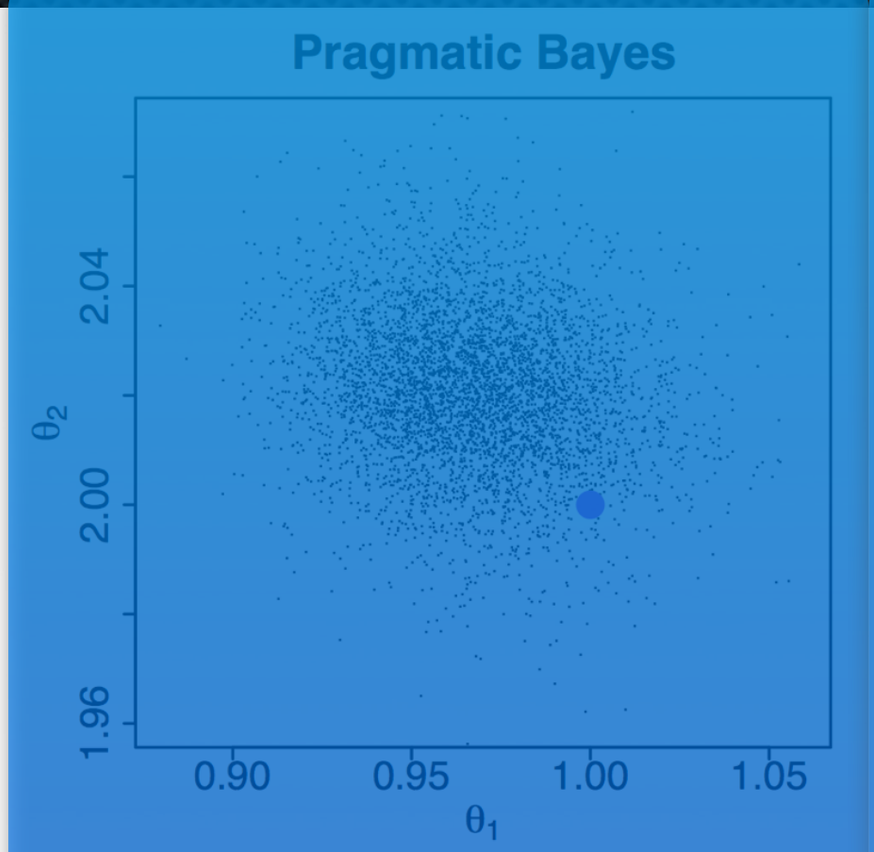

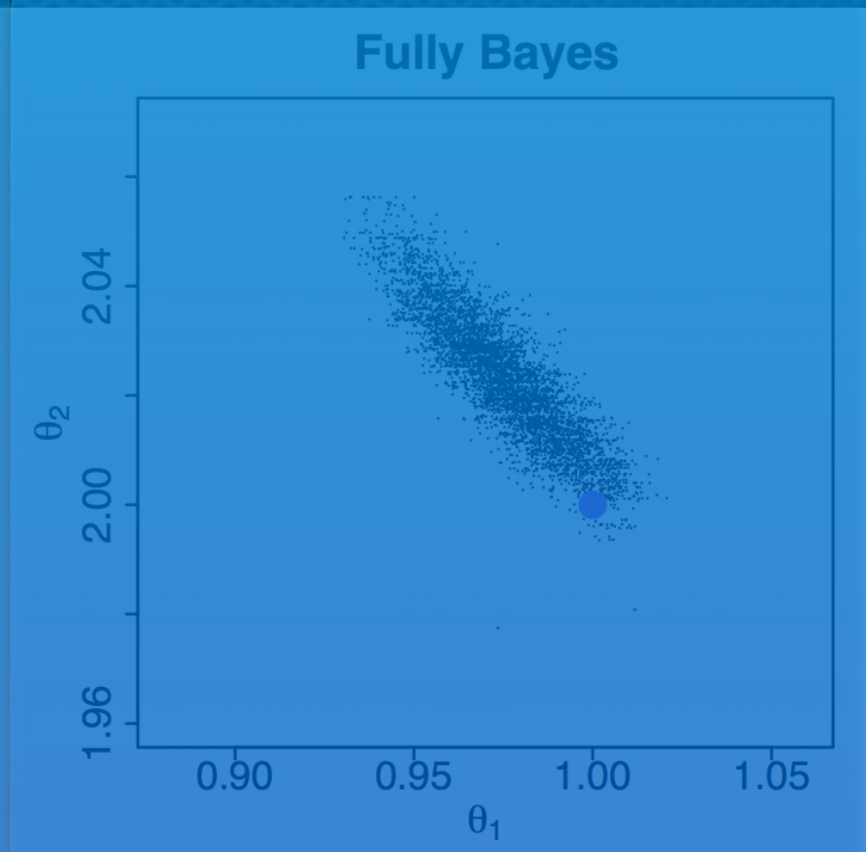

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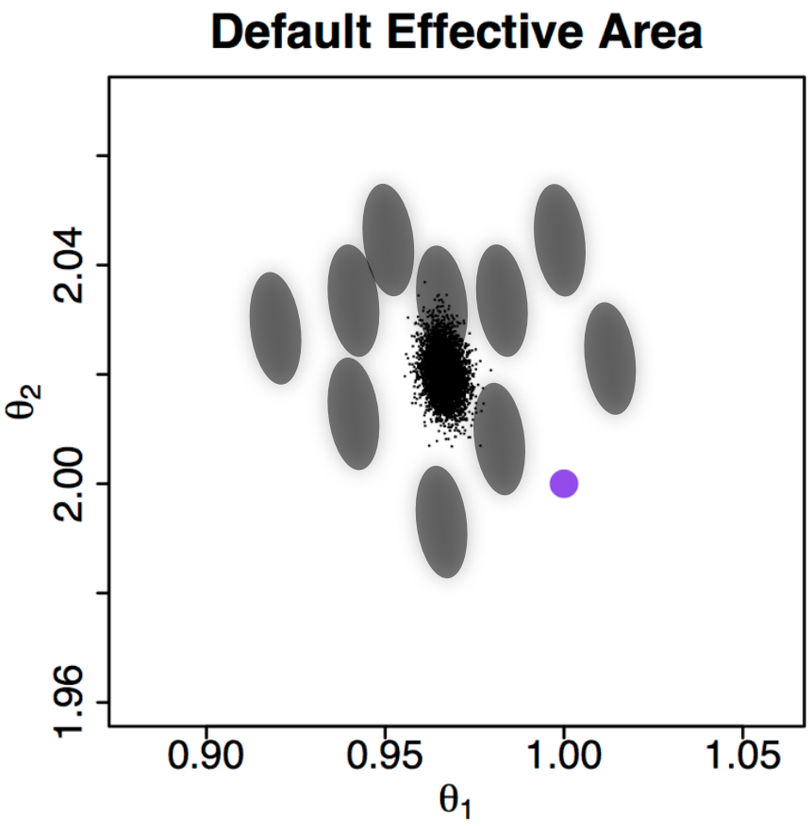

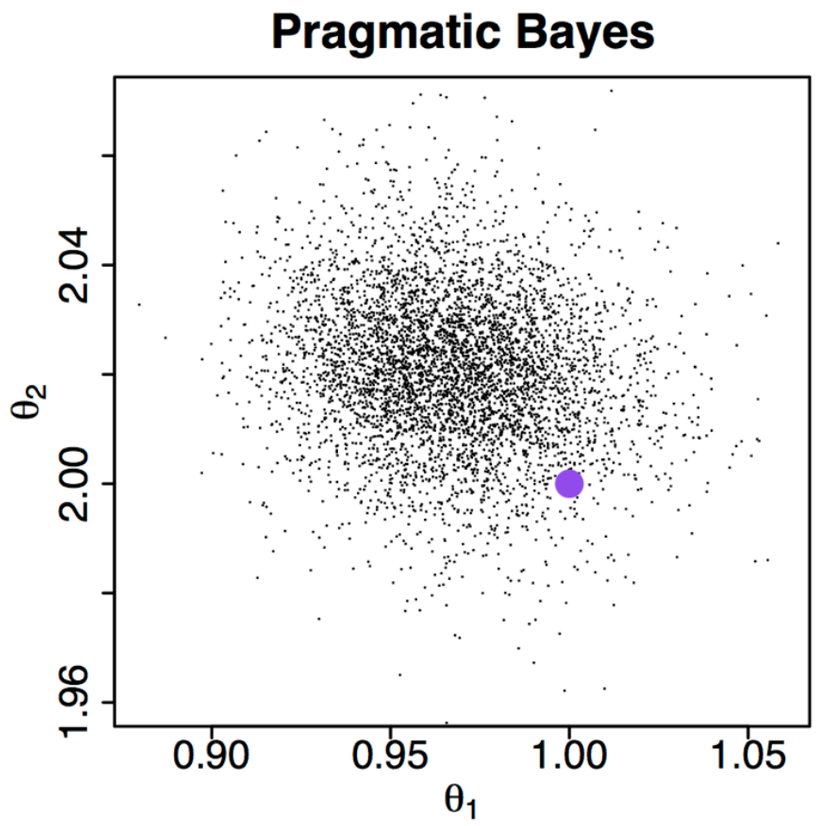

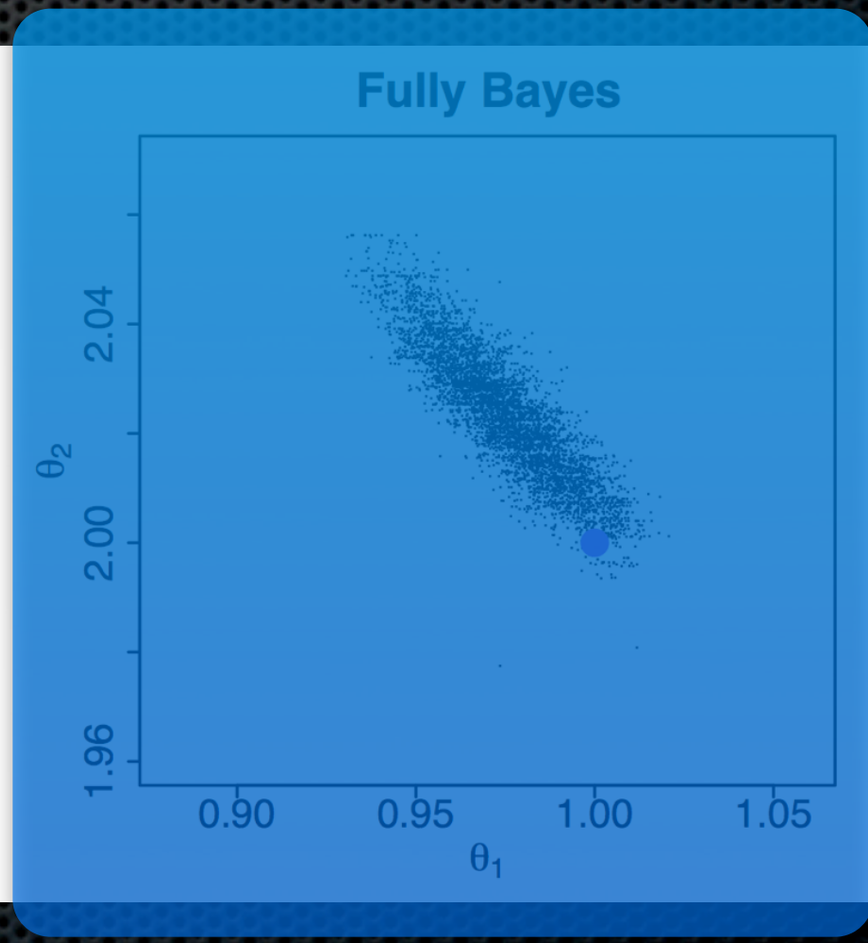

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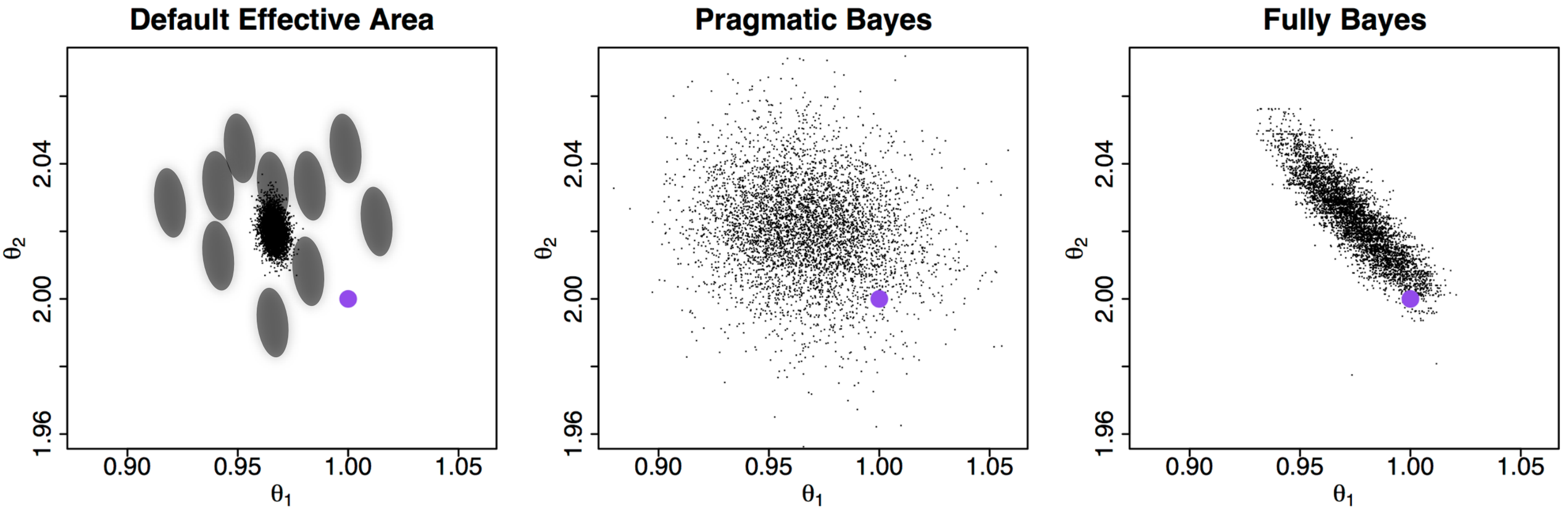

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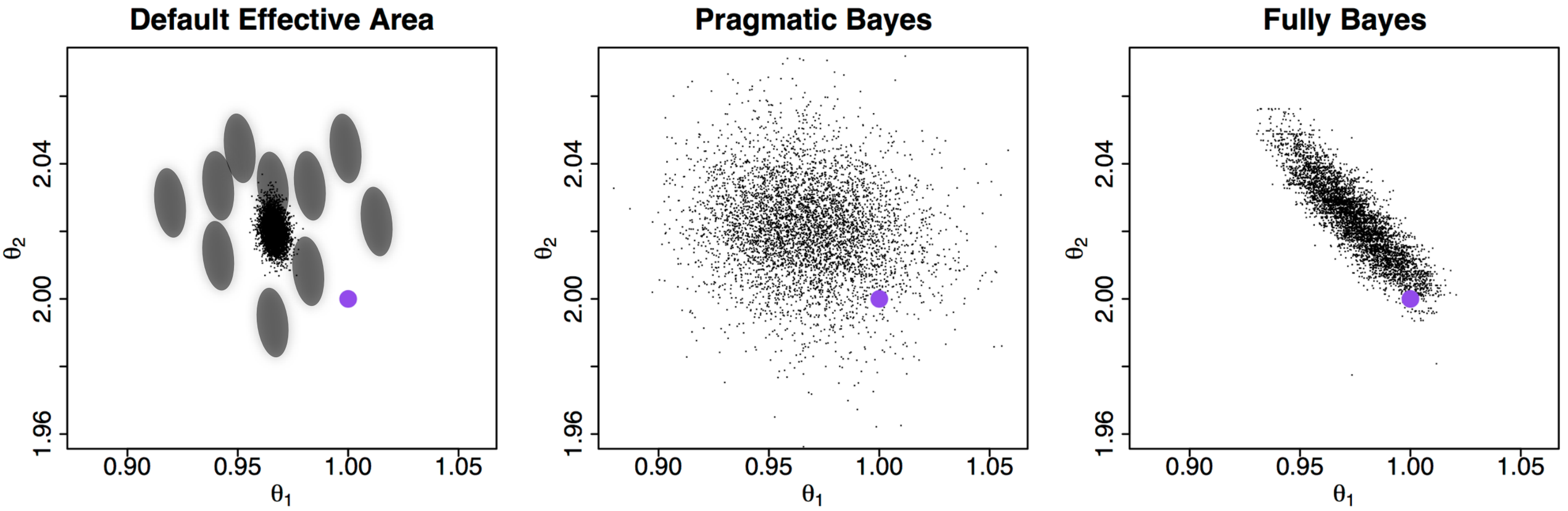

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