Uncertainty Quantification of Object Boundaries Extracted from Spatial Point Patterns

Thomas C. M. Lee

Department of Statistics
University of California, Davis

Joint work with Minjie Fan, Vinay L. Kashyap, David van Dyk, Jue Wang and Andreas Zezas

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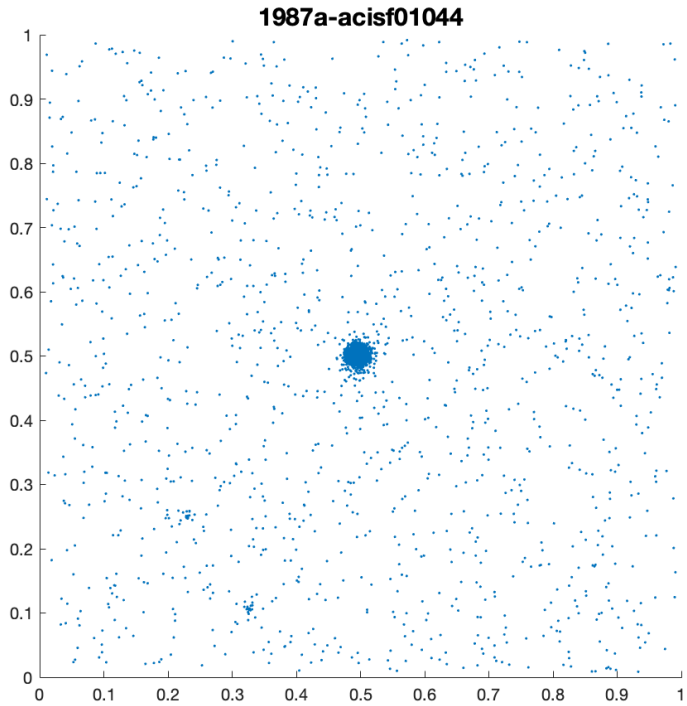
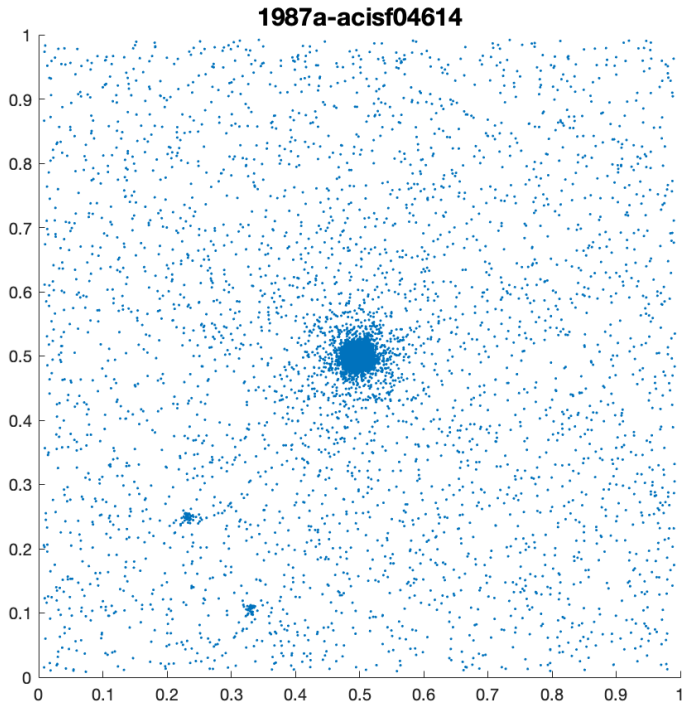
Motivating problem/data: any difference?





observed: 2001-04-25

observed: 2004-01-02

Figure 1: Supernova1987a captured by Advanced CCD Imaging Spectrometer

Segmentation using Seeded Region Growing on Graph (SRGonG)

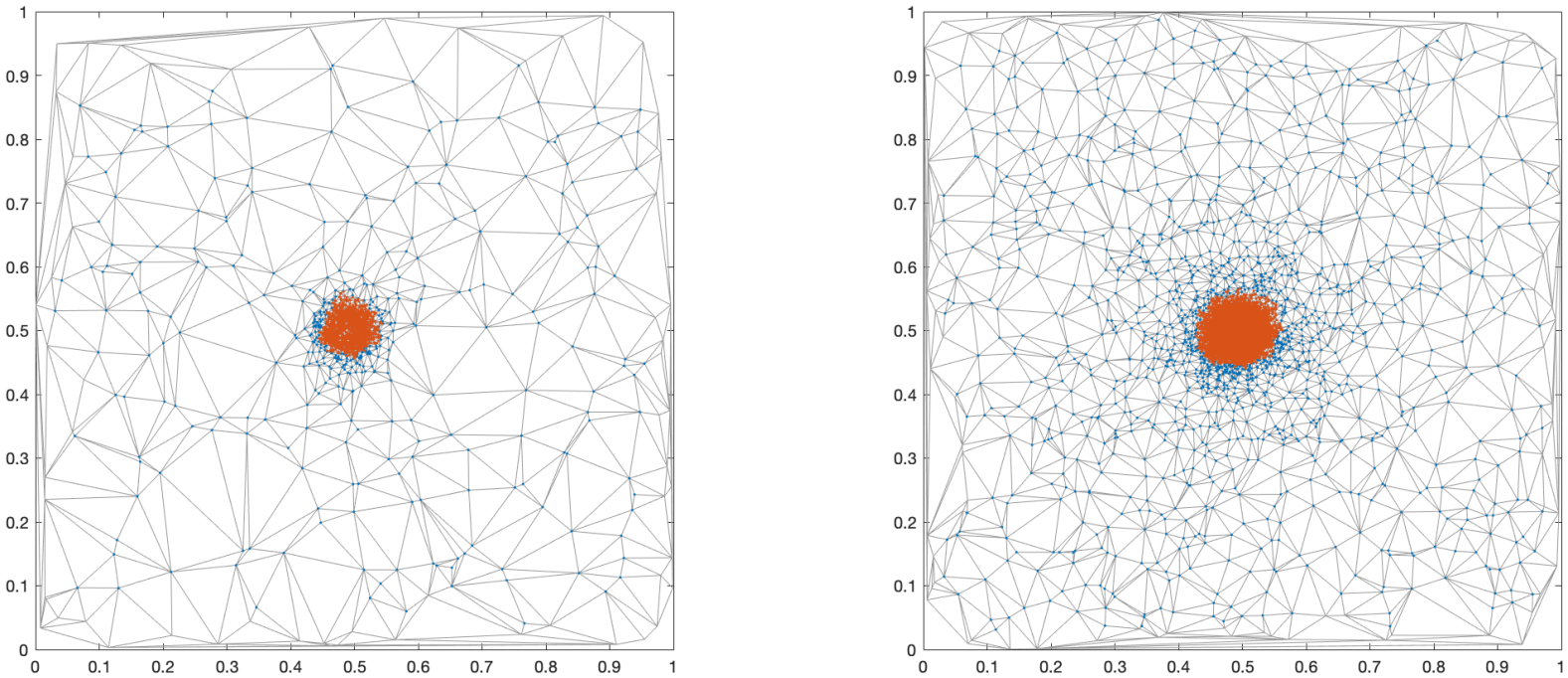


Figure 2: SRGonG segmentation results of Supernova1987a

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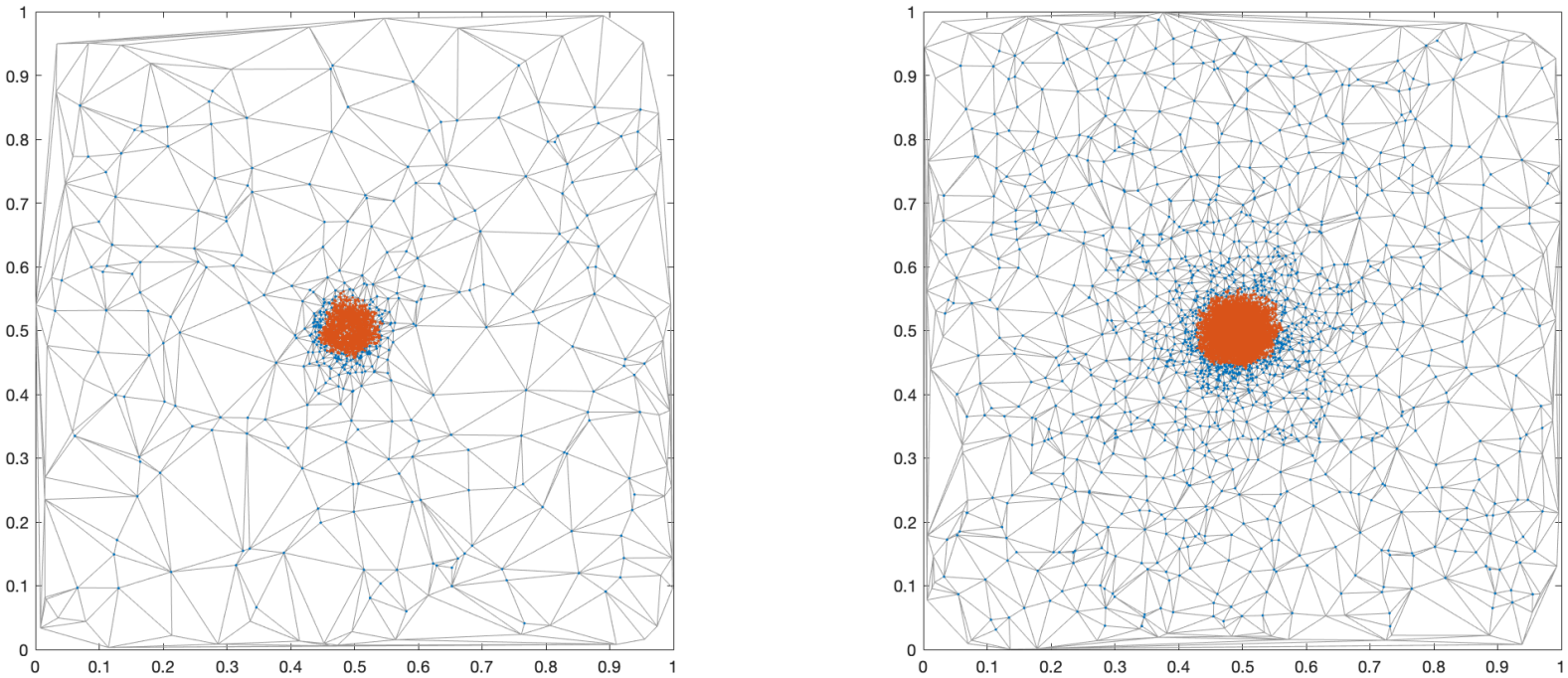

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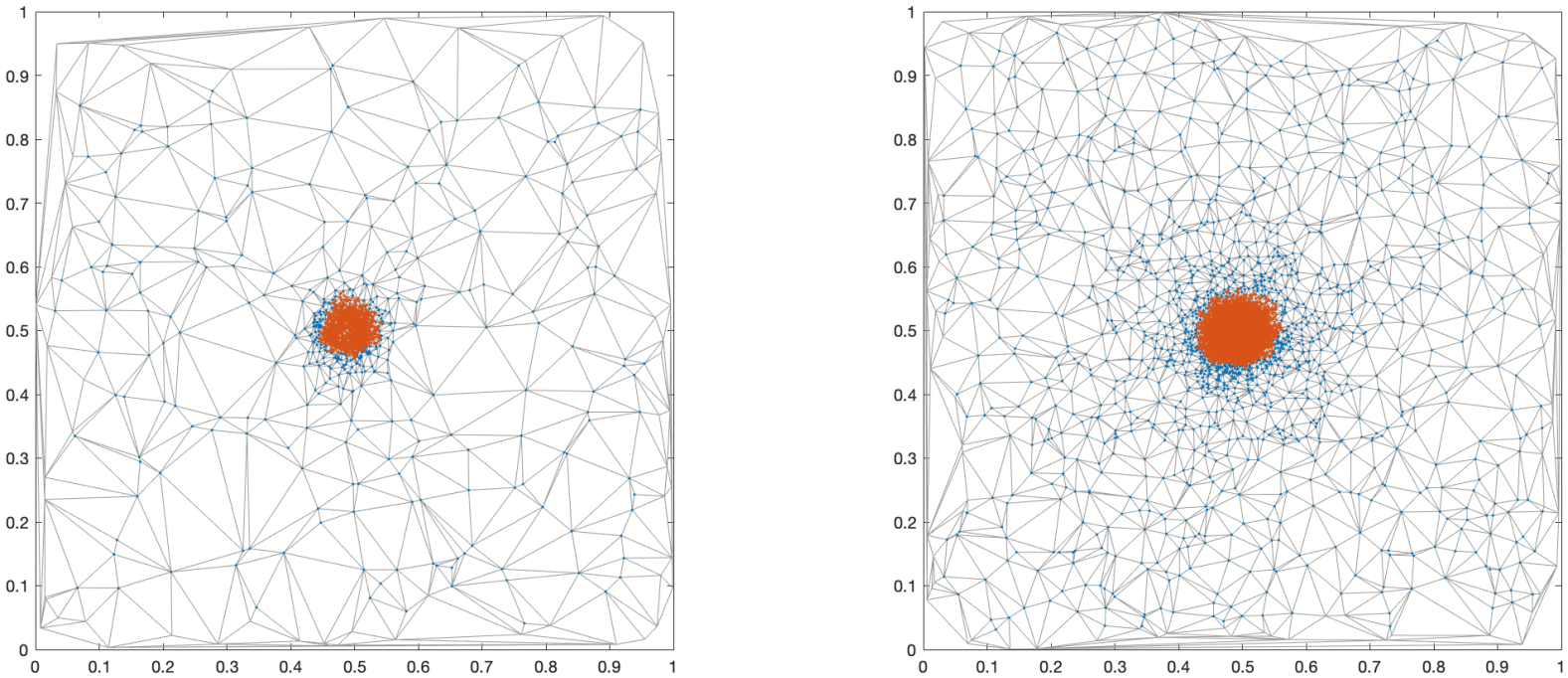

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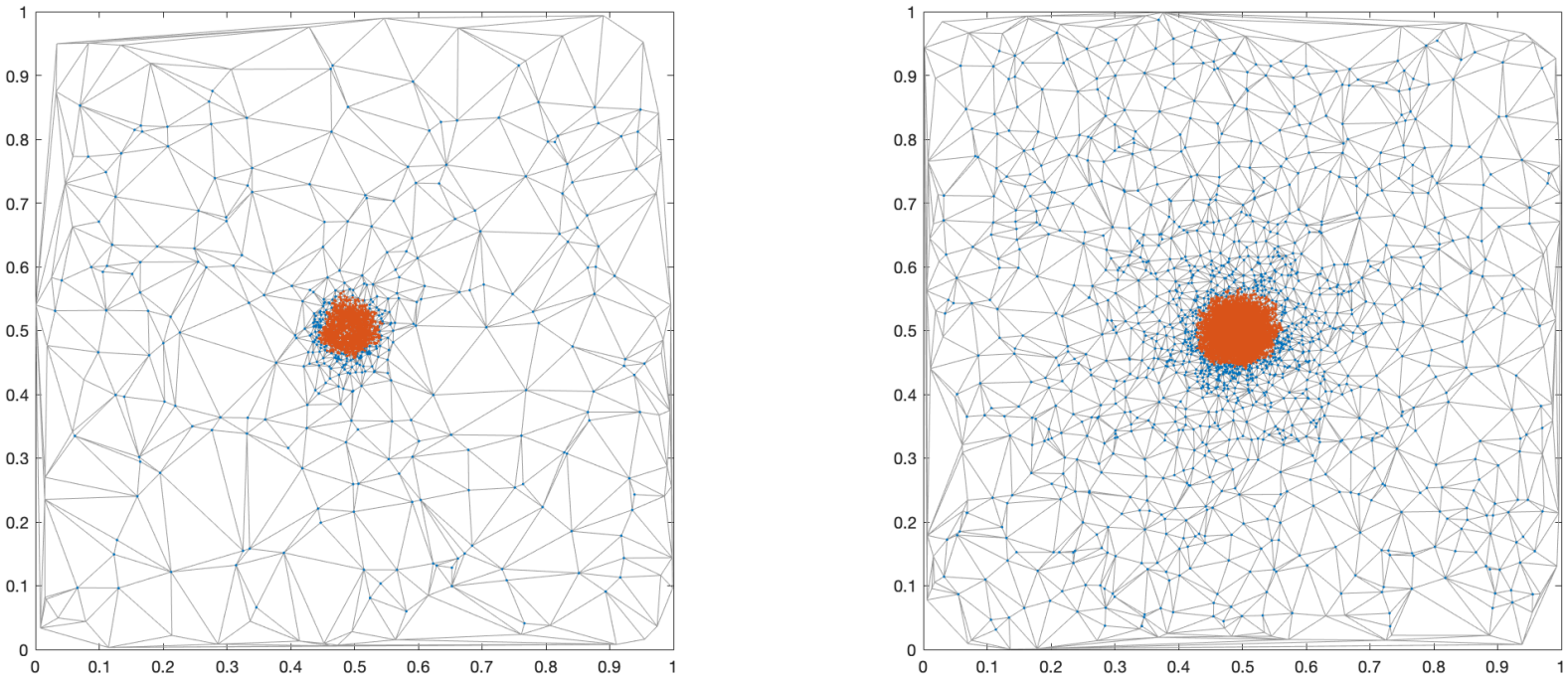

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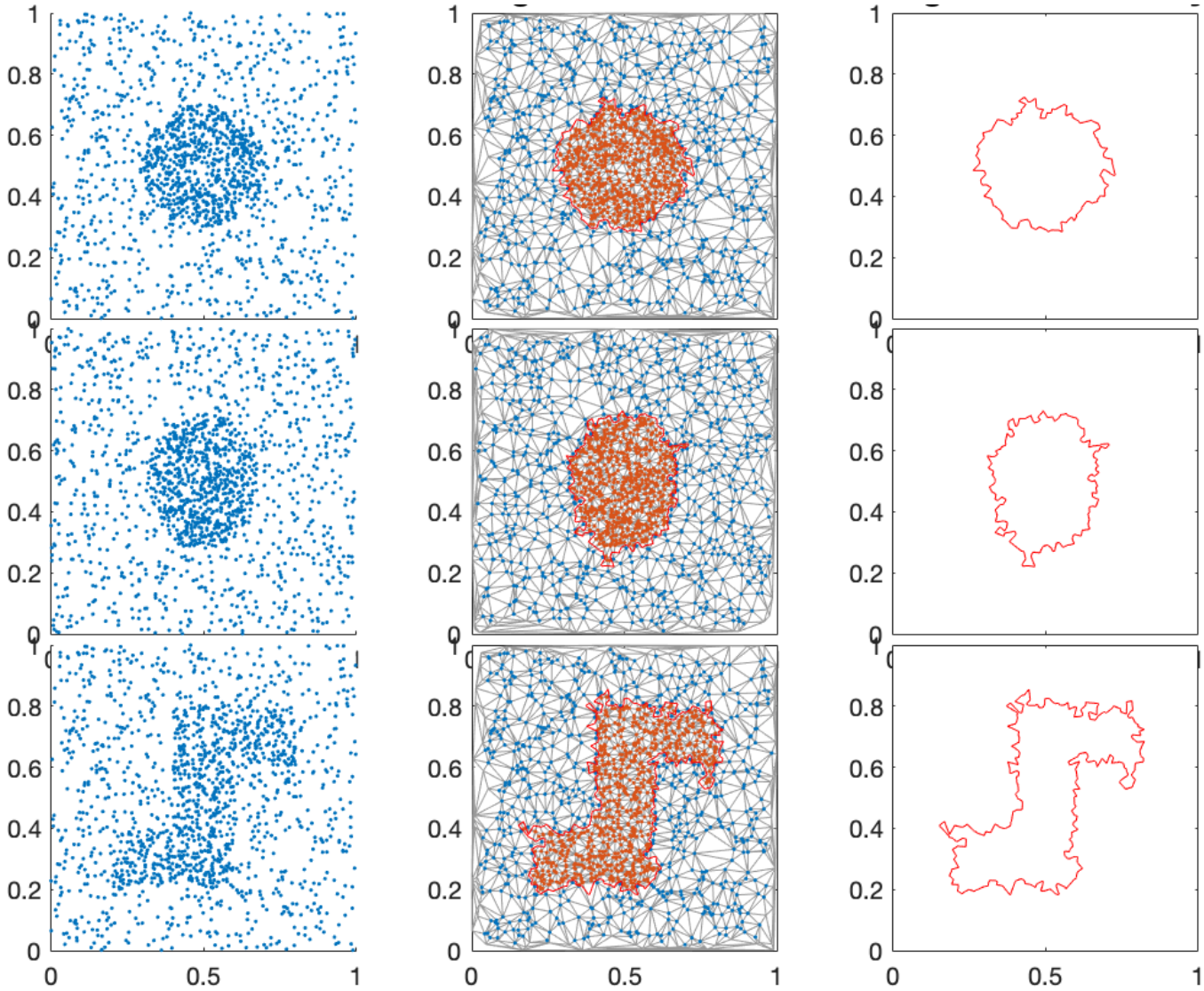
- model with a 2D inhomogeneous Poisson process
- we use SRGonG paired with AIC or BIC

More examples of SRGonG segmentation



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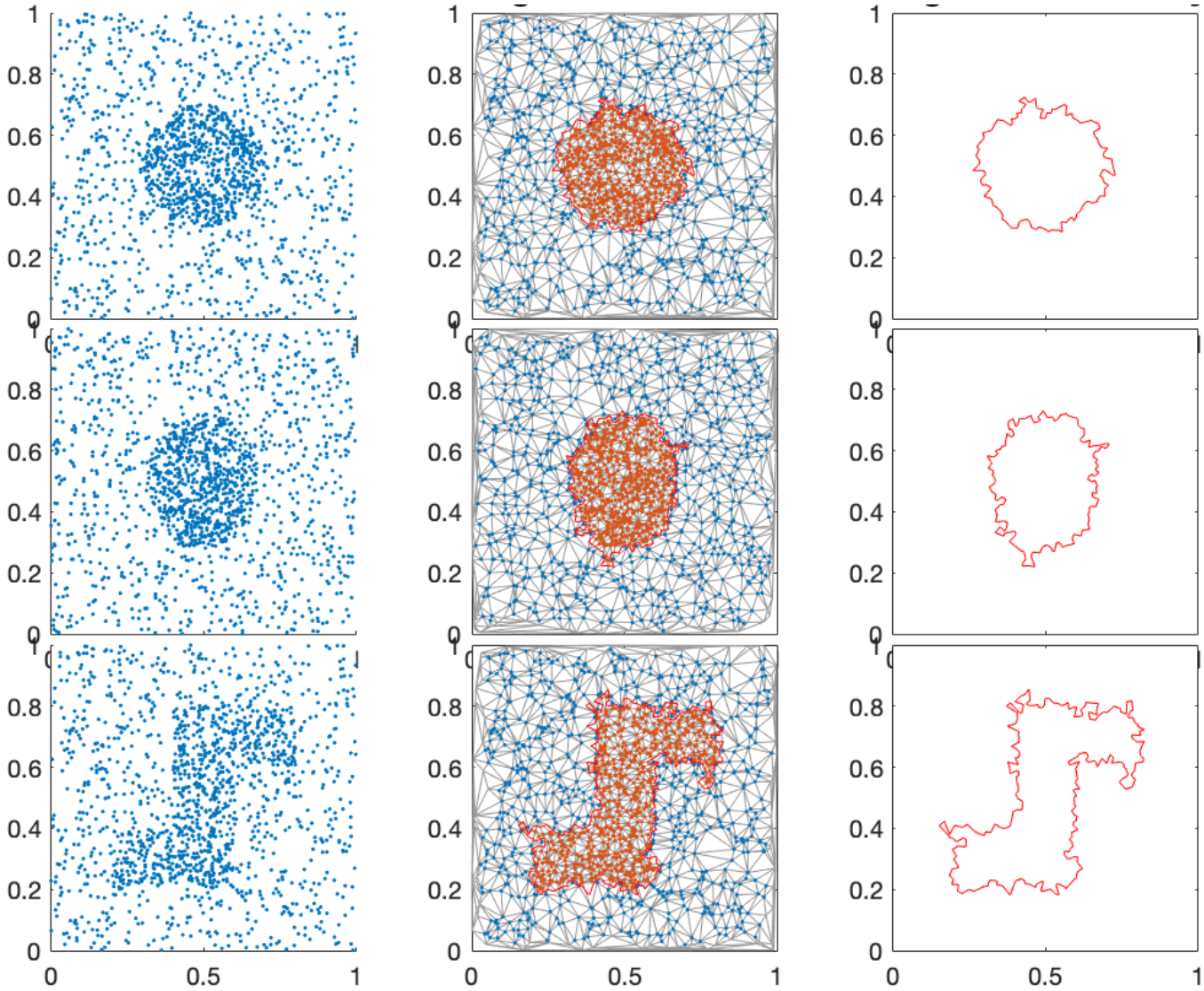

- here we will mostly deal with one object

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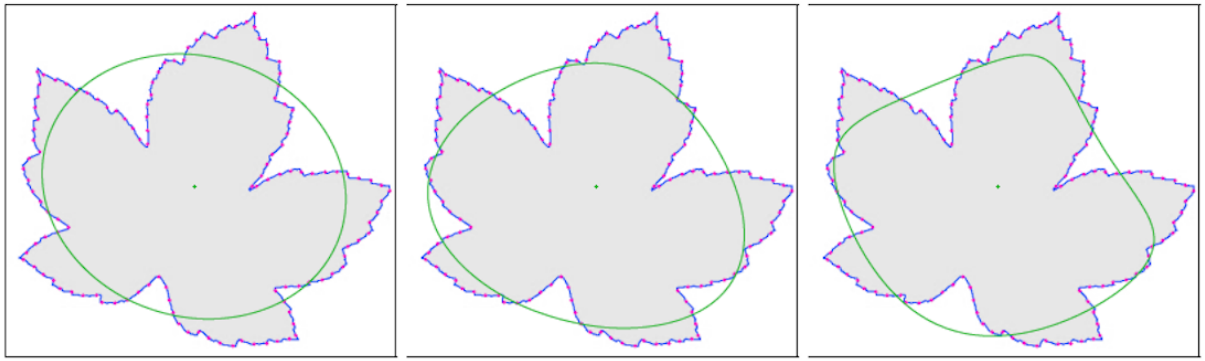
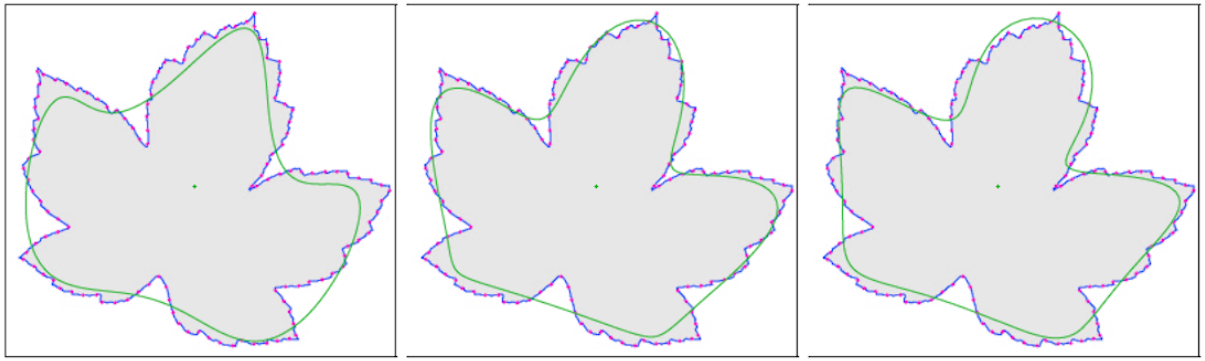
- need to determine how many pairs of sine/cosine bases

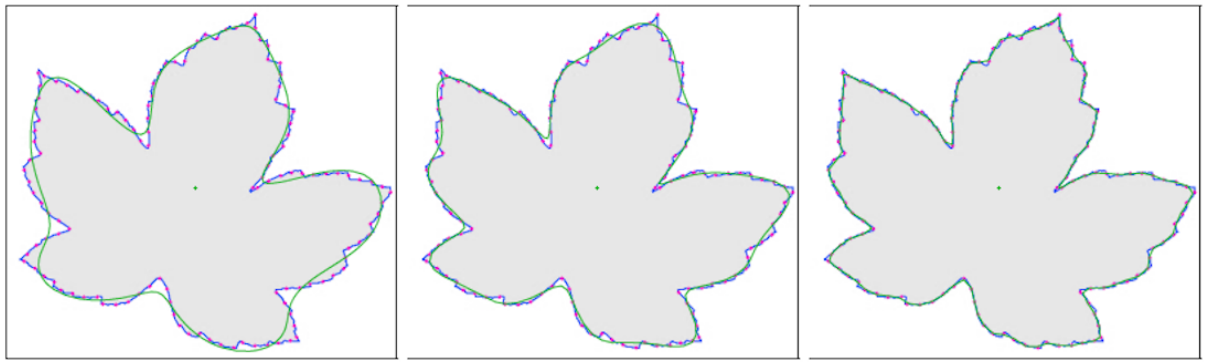
How many pairs?



(a) 1 pair (b) 2 pairs (c) 3 pairs

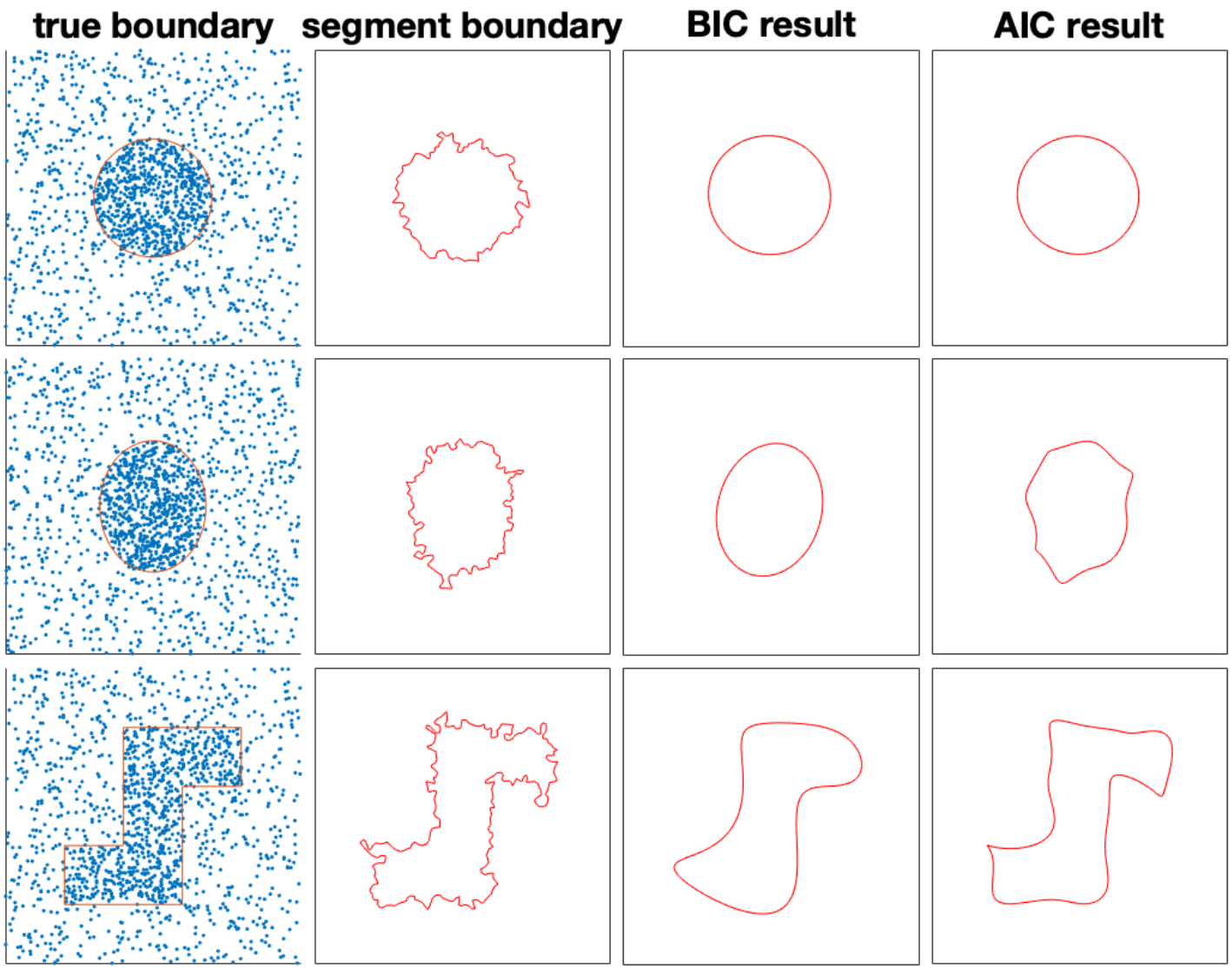


(d) 4 pairs (e) 5 pairs (f) 6 pairs



(j) 10 pairs (k) 15 pairs (l) 40 pairs

Can be chosen by (pseudo) AIC/BIC with profile likelihood



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- we do Monte Carlo testing using non-parametrically bootstrapped data

Simulation results - power curves (ν: sample size; η: snr)

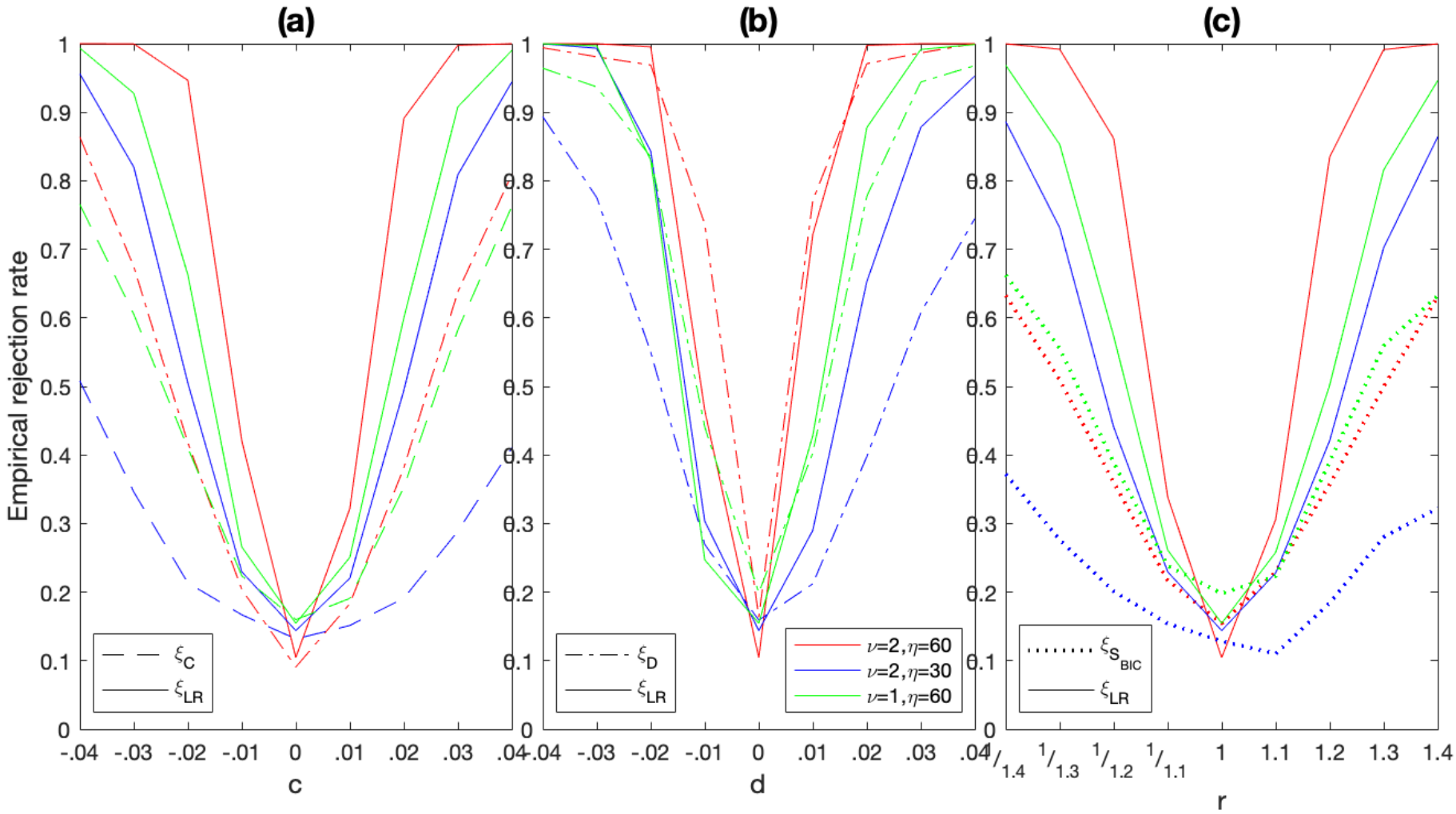
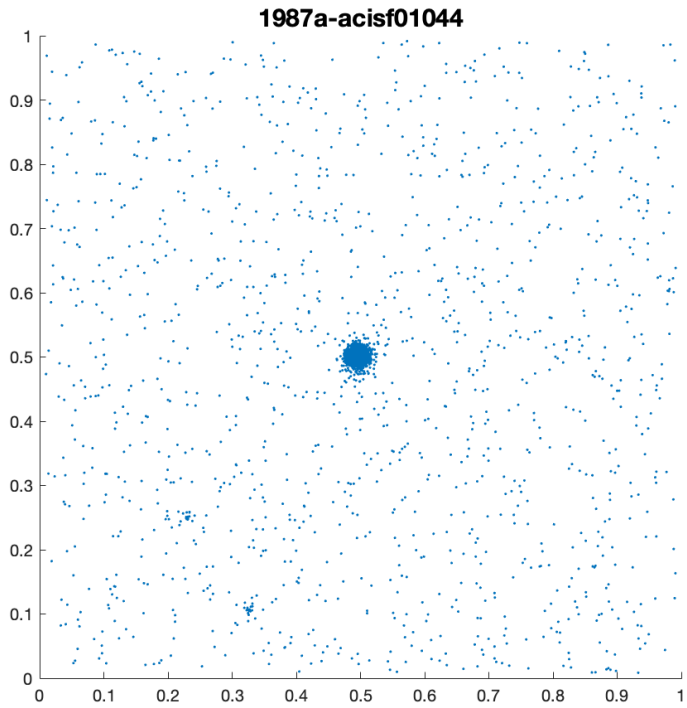


Figure 3: Test for changes in (a) locaton, (b) size, and (c) eccentricity

Back to the motivating problem/data



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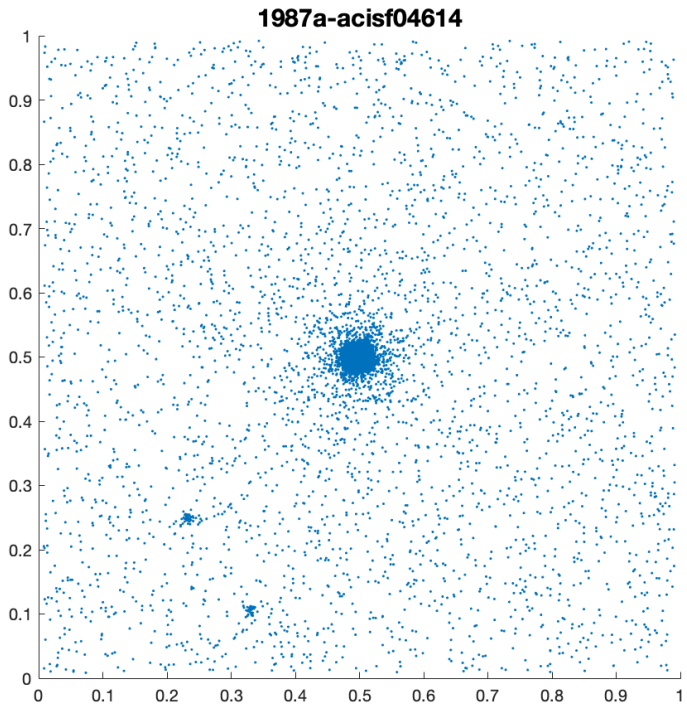

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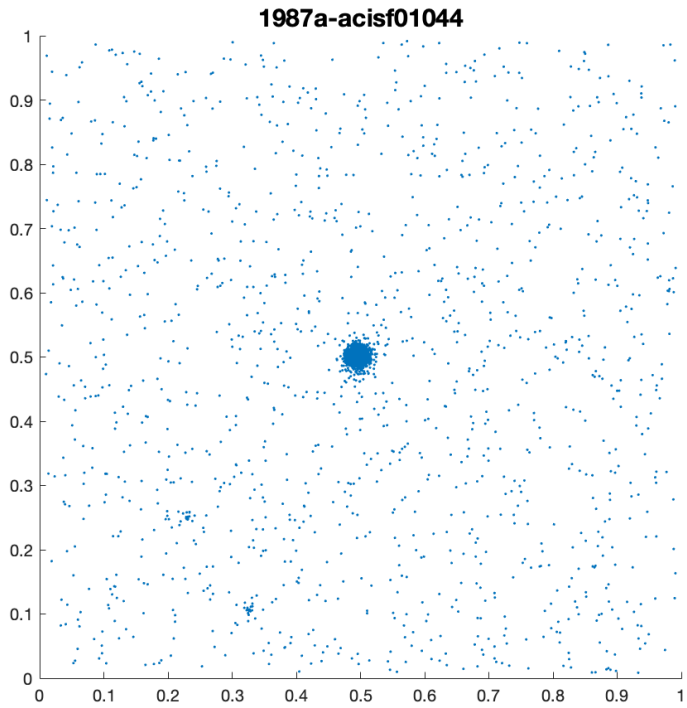

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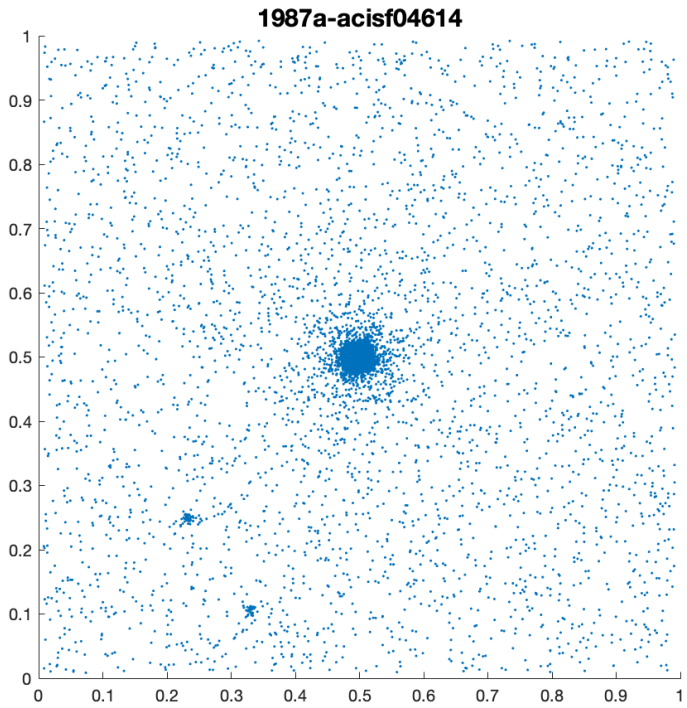

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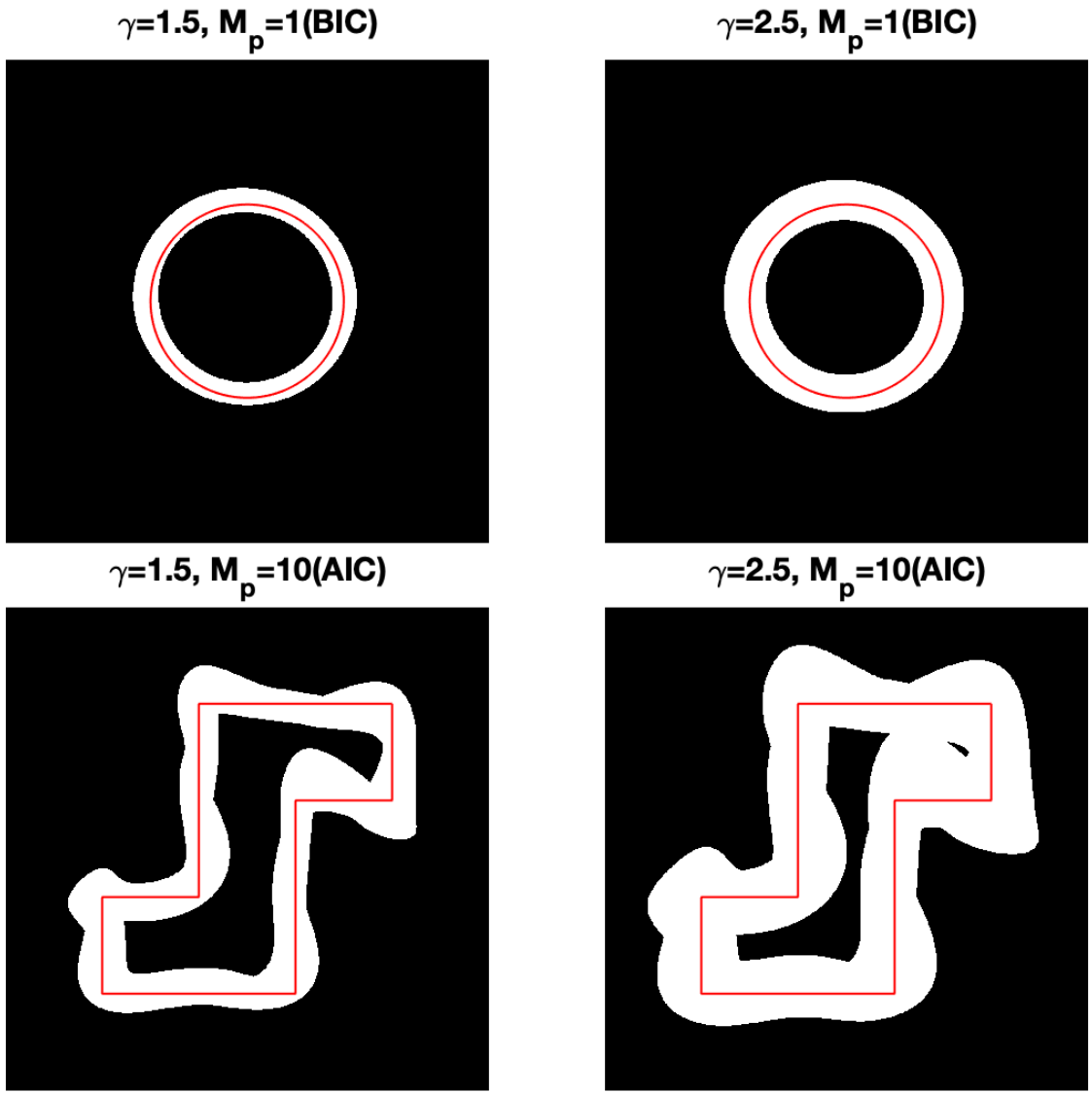
- Test results: there is change in size, but no critical evidence to conclude changes in location or shape.

The notation of "confidence regions:" preliminary results



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- major challenge: more than 1 astonomical object

Data: ngc2300 XMM ("donut")