Statistical Models for Flaring Detection in Astronomical Gamma-ray Light Curves

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Joint work with:
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Data: a light curve of n flux measurements $\{y_{t_i}\}_{i=1}^n$ observed at times $(t_1, \ldots, t_n)$, with $\Delta_i = t_i - t_{i-1}$ not constant.

time t_i	flux y_{t_i} (10^{-7})	error ζ_{t_i} (10^{-7})
54684.00	5.58	2.07
54693.60	1.95	0.83
54695.60	7.56	1.69
54697.60	4.36	1.23
54699.60	5.58	0.92
54701.60	5.23	1.29
54703.60	5.18	1.39
$\vdots$	$\vdots$	$\vdots$

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Goals:

1. derive a statistical model to accurately describe the dynamics of the source emission activity.
2. separate the different states of variability at the base of the light curve, with particular attention to the flares.

Model 1: continuous time hidden Markov model

We model the joint distribution of (Y_{t_i}, S_{t_i}), where:

- $S_{t_i} \in \{1, \ldots, \mathrm{S}\}$ is a latent continuous-time Markov process with initial probability $\boldsymbol{\delta}$ and generator matrix $\mathbf{Q}$.
- Y_{t_i} is the flux at the i-th observation time. We assume that it depends on both the previous flux measurement and the current value of the latent state:

$$Y_{t_i} | S_{t_i}, Y_{t_{i-1}} \sim \mathcal{N}(\mu^*_{i,s}, \sigma^{2*}_{i,s}),$$

where $\mu^*_{i,s}$ and $\sigma^{2*}_{i,s}$ are the mean and the variance of an OU-process parametrized by $(\mu_s, \sigma^2_s, \tau_s)$.

The statistical model

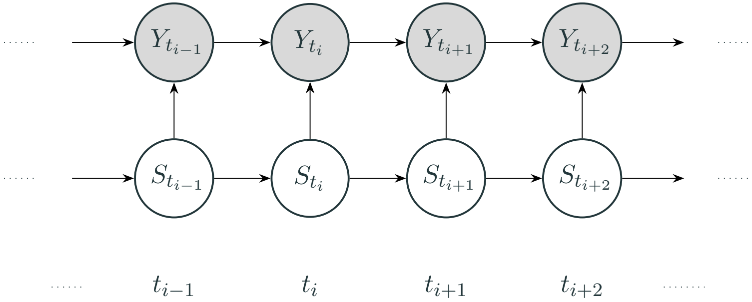

Figure 1: DAG of the continuous time hidden Markov model. Grey circles are the data and white circles represent the latent Markov process.

- S_{t_i} is unknown, and so the inference is carried out using the EM algorithm.
- The distributions of the parameter estimators are assessed using the bootstrap.

Case study: Blasar PKS 1510-05

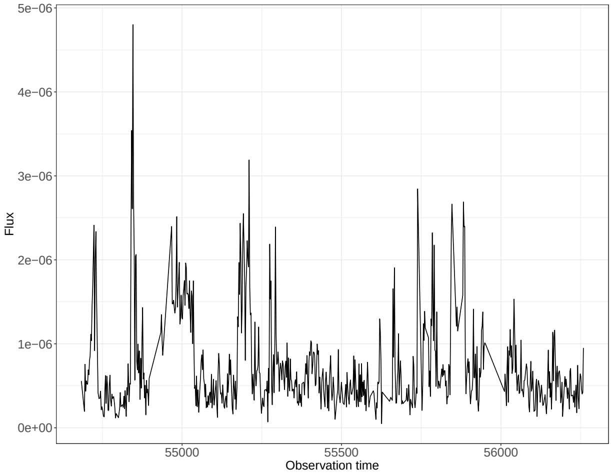

Figure 2: The γ-ray light curve from the blasar PKS 1510-05 recorded by the *Fermi* LAT telescope over 630 observation times. The most frequent time gap is $\Delta_i = 2$, and the largest is $\Delta_i = 60$.

PSF 1510-05: Model fitting

- When $s = 1$, we fit an OU process.
- When $s = 2$, we fit a log-OU process.

Table 1: Estimates of the model parameters in the two latent states. From left to right: mean, square of the volatility, speed of mean reversion and probability to remain in the same state after an interval $\Delta_i = 2$. The standard errors obtained with $B = 200$ bootstrap replicates are given in parenthesis.

	$\hat{\mu}_s$	$\hat{\sigma}_s^2$	$\hat{\tau}_s$	$\hat{p}_{s,s}(\Delta_t = 2)$
$s = 1$	$4.69 \cdot 10^{-7}$	$5.563 \cdot 10^{-14}$	0.699	0.952
	$(1.602 \cdot 10^{-8})$	$(8.822 \cdot 10^{-15})$	(0.08)	(0.012)
$s = 2$	-13.443	0.172	0.522	0.868
	(0.066)	(0.032)	(0.119)	(0.039)

PSF 1510-05: Model fitting

Table 2: Mean and the variance of the limit distributions in the two states.

	$\lim_{\Delta\to\infty} \mathbb{E}_Y$	$\lim_{\Delta\to\infty} \mathbb{V}_Y$
$s = 1$	$4.69 \cdot 10^{-7}$	$3.977 \cdot 10^{-14}$
$s = 2$	$1.576 \cdot 10^{-6}$	$4.446 \cdot 10^{-13}$

PKS 1510-05: Model fitting

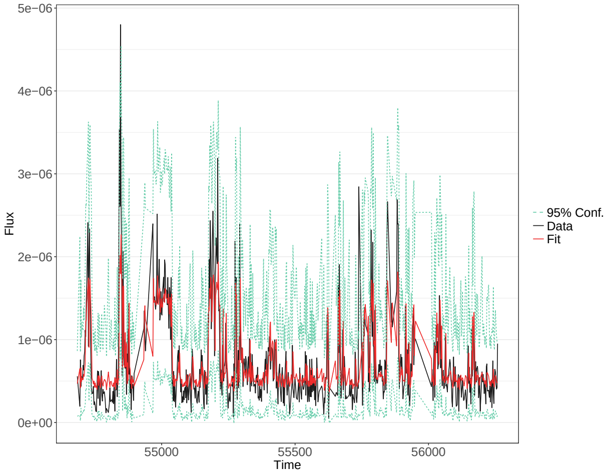

Figure 3: Light curve of the blazar PKS 1510-05 (solid black line) against the mean (solid red line) and 95% confidence interval (dashed lines) of the predictive density.

PKS 1510-05: Model fitting

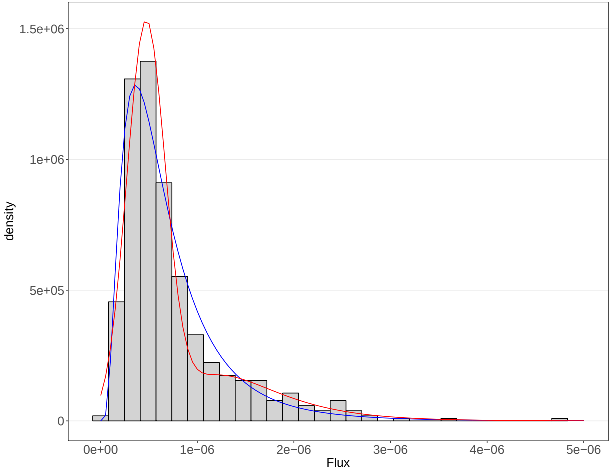

Figure 4: Histogram of the flux compared to the limit distribution of the proposed model (red line) and of a single log-OU process (blue line).

PKS 1510-05: Residuals

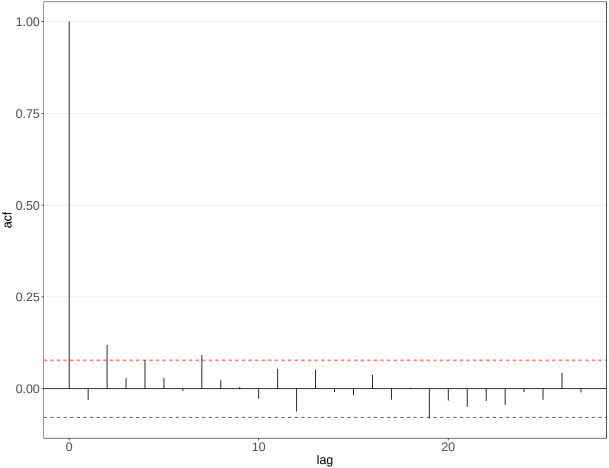

Figure 5: Autocorrelation function of the model residuals.

PKS 1510-05: Flaring probabilities

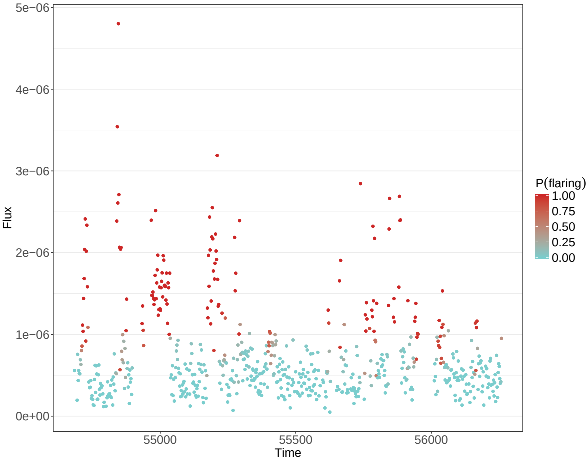

Figure 6: Plot of the classification of the flux measurements. For every data point i, the colour represents the estimated probability of being a flare. A shift toward red states that the observation is more probable to come from the flaring activity.

PKS 1510-05: Maximum of the process

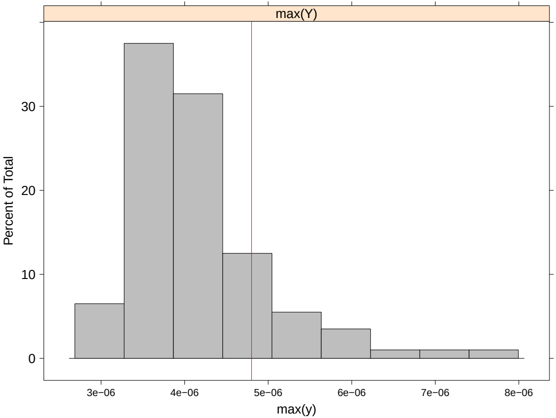

Figure 7: Distribution of $\max(Y)$ based on $B = 200$ bootstrap replicates. The red line corresponds to the maximum in the observed light curve $\mathcal{Y}$. The proportion of bootstrap values larger than the observed maximum is 0.16.

Model 2: state-space

Main idea: the observed flux measurement y_{t_i} comes from a real flux measurement, x_{t_i}, which was observed with an error term ε_{t_i}, and this error has known variance ζ_{t_i} given by the telescope.

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$$y_{t_i} = x_{t_i} + \varepsilon_{t_i} \qquad \varepsilon_{t_i} \sim \mathcal{N}(0, \zeta_{t_i}),$$

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State equation:

$$x_{t_i} = e^{-\tau\Delta_i} x_{t_{i-1}} + \mu\left(1 - e^{-\tau\Delta_i}\right) + \eta_{t_i}, \quad \eta_{t_i} \sim \mathcal{N}\left\{0, \frac{\sigma^2\left(1 - e^{-2\tau\Delta_i}\right)}{2\tau}\right\}$$

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The above structure can be thought of as a continuous time Gaussian state space-model, and can be estimated using the *Kalman filter* (Durbin and Koopman, 2012)

A sketch of the Kalman filter

In order to perform the parameter estimate, the filter performs the following two steps:

1. compute the expectation and the variance of the following distributions:

$$X_{t_i}|Y_{t_{i-1}} \sim \mathcal{N}(A_i, P_i), \qquad \text{prediction}$$
$$X_{t_i}|Y_{t_i} \sim \mathcal{N}(A_{i|i}, P_{i|i}), \qquad \text{filtering}$$

2. compute the log-likelihood of the model on the data, based on the fact that
$$Y_{t_i} \sim \mathcal{N}(A_i, P_i + \zeta_{t_i}).$$

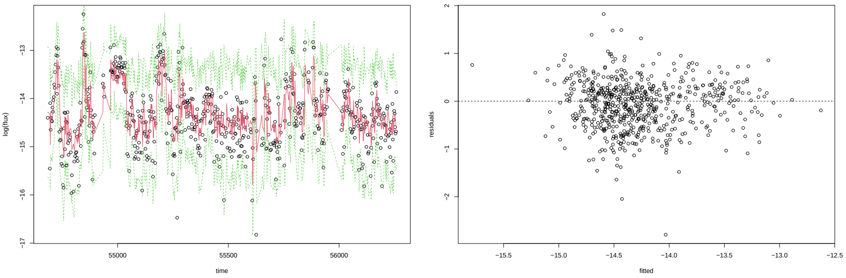

Figure 8: Top: model fitting (red line) and 95% prediction interval (green lines). Bottom: fitted vs residuals.

Table 3: Mean and the variance of the limit distributions in the original scale.

$\lim_{\Delta\to\infty} \mathbb{E}_Y$	$\lim_{\Delta\to\infty} \mathbb{V}_Y$
$7.53 \cdot 10^{-7}$	$2.48 \cdot 10^{-13}$

Next step: state-space hidden Markov model

- The next step will consist in putting together the two models described during this presentation:

$$Y_{t_i}|X_{t_i} \sim \mathcal{N}(X_{t_i}, \zeta_{t_i}),$$

$$X_{t_i}|X_{t_{i-1}}, S_{t_i} = s \sim \mathcal{N}(\mu^*_{i,s}, \sigma^{2*}_{i,s}),$$

$$\mathbb{P}(S_{t_i} = s|S_{t_{i-1}} = s') = \{\exp(\mathbf{Q}\Delta_i)\}_{ss'}.$$

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Thank you!