

Functional summary statistics for Poisson processes on convex shapes in three dimensions

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June 2019

Current methodologies in spatial point pattern analysis have been developed for planar and spatial data. Although this theory is applicable to a wide variety of practical applications, for example tree patterns in geology, many point patterns are not observed in Euclidean space. For example in microbiology, researchers are interested in patterns arising on the cellular membranes of bacteria. In this scenario point patterns are restricted to specific shapes such as spheres and ellipsoids. Research has only recently extended the theory to point pattern analysis on a sphere with many other shapes left unexplored, mainly due to the lack of isometries existing for general metric spaces. Our work discusses extensions of typical functional summary statistics, such as Ripley's K -function, for Poisson processes defined on convex shapes in three dimensions. By using the invariance of Poisson processes under transformations between metric spaces (known as the Mapping Theorem) we can transform a Poisson process from any convex shape to a Poisson process on the unit sphere and take advantage of its rotational symmetries to construct functional summary statistics. This allows us to determine whether patterns exhibit complete spatial randomness or determine if there exists spatial preference on the original convex space. In this talk we will present this methodology and discuss its potential application within astrophysics.