Time Delay Lens Modeling Challenge for the Hubble Constant Estimation

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Introduction

Video Credit: Science, American Association for the Advancement of Science

The Hubble constant H_0 represents the current expansion rate of the Universe, as well as the age ($= H_0^{-1}$), size, and density of the Universe.

Introduction (cont.)

But there have been several different estimates of H_0 from various methods.

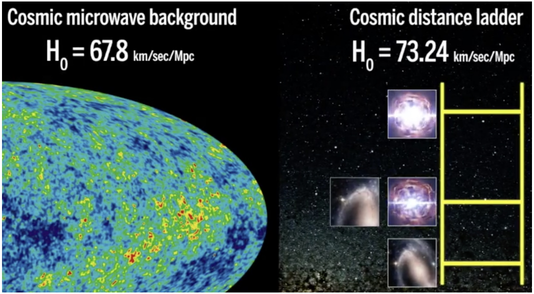


Image Credit: Science, American Association for the Advancement of Science

The most recent estimates from these two methods are

- CMB (Plank collaboration, 2016): 67.8 ± 0.9 km $\text{s}^{-1}\text{Mpc}^{-1}$.
- CDL (Reiss et al., 2016): 74.3 ± 2.1 km $\text{s}^{-1}\text{Mpc}^{-1}$.

Is this difference true (new physics) or not (within statistical uncertainty)?
Improving statistical accuracy or double-checking by independent methods.

Time delay cosmography

Quasar is a highly luminous galaxy hosting a supermassive black hole at the center. Since it is extremely bright, it can be seen at a great distance.

Video Credit: Space.com

Time delay cosmography (cont.)

Video source: https://www.youtube.com/watch?v=iE8x9kDHCFo

Strong gravitational lensing: The strong gravitational field of the intervening galaxy bends the light rays towards the Earth (like a lens), and thus we see multiple images of the same quasar in the sky.

Time delay cosmography (cont.)

Credit: NASA's Goddard Space Flight Center

Time delay: Light rays take different routes and travel through different gravitational potential, and thus their arrival times can differ → time delay!

Time delay cosmography (cont.)

Inference on H_o via an equation for additional travel distance (Refsdal, 1964).

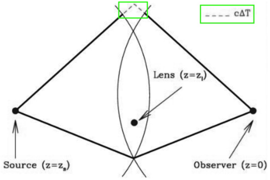
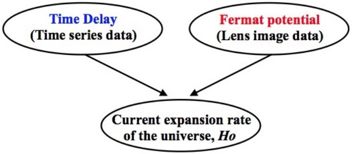


Image Credit: Tommaso Treu (UCLA) in "Dark Matter and Strong Lensing (2014)"

Speed of light (c) $\times$ Time delay (Δt_{ij})
$=$ Time delay distance ($D_{\Delta t}(H_o, z, \Omega)$) $\times$ Fermat potential difference $(\Delta\phi_{ij})$



Closed-form marginal posterior of H_o

Since $\Delta\phi_{ij} = \frac{c\Delta t_{ij}}{D_{\Delta t}(H_o, z, \Omega)}$, Marshall+ (2016) suggest (with fixed z and Ω)

$$\Delta\hat{\phi}_{ij} \mid \Delta t_{ij}, H_o \sim \mathrm{N}\left[\frac{c\Delta t_{ij}}{D_{\Delta t}(H_o)},\ \sigma^2_{\Delta\hat{\phi}_{ij}}\right],$$

$$\Delta t_{ij} \sim \mathrm{N}(\Delta\hat{t}_{ij},\ \sigma^2_{\Delta\hat{t}_{ij}}).$$

$$\text{Marginally,}\ \ \Delta\hat{\phi}_{ij} \mid H_o \sim \mathrm{N}\left[\frac{c\Delta\hat{t}_{ij}}{D_{\Delta t}(H_o)},\ \frac{c^2}{D^2_{\Delta t}(H_o)}\sigma^2_{\Delta\hat{t}_{ij}} + \sigma^2_{\Delta\hat{\phi}_{ij}}\right].$$

All but H_o ($\sim \mathrm{Unif}[50,\ 90]$ a priori) are known or (at least) estimable!

- $\Delta\hat{\phi}_{ij}$: Fermat potential difference estimate between images i & j.
- $\sigma^2_{\Delta\hat{\phi}_{ij}}$: An uncertainty estimate (variance) of $\Delta\hat{\phi}_{ij}$.
- $\Delta\hat{t}_{ij}$: A time delay estimate between images i and image j.
- $\sigma^2_{\Delta\hat{t}_{ij}}$: An uncertainty estimate (variance) of $\Delta\hat{t}_{ij}$.
- $D_{\Delta t}(H_o)$: The time delay distance, a deterministic function of H_o.

Data for Time Delay

Data for a doubly-lensed quasar are two time series (light curves) with known measurement errors.

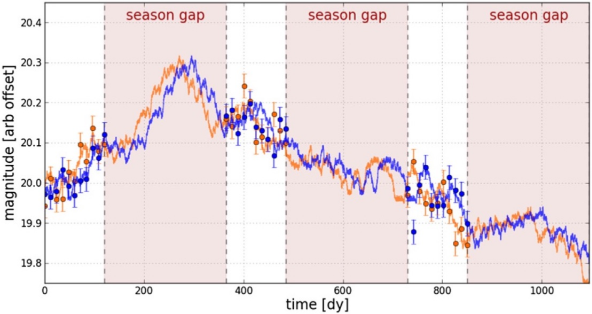


Image Credit: Dobler et al. (2015)

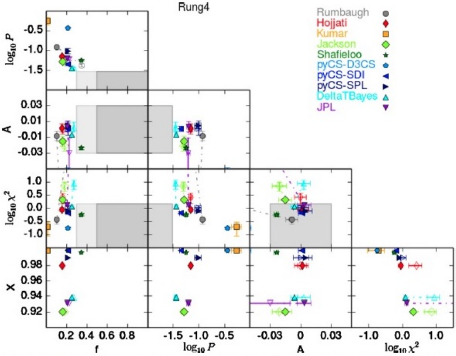
We can estimate Δ by the horizontal shift between two time series.

Time Delay Challenge

Time Delay Challenge (Dobler et al., 2015; Liao et al., 2015)

- A blind competition held by 8 astrophysicists from 2013 to 2014.
- Goal was to improve existing estimation methods.
- 5,000+ simulated data sets with some time delays.
- 13 teams blindly analyzed the simulated data sets.

Image Credit: HBO website



Time Delay Lens Modeling Challenge

Another blind competition to improve lens-modeling methods (Ding+, 2018+).

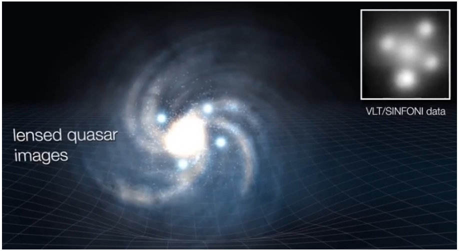


Image Credit: https://www.youtube.com/watch?v=iE8x9kDHCFo

Modeling the lens: Lens mass $\rightarrow$ lens potential $\rightarrow$ Fermat potential.
(The mass density is the second derivative of the lens potential.)

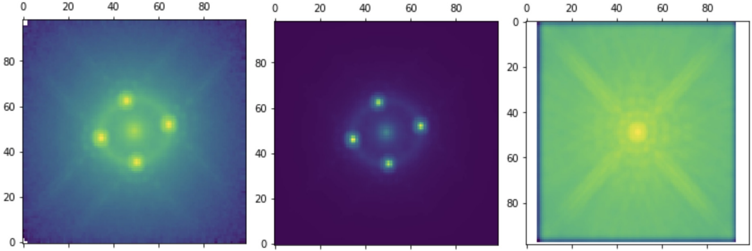
Outline of TDLMC

The time delay lens modeling challenge is a three-step blind competition composed of four rungs. Each rung shares the same (simulated) Hubble constant. The difficulty increases as we move up higher rungs.

- Rung 0: The true H_o is disclosed for participant's reference. Two images, one for a doubly-lensed image and the other for a quadruply-lensed image. The point spread function is provided.
- Rung 1: 16 images. Due was Sep 8. Real galaxy images for realistic surface brightness are used for simulations.
- Rung 2: 16 images. Due is Jan 8. On top of Rung 1's difficulty, a guess of the point spread function is provided for each image.
- Rung 3: 16 images. Due is May 8. In addition to all challenges in Rungs 1 and 2, images are generated by massive early-type galaxies.

Image Data

Image data (from the left): (i) Light intensity (brightness) in 100×100 pixels, (ii) measurement errors, (iii) point spread function (used in (i)).



Lens modeling

We model (i) lens mass, (ii) lens brightness, and (iii) source brightness.

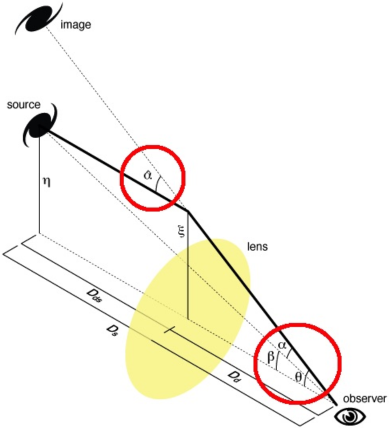


Image Credit: Michael Sachs (from Wiki)

Angular positions (unknown param.)

β: Source position in the absence of the lens.
θ: Lensed image position.
$\hat{\alpha}$: Deflection angle.
$\alpha(\theta)$: Scaled deflection angle for the image at θ.
$\beta = \theta - \alpha(\theta)$: The lens equation.

Given the lens mass distribution, we can infer $\alpha(\theta)$ and β.

Lens modeling (cont.)

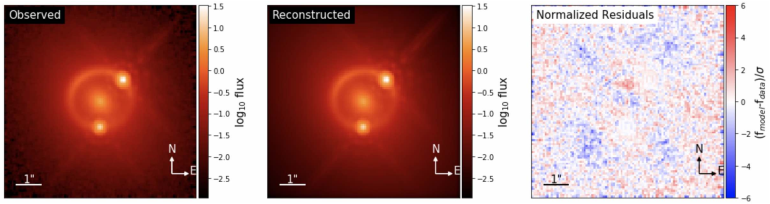
Outline of lens modeling:

1. Setting (choosing) a lens mass density function, $\Sigma(D_d\theta)$.
2. Deriving a dimensionless surface mass density, $\kappa(\theta) = \Sigma(D_d\theta)/\Sigma_{\rm cr}$, where $\Sigma_{\rm cr}$ is the critical surface mass density. For example, with an elliptical power-law mass density,
$$\kappa(\theta_{i1}, \theta_{i2}) = \frac{3-\gamma'}{2}\left(\frac{\sqrt{q\theta_{i1}^2 + \theta_{i2}^2/q}}{\theta_E}\right)^{1-\gamma'},$$
where θ_E is the radius of Einstein ring, q is the ellipticity, and γ' is the radial power-law slope.
3. Computing $\alpha(\theta) = \frac{1}{\pi}\int_{\mathbb{R}^2} d^2\theta' \kappa(\theta') \frac{\theta-\theta'}{|\theta-\theta'|^2}$.
4. Computing lens potential: $\psi(\theta) = \frac{1}{\pi}\int_{\mathbb{R}^2} d^2\theta' \kappa(\theta') \log|\theta - \theta'|$.
5. Finally, the Fermat potential is computed as $\phi(\theta) = \alpha(\theta)^2/2 - \psi(\theta)$.

Lens modeling (cont.)

The number of unknown parameters is 22 (double) or 28 (quad).

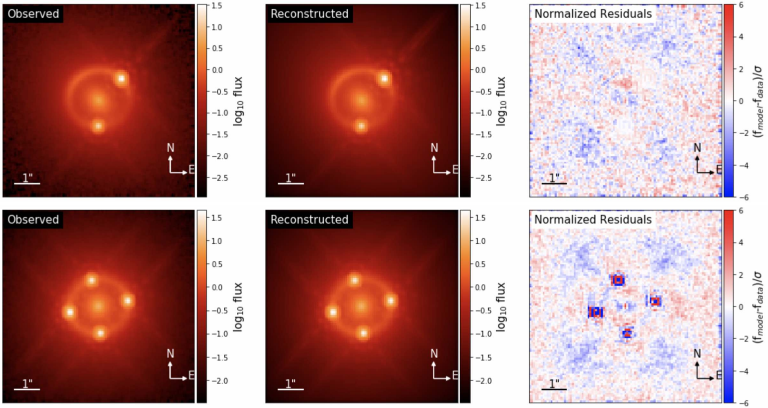
We use a Python package `lenstronomy` (Birrer+, 2015, 2016, 2018) to fit a lens model on the image data. Fitting the model is a two-step procedure; (i) particle swarm optimization to find a global optimum of 20–26 parameters; (ii) MCMC initialized at the global optimum.



Given the observed data (1st), it reconstructs the image (estimate) based on the fitted model (2nd), and shows a residual plot (3rd = 2nd − 1st).

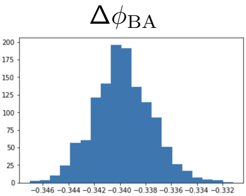
Result of Rung 0

Observed images (1st column), estimated images (2nd column), and residuals (3rd column).

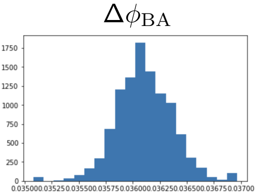


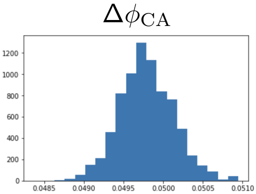
Result of Rung 0 (cont.)

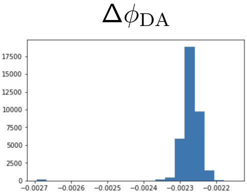
Posterior of one Fermat potential difference from a double-image.



Posteriors of three Fermat potential differences from a quad-image.







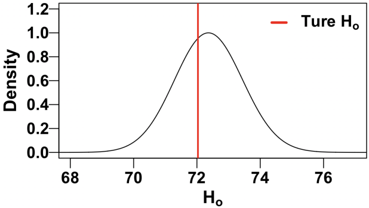
Result of Rung 0 (cont.)

The marginal posterior distribution of H_o is closed-form.

$$\Delta\hat{\phi}_{ij} \mid H_o \sim \mathrm{N}\left[\frac{c\Delta\hat{t}_{ij}}{D_{\Delta t}(H_o)},\ \frac{c^2}{D^2_{\Delta t}(H_o)}\sigma^2_{\Delta\hat{t}_{ij}} + \sigma^2_{\Delta\hat{\phi}_{ij}}\right].$$

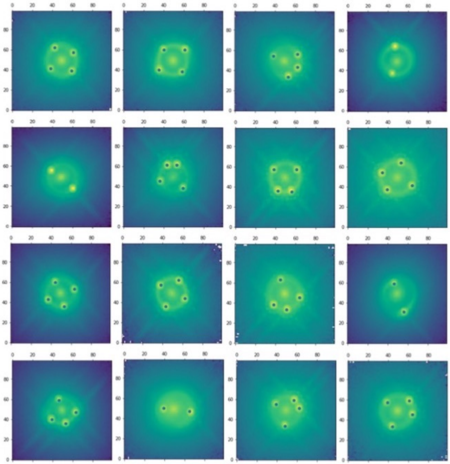
$$H_o \sim \mathrm{Unif}(50,\ 90).$$

The resulting posterior of H_o based on the four pairs of $\Delta\hat{\phi}_{ij}$ and $\sigma^2_{\Delta\hat{\phi}_{ij}}$ (time delays $\Delta\hat{t}_{ij}$ and their uncertainties $\sigma^2_{\Delta\hat{t}_{ij}}$ are given):



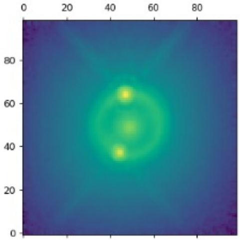
Result of Rung 1

16 lens image data sets (simulated under the same H_o) to be analyzed.



Result of Rung 1 (cont.)

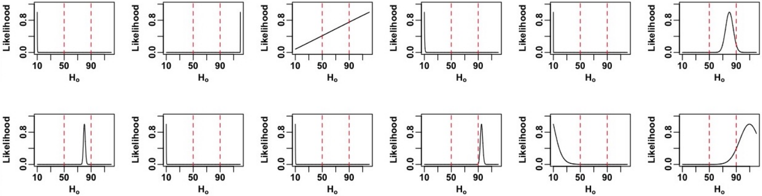
Analytic sequence for **one** image data set as an example.



1. We fit our model on this image data set with 12 variations each for a combination of four different values of point spread function error inflation (1%, 5%, 10%, 20%) and three different lens light models (1, 2 or 3 lens light models) → 12 Fermat potential difference estimates and their uncertainties.

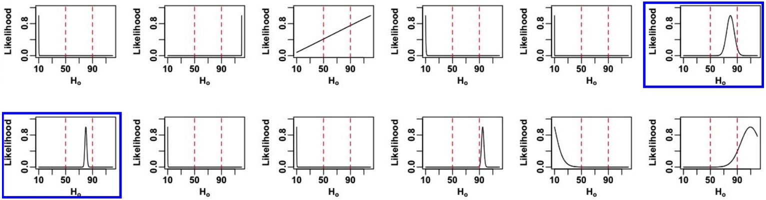
Result of Rung 1 (cont.)

2. We derive the posterior of H_o using each pair of Fermat potential difference estimate and uncertainty, leading to 12 posteriors of H_o:



Result of Rung 1 (cont.)

3. We collect pairs of Fermat potential estimate and uncertainty that result in the posterior mode of H_o between 50 and 90 (between red vertical dashed lines).



Result of Rung 1 (cont.)

4. We take an average of the collected pairs in three ways:

(1) Weighted average and variance

$$\Delta\hat{\phi}_{AB} = \frac{\sum_{i=6}^{7} \Delta\hat{\phi}_{AB}^{(i)} / \sigma^2_{\Delta\hat{\phi}_{AB}^{(i)}}}{\sum_{i=6}^{7} 1/\sigma^2_{\Delta\hat{\phi}_{AB}^{(i)}}} \quad \text{and} \quad \sigma^2_{\Delta\hat{\phi}_{AB}} = \frac{1}{\sum_{i=6}^{7} 1/\sigma^2_{\Delta\hat{\phi}_{AB}^{(i)}}}.$$

(2) Sample mean of estimates, and sample mean of variance

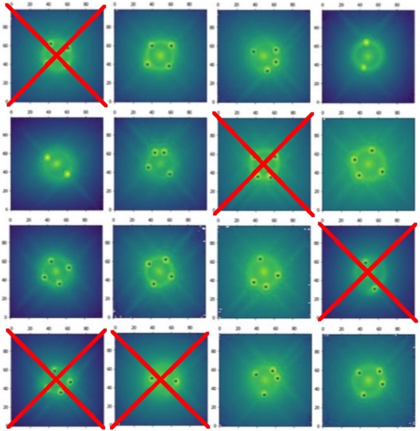
$$\Delta\hat{\phi}_{AB} = \frac{1}{2}\sum_{i=6}^{7} \Delta\hat{\phi}_{AB}^{(i)} \quad \text{and} \quad \sigma^2_{\Delta\hat{\phi}_{AB}} = \frac{1}{2}\sum_{i=6}^{7} \sigma^2_{\Delta\hat{\phi}_{AB}^{(i)}}.$$

(3) Sample mean of estimates, and sample variance of estimates

$$\Delta\hat{\phi}_{AB} = \frac{1}{2}\sum_{i=6}^{7} \Delta\hat{\phi}_{AB}^{(i)} \quad \text{and} \quad \sigma^2_{\Delta\hat{\phi}_{AB}} = \sum_{i=6}^{7} (\Delta\hat{\phi}_{AB}^{(i)} - \Delta\hat{\phi}_{AB})^2.$$

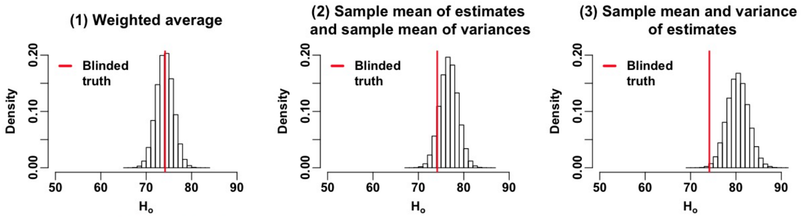
Result of Rung 1 (cont.)

We applied the estimation routine to 16 images and could successfully analyze 11 images out of 16, leading to 25 Fermat potential difference estimates and their uncertainties.



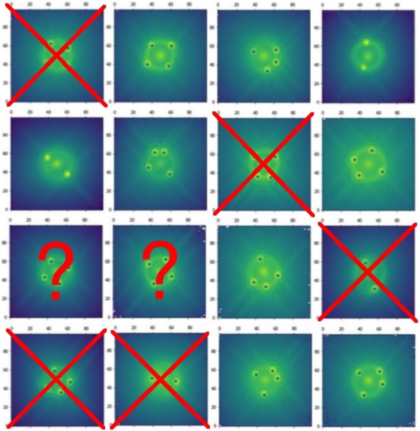
Result of Rung 1 (cont.)

The following three estimates are reported:



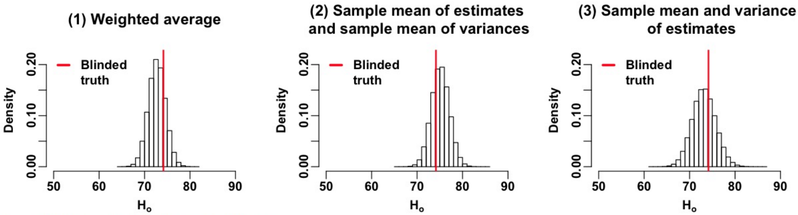
Result of Rung 1 (cont.)

The two lenses below (marked by red question marks) result in the H_o estimates close to 90. What about removing them?



Result of Rung 1 (cont.)

The following three estimates are additionally reported:



Concluding remarks

Our contribution is to provide a way to combine Fermat potential difference estimates obtained from independent image data sets.

- The weighted average method works pretty well.
- The third way to make the representative estimate, i.e., the sample mean and variance of estimates (not using the uncertainty estimates) will not be used for rung 2.
- For rung 2, I will not put my personal curiosity (no additional three submissions), trusting what the data tell us.
- The due for rung 2 is Jan 5.