Coded-Aperture Imaging

JaeSub Hong
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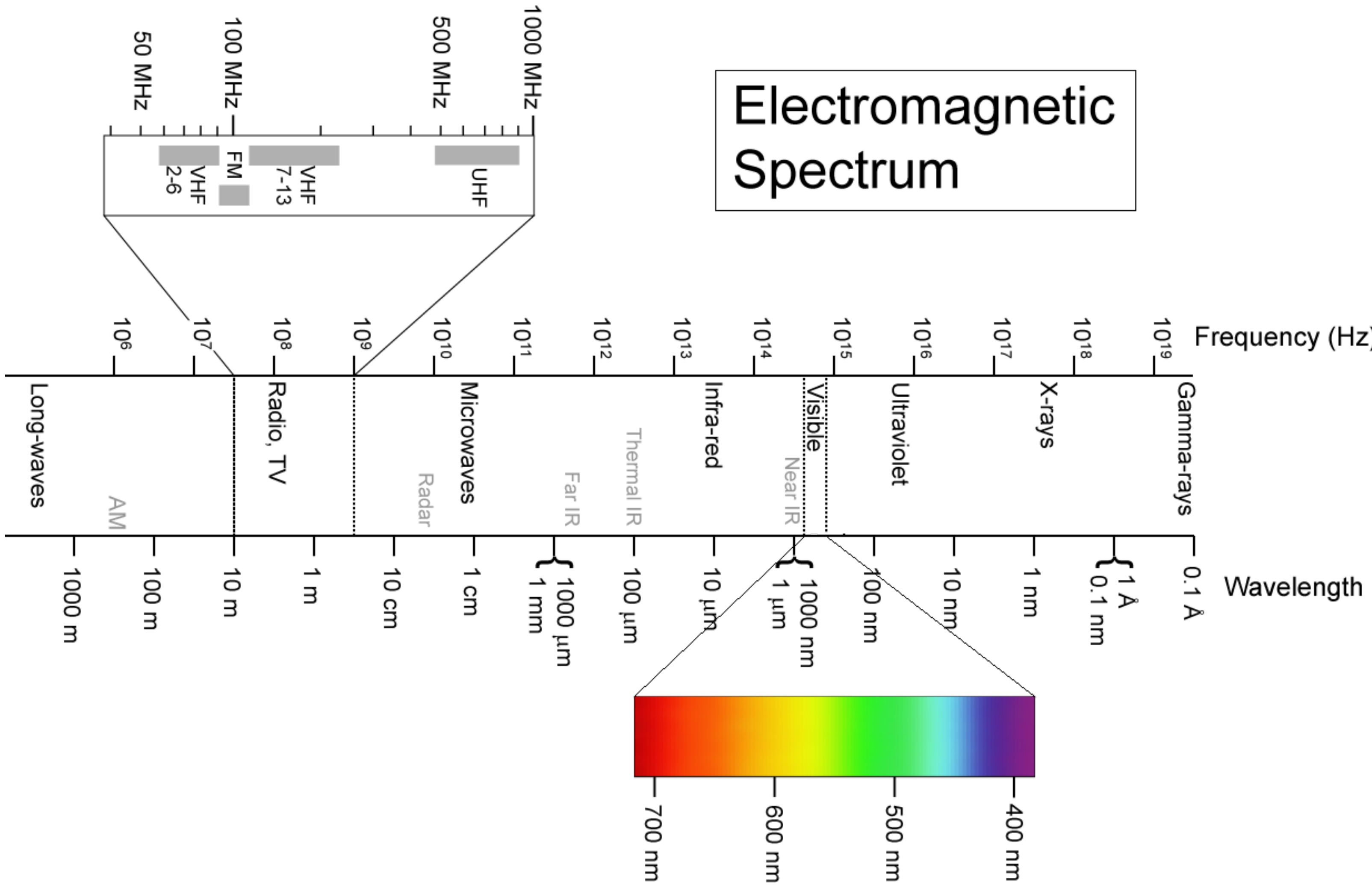
MIT

Outline

1. A Brief History of X-ray Astronomy

2. Coded Aperture Imaging

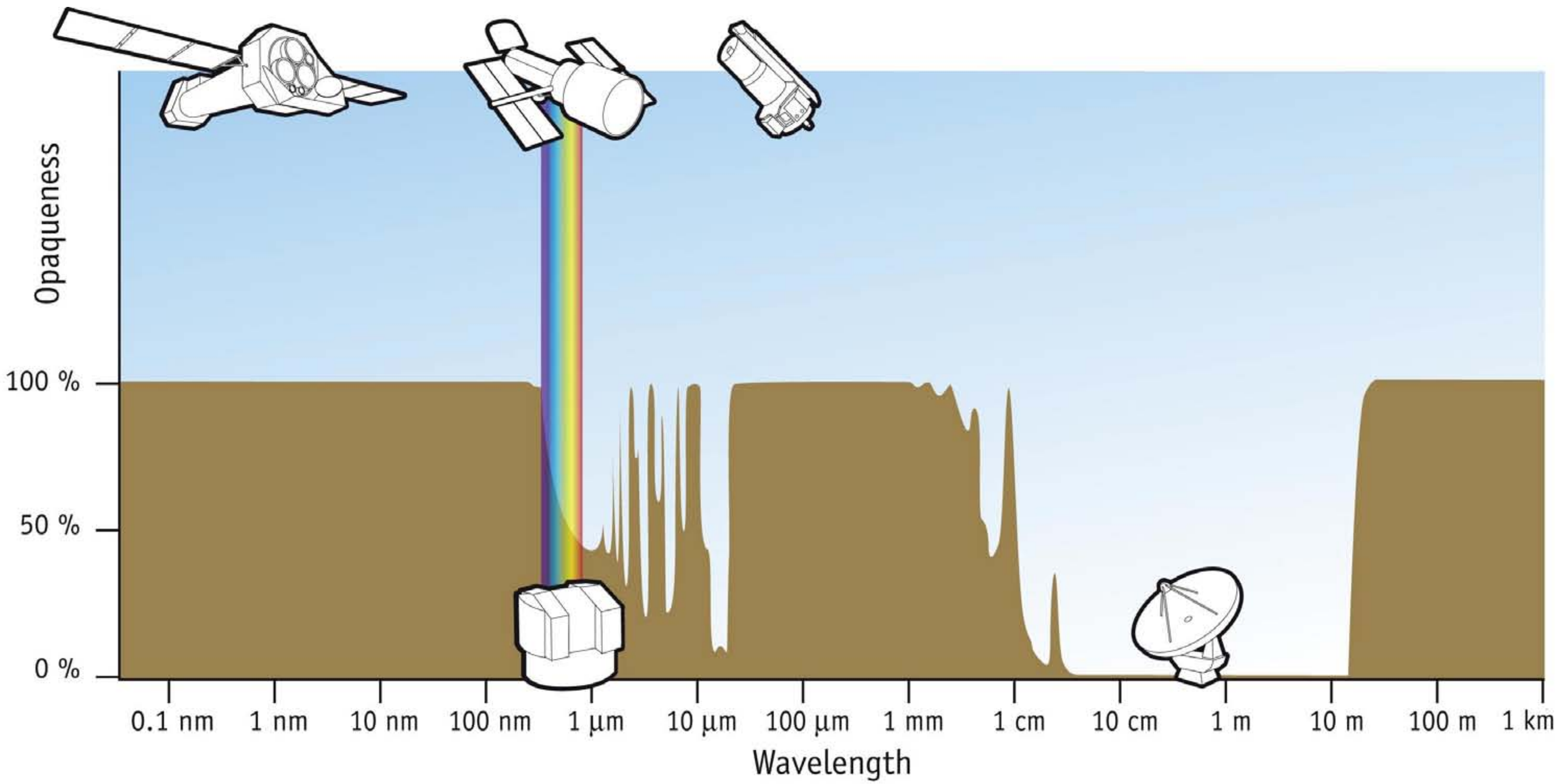
What is an X-ray?



Discovery of X-rays

**Print of Wilhelm Röntgen's first "medical" X-ray,
of his wife's hand,
taken on 22 December 1895 [Wiki]**

X-rays from Sky



Beginning of X-ray Astronomy or not?

Bumper V-2 Rocket Launch, July 24, 1950 [Wiki]

Beginning of X-ray Astronomy

- On June 12, 1962, an Aerobee 150 rocket was launched for an attempt to observe X-rays from the moon.

Riccardo Giacconi

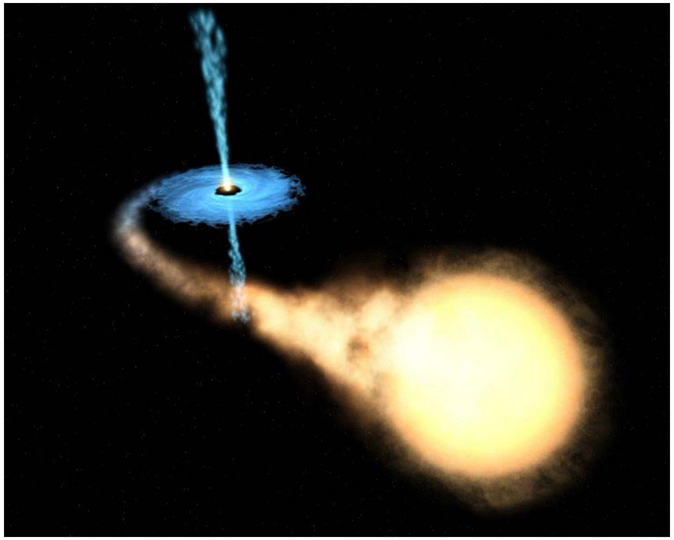
Beginning of X-ray Astronomy

- On June 12, 1962, an Aerobee 150 rocket was launched for an attempt to observe X-rays from the moon.

No chance! But …

- The instrumentation was not equipped with collimation to restrict the field of view narrowly.

Riccardo Giacconi



- It detected the first X-rays from another celestial source (Scorpius X-1) at J1950 RA $16^h\ 15^m$ Dec -15.2°.

- Sco X-1 is a Low Mass X-ray Binary with a Neutron Star [Wiki].

How Bright?

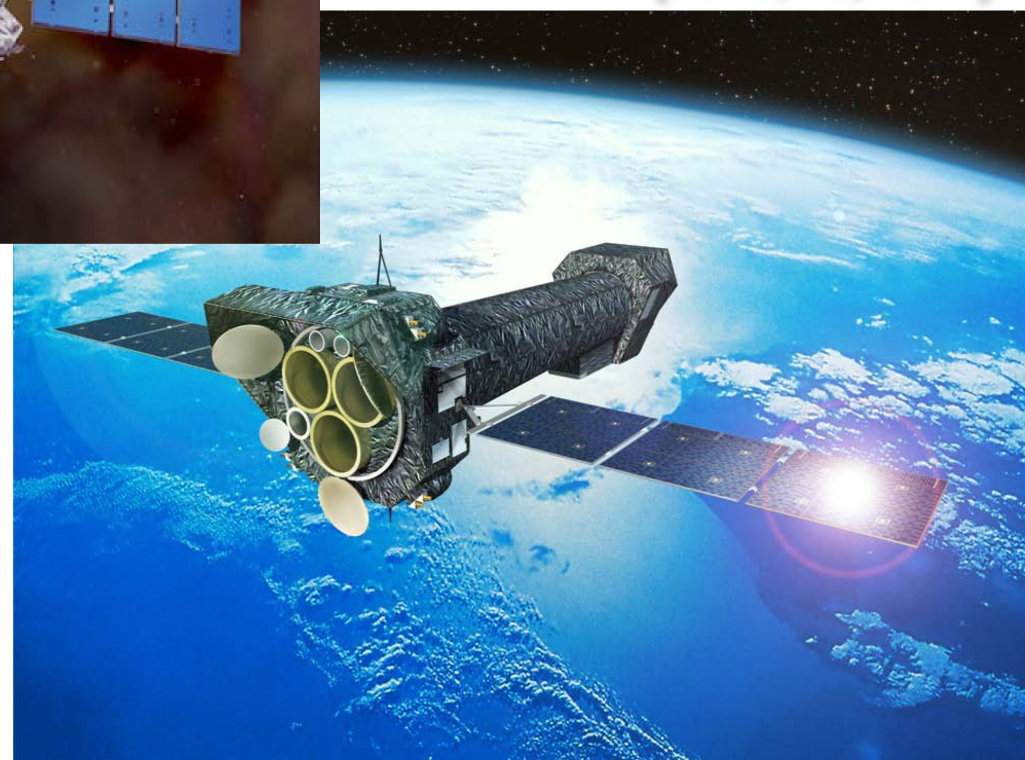
X-ray luminosity of celestial objects

Moon	~10^{12} erg/s	~100 kW
Sun	~10^{27} erg/s	~10^{9} TW
X-ray Binaries	~10^{38} erg/s	~10^{20} TW
Our Galaxy	~10^{39} erg/s	~10^{21} TW
Supernova	~10^{41} erg/s	~10^{23} TW
Active Galactic Nuclei	~10^{47} erg/s	~10^{29} TW
Gamma-Ray Bursts	~10^{52} erg/s	~10^{34} TW

X-ray Telescopes

Chandra X-ray Observatory
(1999/07/22 -)

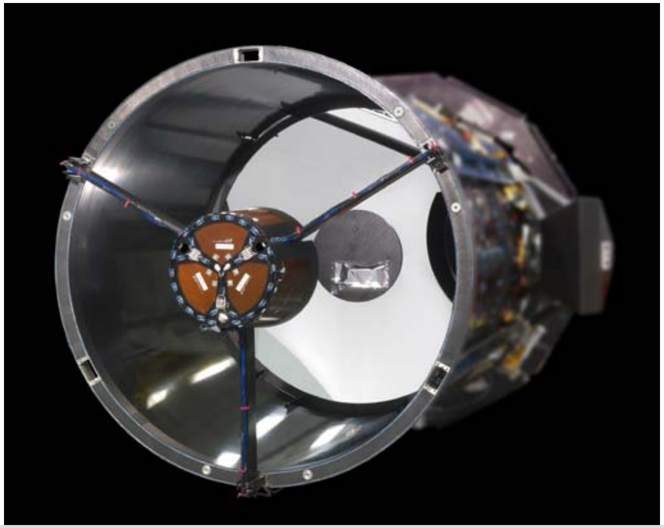
XMM-Newton
(1999/12/10 -)

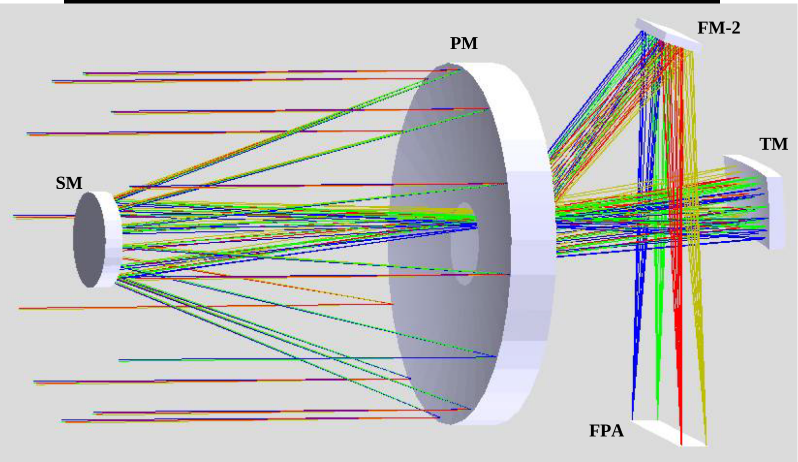



G292.0+1.8

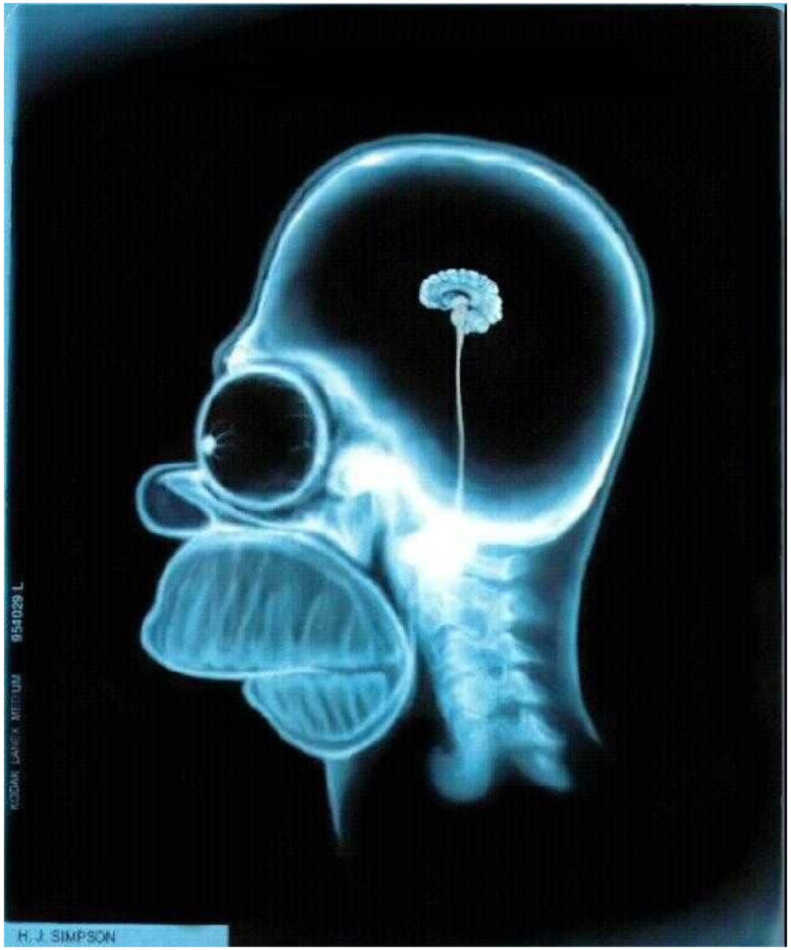

KEPLER'S SUPERNOVA REMNANT

Optical Telescope Assembly
Normal Incidence Optics





How do you build X-ray Telescopes?

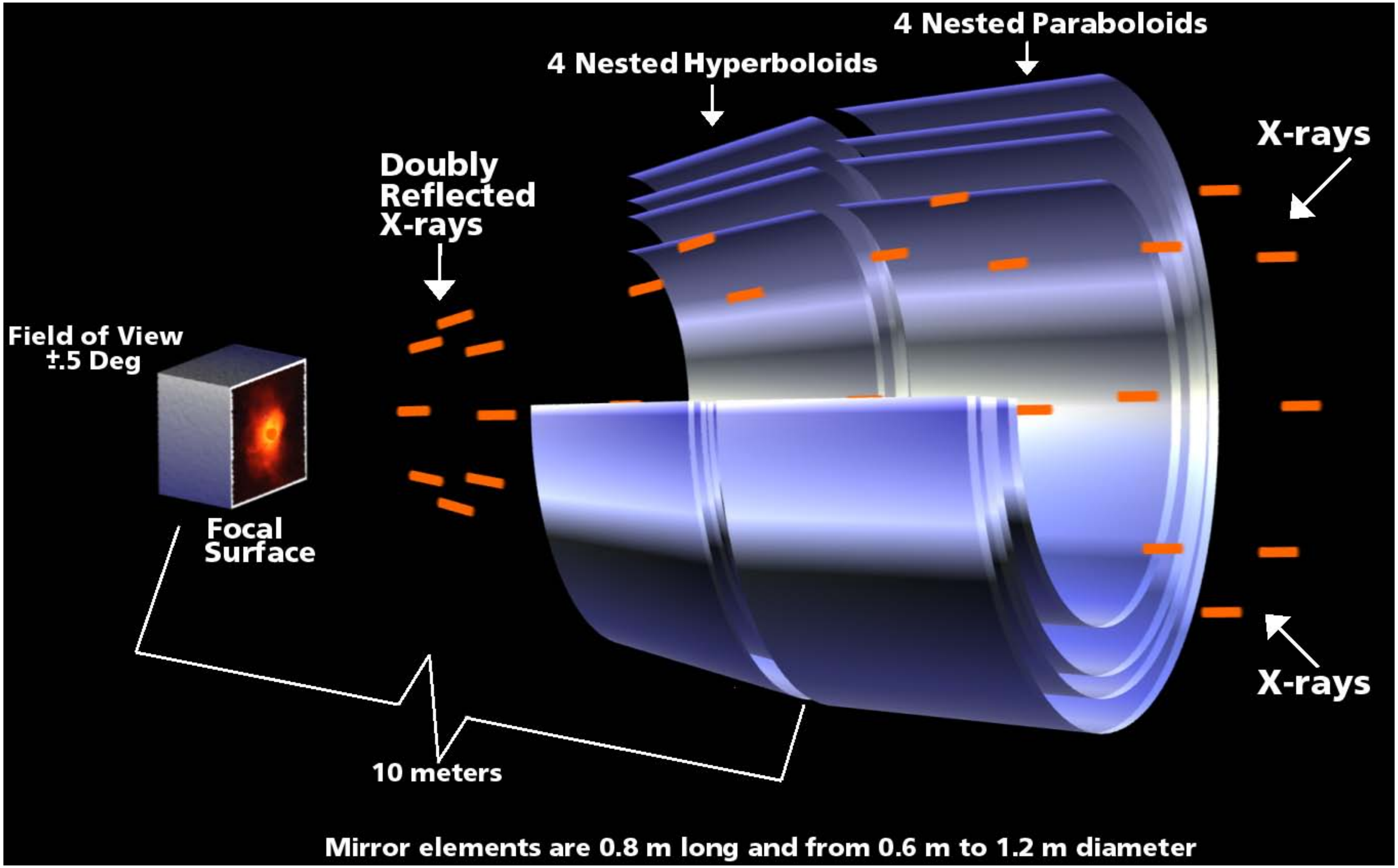


Inferior Mirage

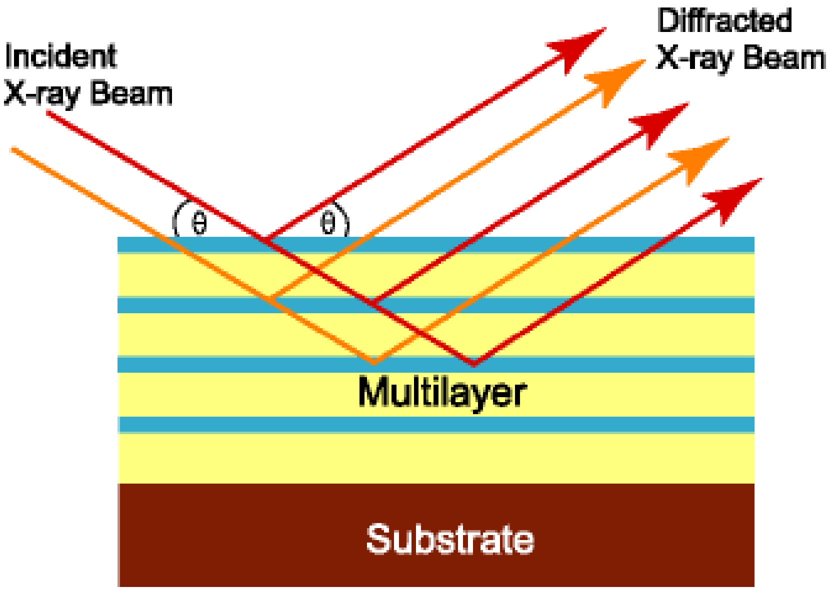
Shallow Angle Reflection

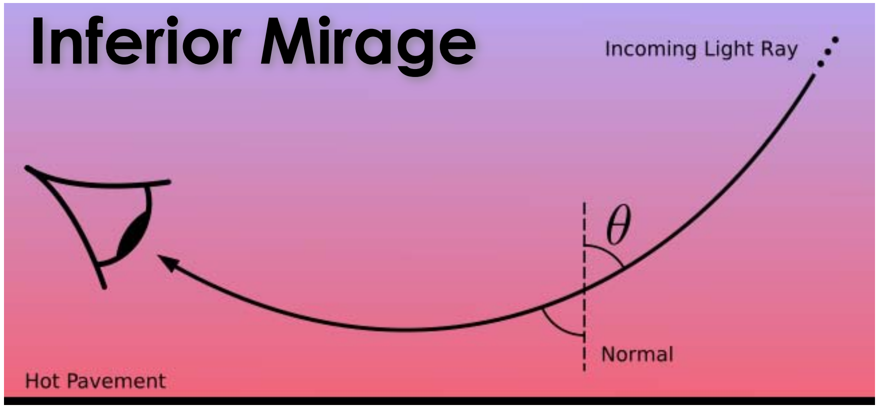
Chandra X-ray Observatory

Grazing Incidence Optics: up to ~10 keV



Grazing Incidence + Multi-Layer Optics
Up to ~70 – 80 keV

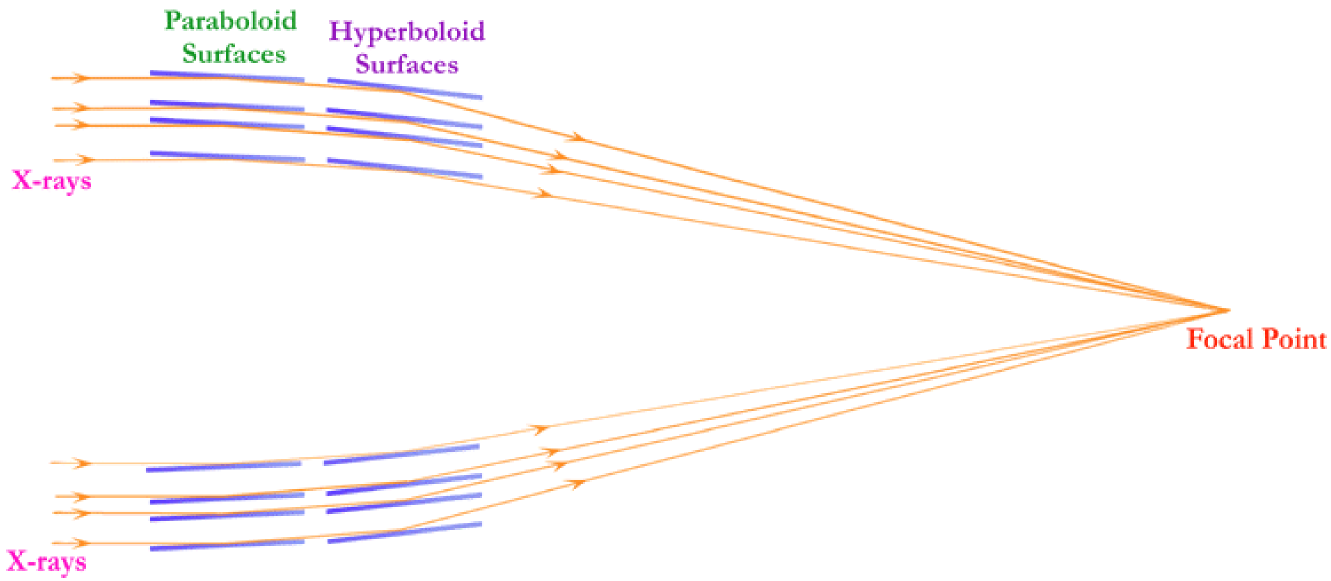




The Nuclear Spectroscopic Telescope Array
(NuSTAR) 2012

- IR, Visible, UV: Normal Incidence Optics
- Soft X-ray, Hard X-ray
  - < 10 keV: Grazing incidence
  - < 100 keV: Grazing+MultiLayer Optics

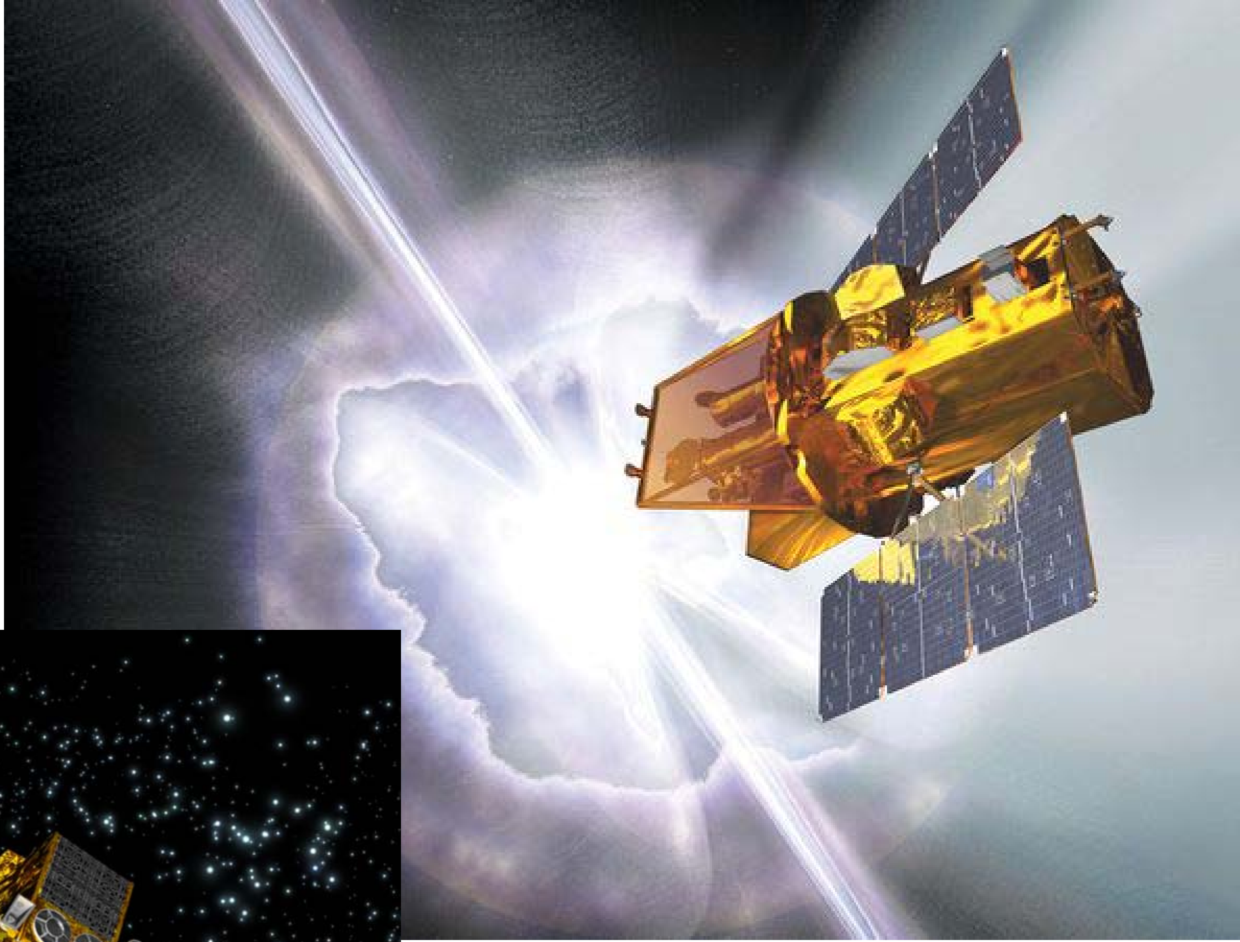
- What about X-rays above 100 keV?
- How to cover wide field?

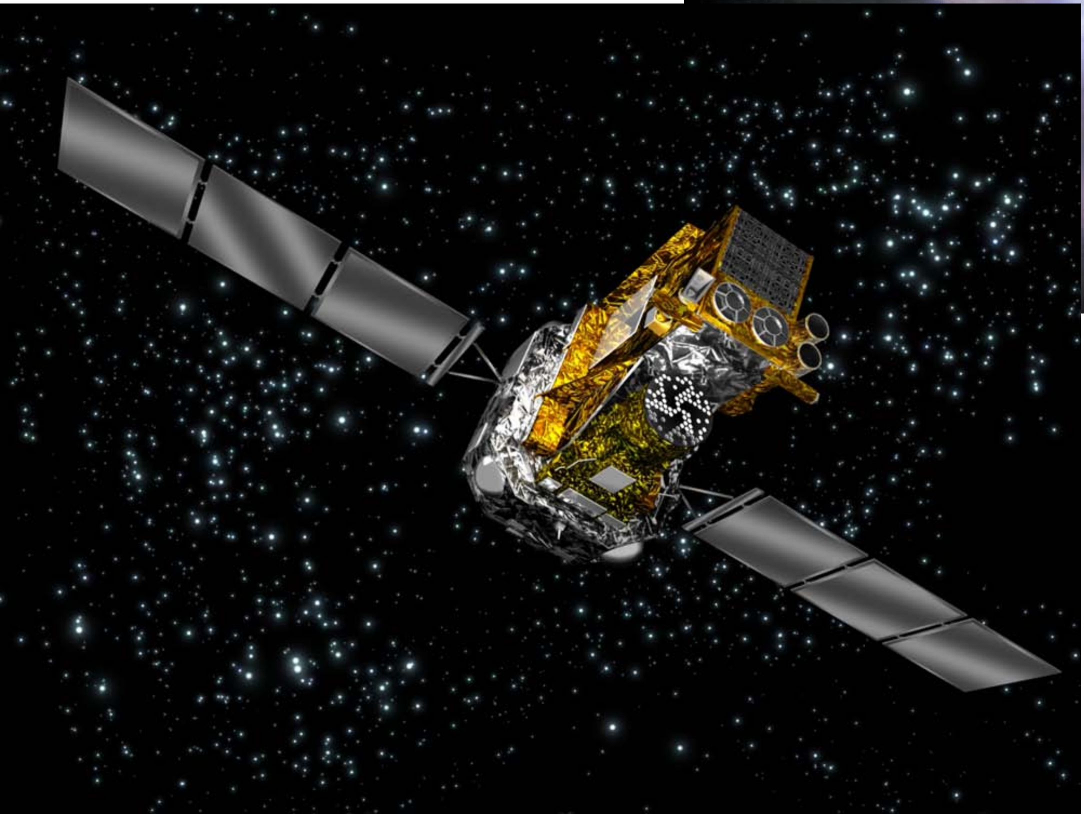


Coded-Aperture Imaging

Coded-Aperture Imagers

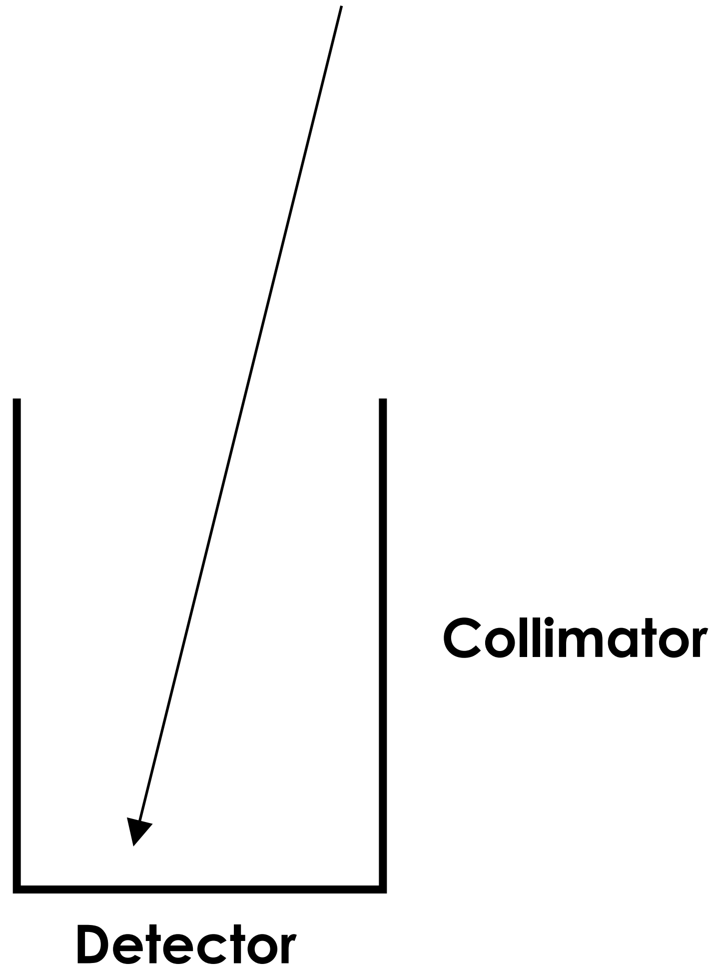
Swift/BAT
2004/11/20 -





INTEGRAL/IBIS & SPI
2002/10/17 -

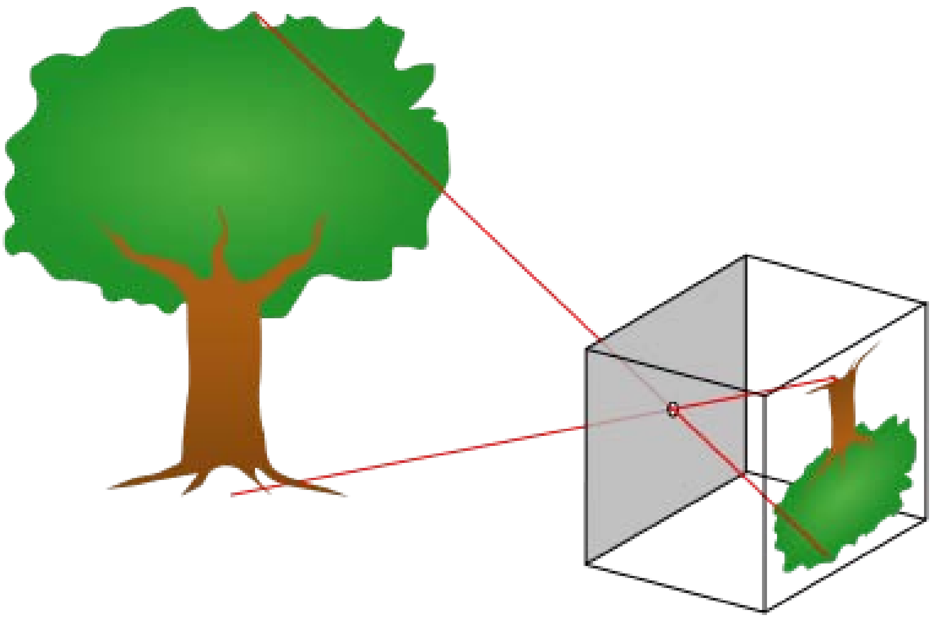
Collimator

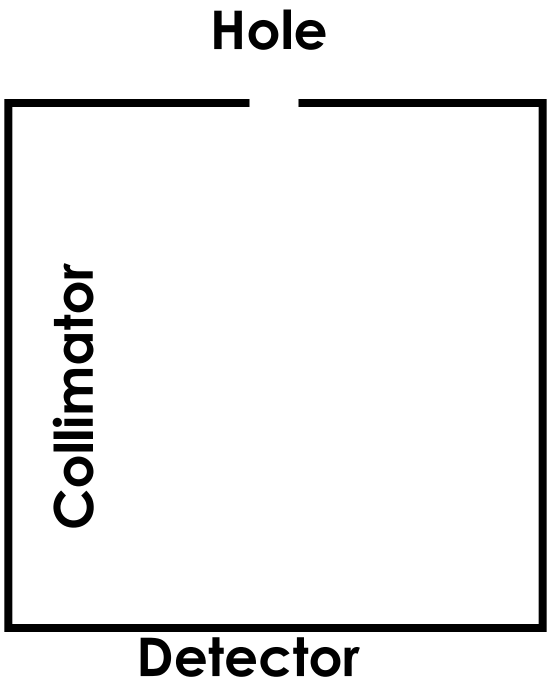


Simple, but not a real imager

Field of View = Angular Resolution

Pin Hole Camera

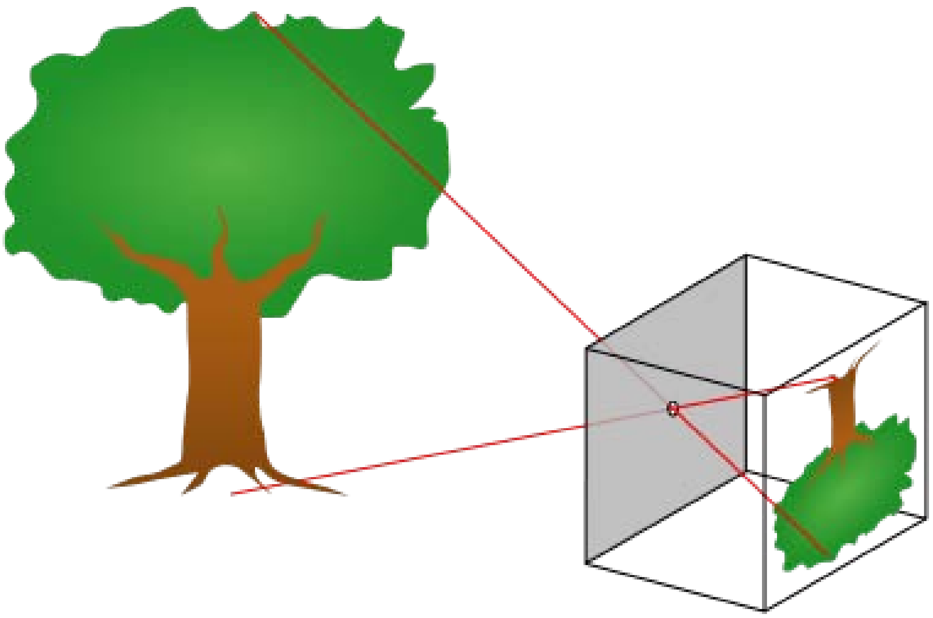


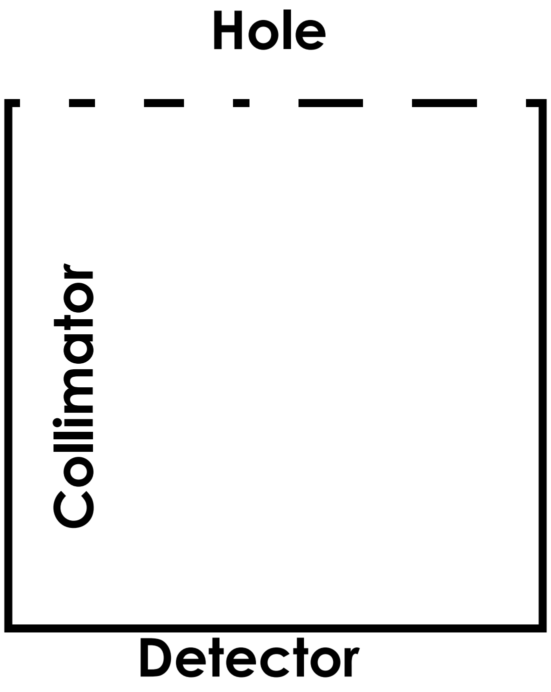


A real imager but extremely inefficient

►Low sensitivity

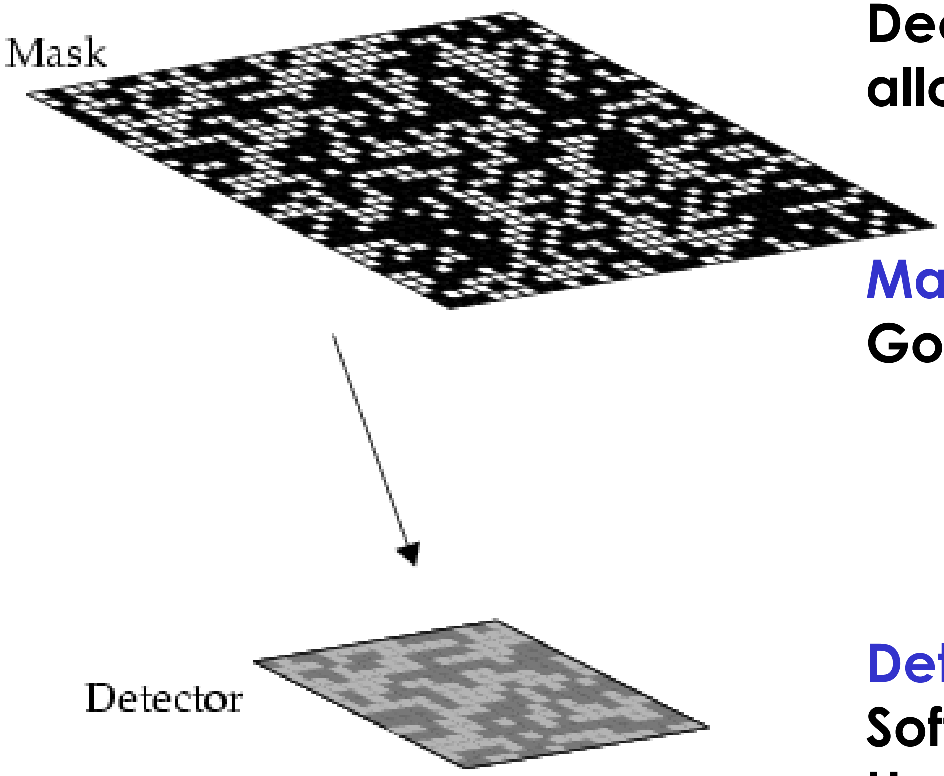
Pin Hole Camera





Mertz & Young (1961);
Dicke (1968)

Coded-Aperture Imaging Telescope

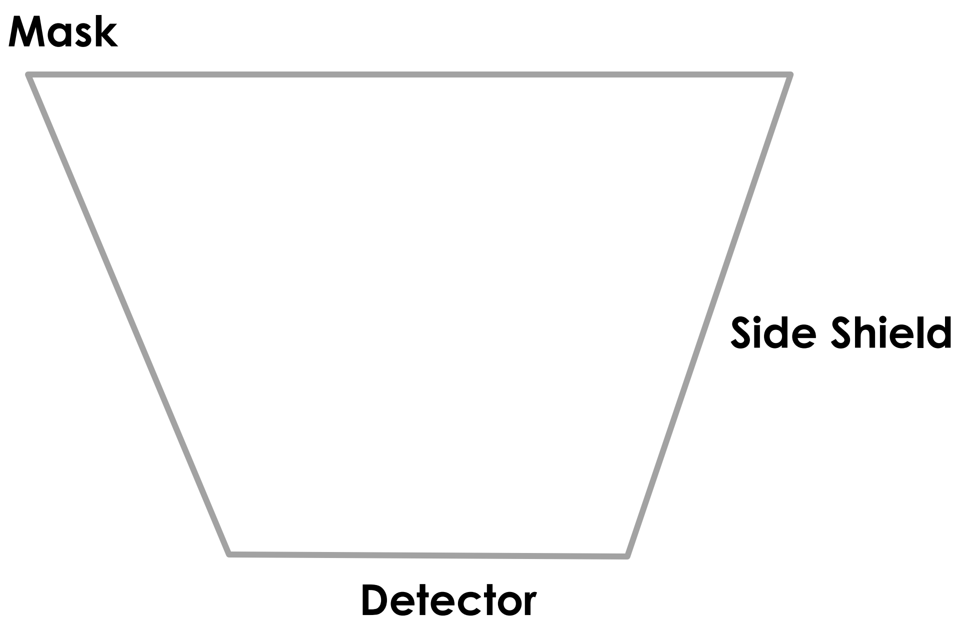


Decoding Shadowgram allows wide-field imaging.

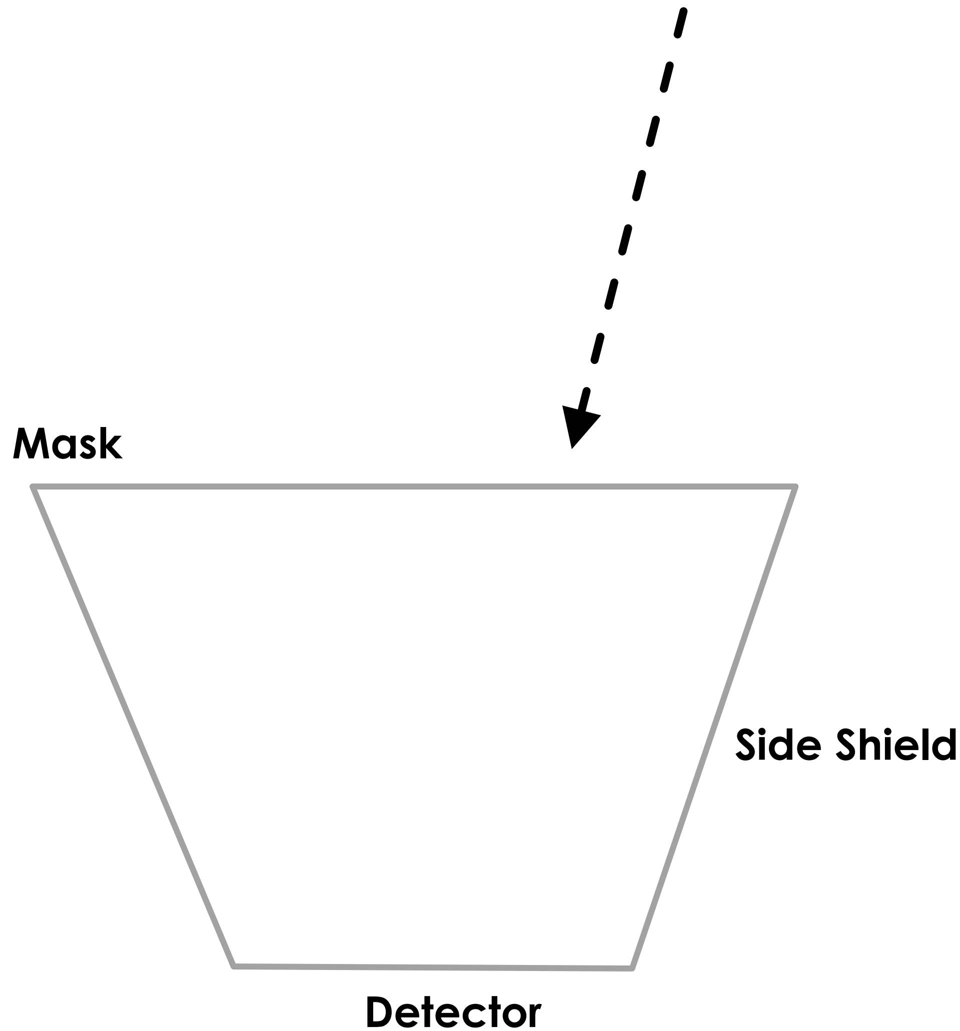
Mask
Gold, Lead, Tungsten, …

Detector
Soft X-ray : X-ray CCD, …
Hard X-ray : CdZnTe, Ge, …

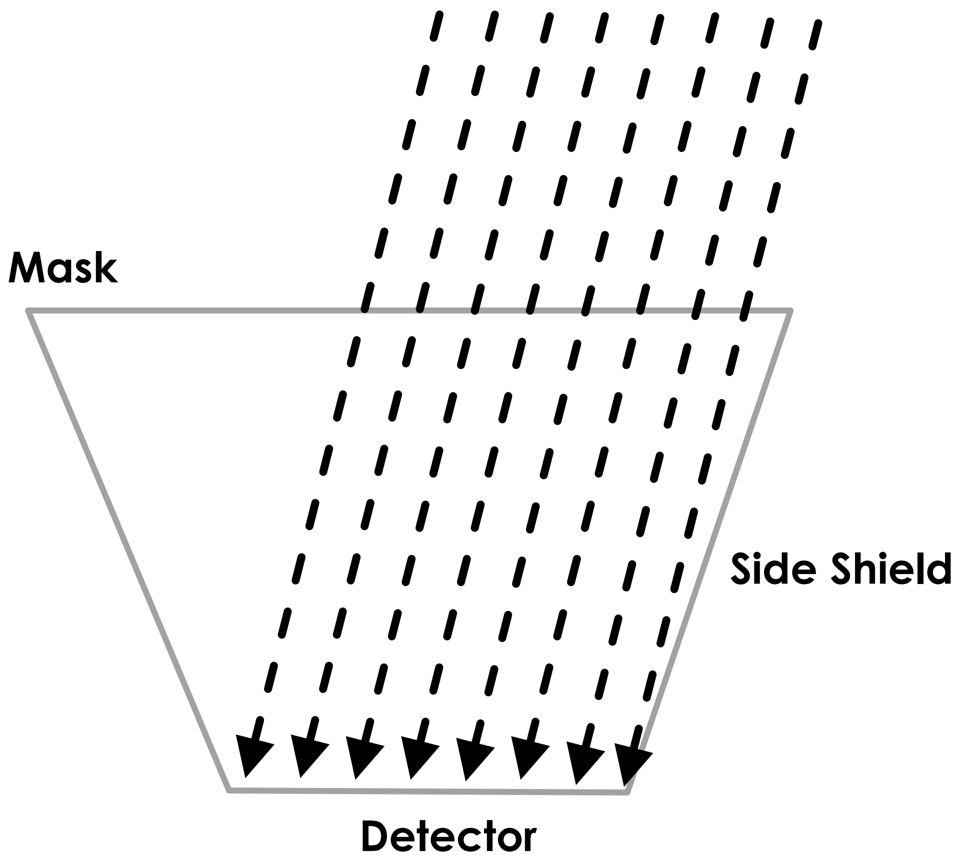
Field of View & Coding Fraction



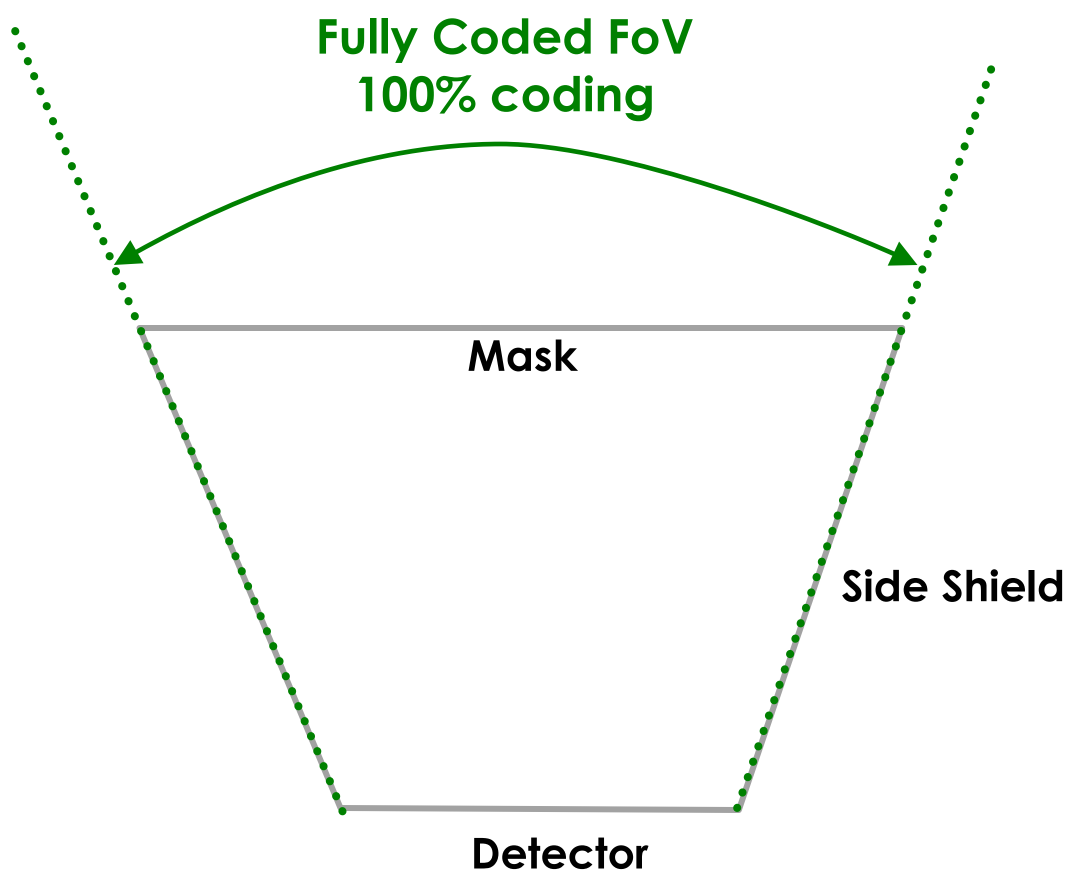
Field of View & Coding Fraction



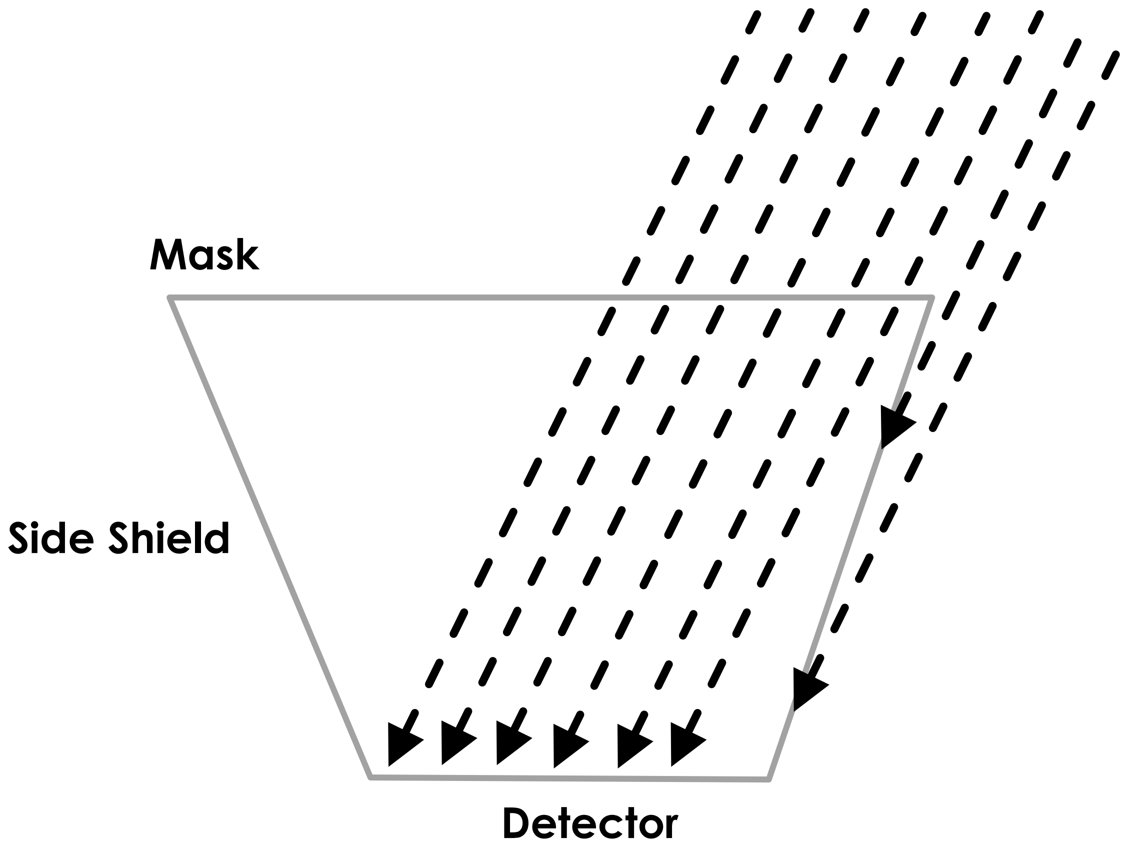
Field of View & Coding Fraction



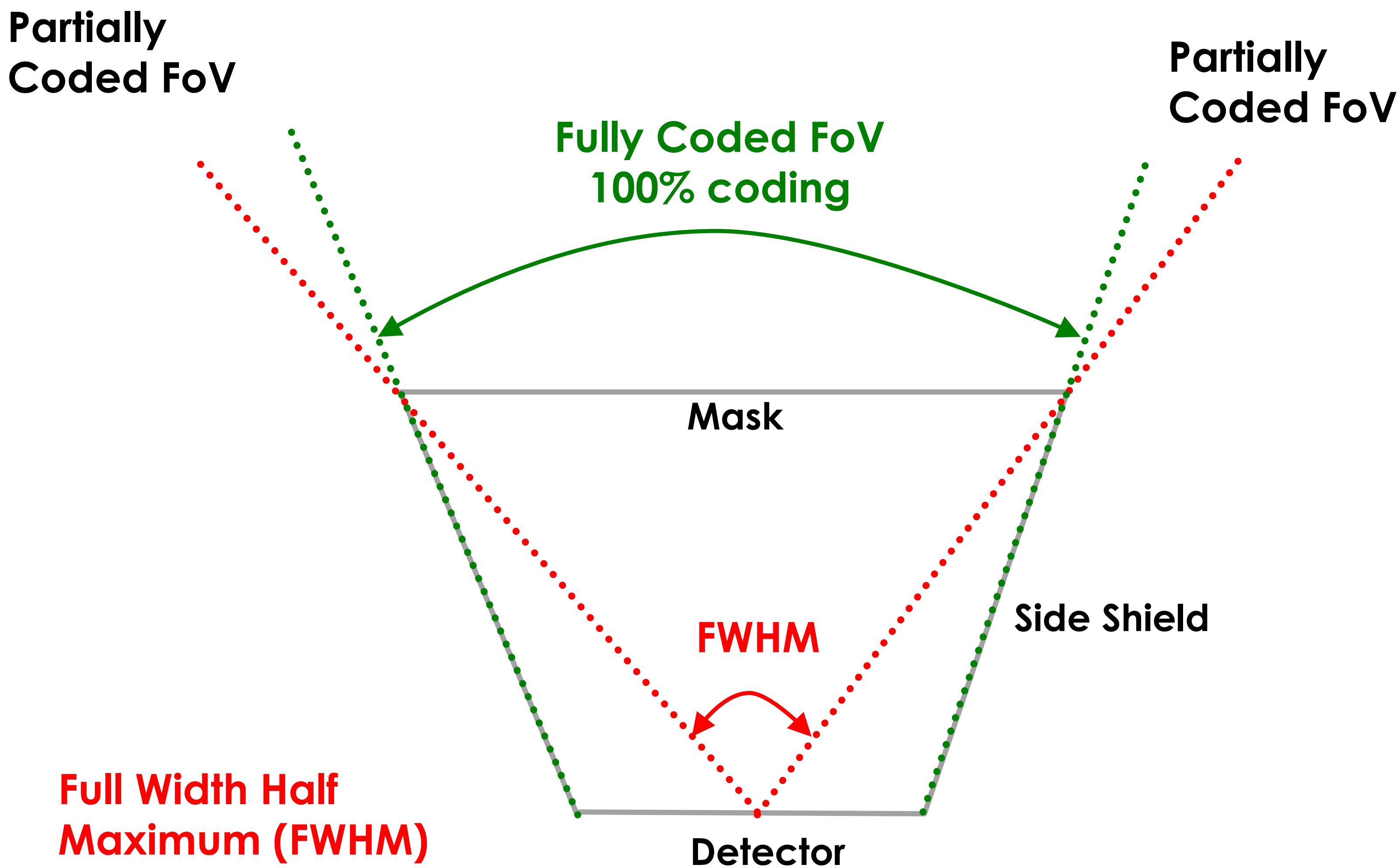
Field of View & Coding Fraction



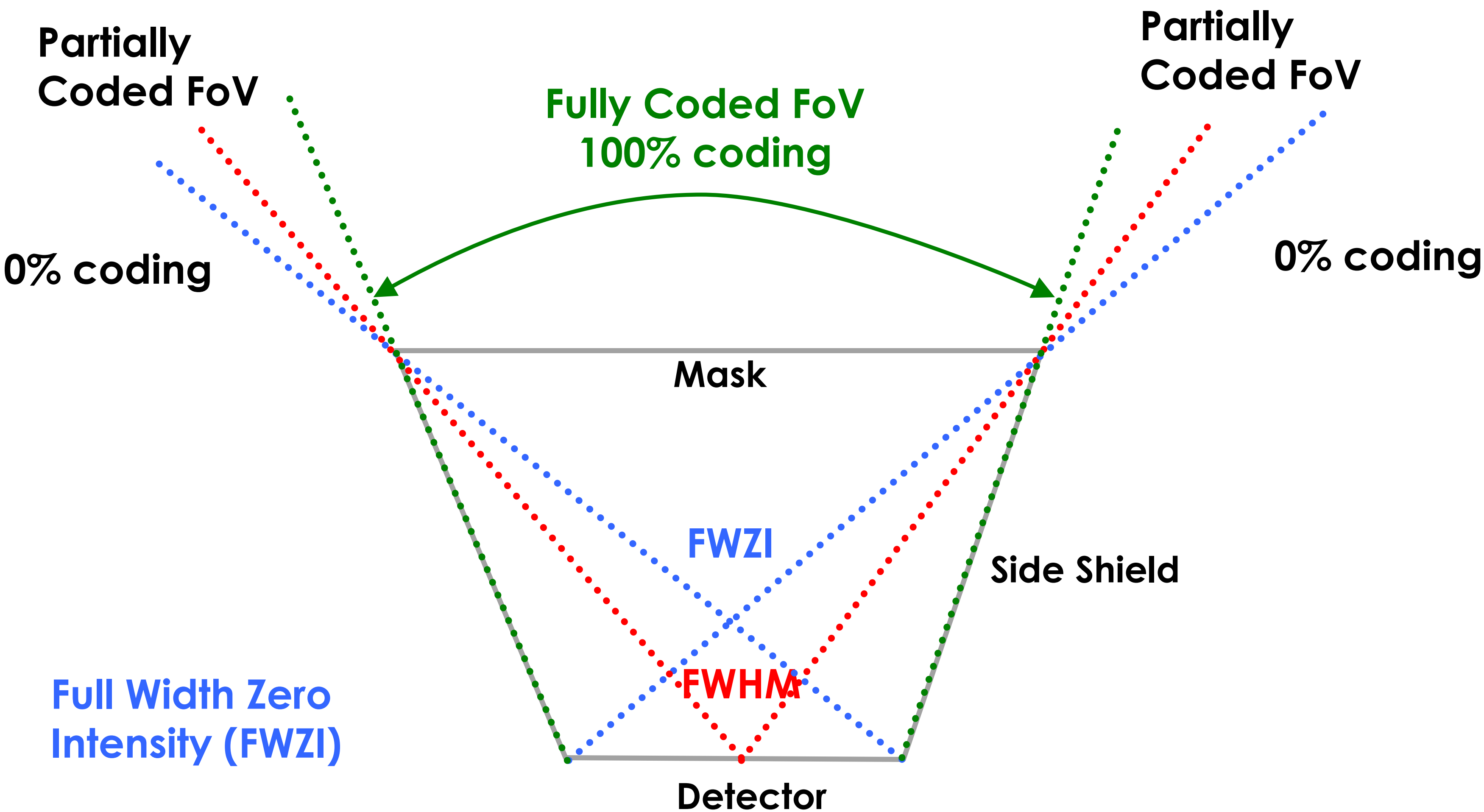
Field of View & Coding Fraction



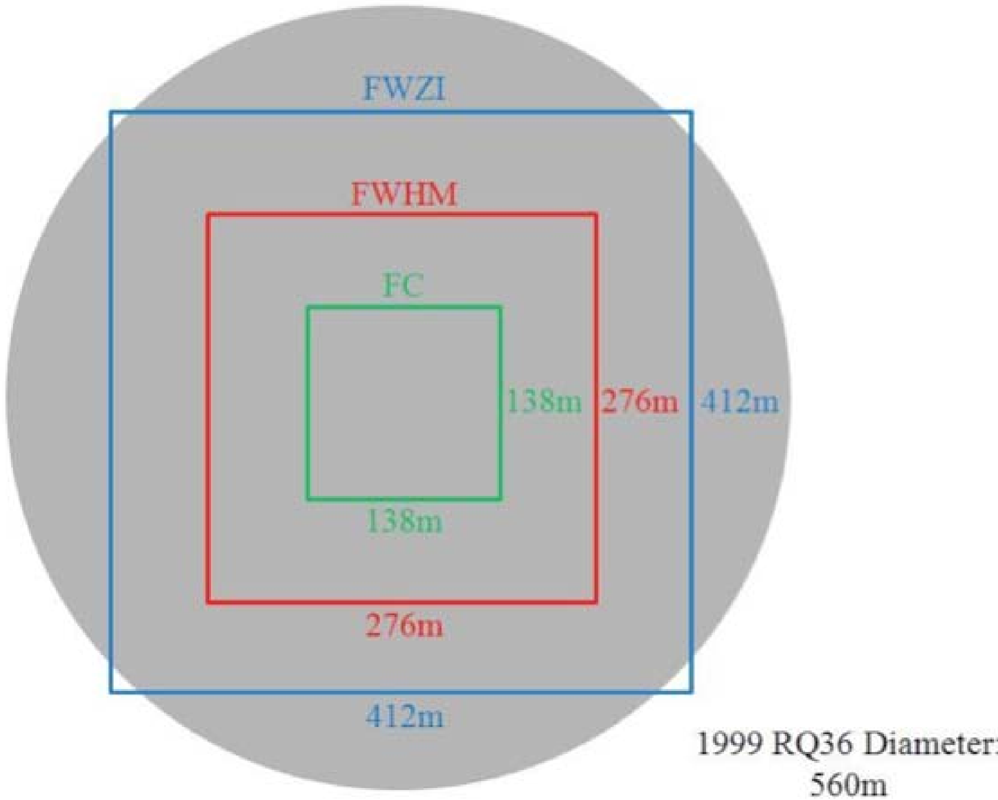
Field of View & Coding Fraction



Field of View & Coding Fraction

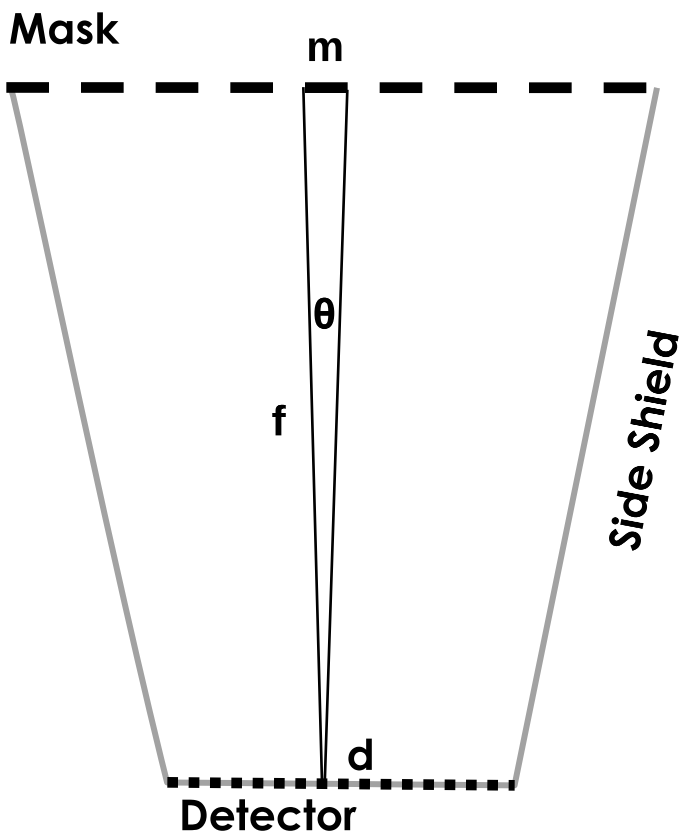


Field of View & Coding Fraction



at a distance of 700m (Phase 5B)

Basics in Coded-Aperture Imaging
Angular Resolution & Localization



mask pixel: m = 1.536 mm
detector pixel: d= 0.768 mm
mask-detector separation: f = 25 cm

Angular Resolution:

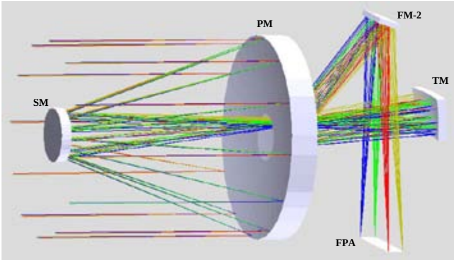
$\theta \sim \text{atan}(m/f) = 21.1'$ (if $d \ll m$)
⤷ 4.3 m at 700 m

$\theta = \text{atan}(\text{sqrt}(m^2+d^2)/f) = 23.6'$
⤷ 4.8 m at 700 m

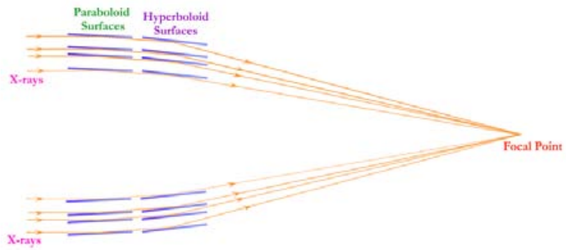
Source Localization:

$\delta = a\,\theta/(\sigma+b) = 2.94'$
for 90% radius, 5σ source,
$a \sim 0.7$, $b \sim 0$

Basics in Coded-Aperture Imaging: Effective Area



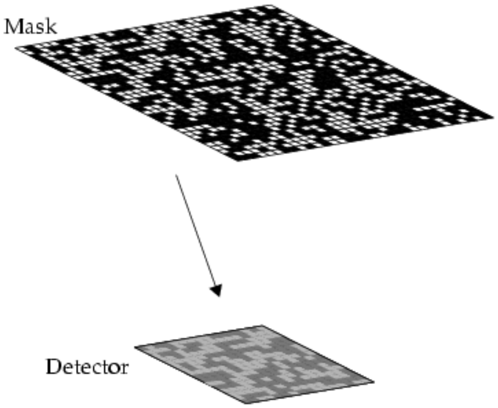
- What determines the sensitivity of a telescope?



- More light collection
  ► More sensitive

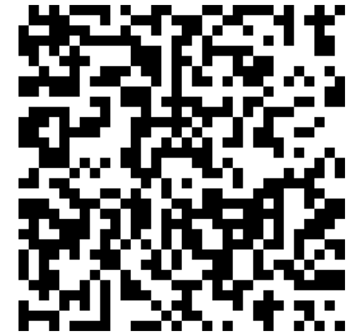
- The size does matter. But the size of what?

  focusing telescopes: mirror size
  non-focusing telescopes: detector size

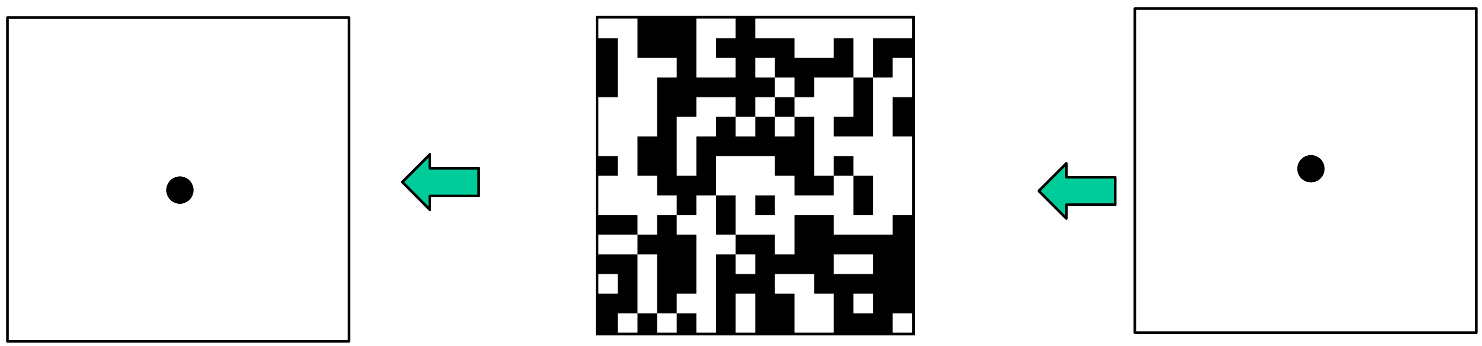


- Geometric Area (A_{geo}) vs Effective Area (A_{eff})
  $A_{eff} = A_{geo} * F_{effic}(E) * F_{atten}(E) * F_{mask}(E) * \ldots$

Image Reconstruction: Simple inversion



Mask (M)



Reconstructed Sky Image ($S'=S$)

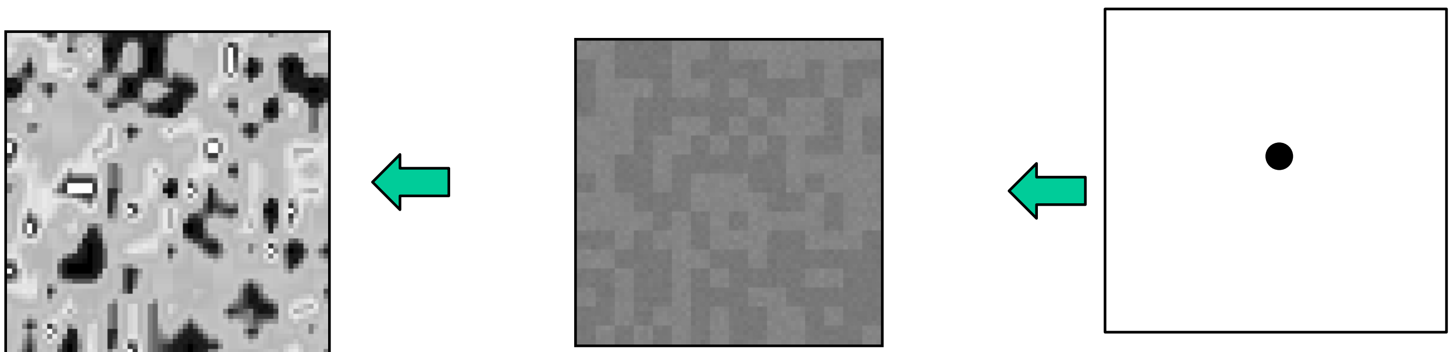
Detector Response (D)

Sky (S)

$$
\begin{aligned}
S' &= M^{-1} \cdot D \\
&= M^{-1} \cdot M \cdot S \\
&= S
\end{aligned}
$$

$$D = M \cdot S$$

Image Reconstruction: Simple inversion



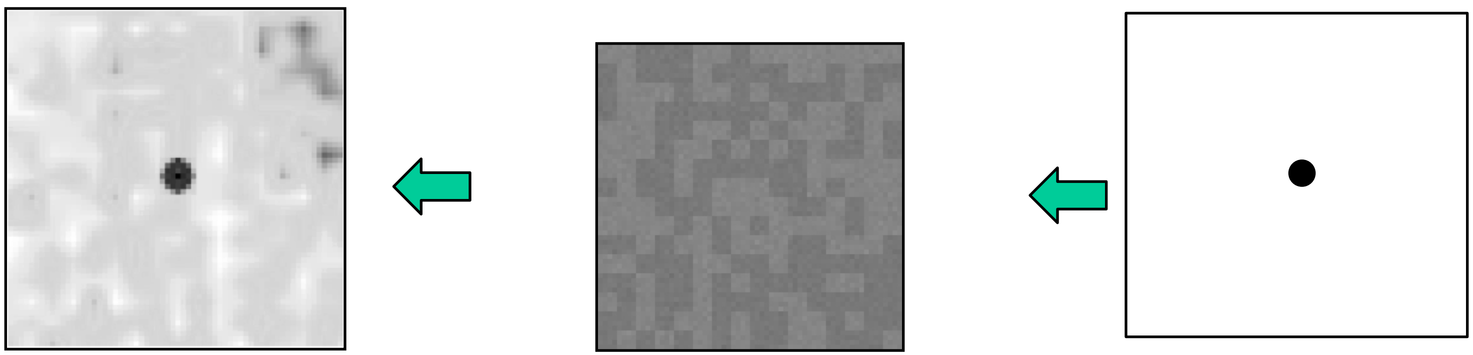
Reconstructed Sky Image (S') — Detector Response (D) — Sky (S)

$$S' = M^{-1} \cdot D$$
$$= S + M^{-1} \cdot O\,(M \cdot S)^{0.5} + M^{-1} \cdot B$$
$$\neq S$$

$$D = M \cdot S + O(M \cdot S)^{0.5} + B$$

M^{-1} is hard to find, sometimes there isn't one.
Inversion introduces Quantum Noise.

Image Reconstruction: Cross Correlation



Reconstructed Sky Image (S')

Detector Response (D)

Sky (S)

$$D = M \cdot S + O(M \cdot S)^{0.5} + B$$

$$S' = M' \cdot D$$

$$\sim S + M' \cdot O(M \cdot S)^{0.5} + M' \cdot B$$

$$\sim S + \text{const}$$

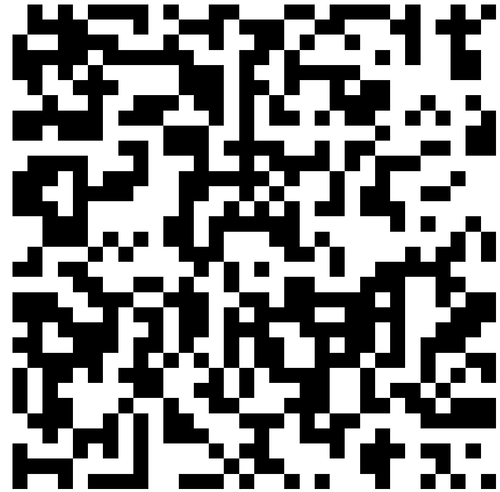
where $M' = a\,M + b$

(e.g. $M' = 2\,M - 1$ for 50% open mask)

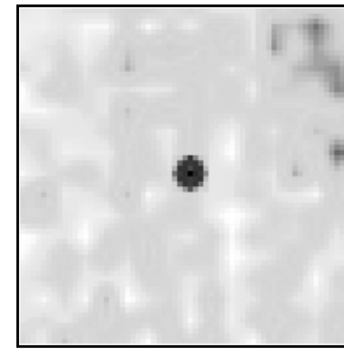
Cross-Correlation allows Fast Fourier Transformation (FFT)

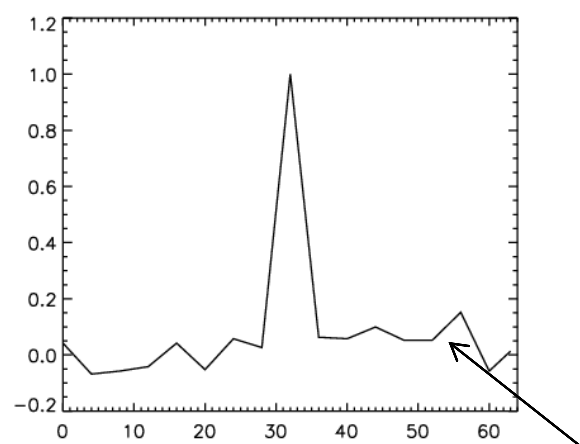
Mask Pattern

- Random Pattern
  - no constraint on mask geometry
  - Introduces coding noise
    - $\sim 1/N^{0.5}$
    - N = number of mask pixels



Point Spread Function





Side Lobe

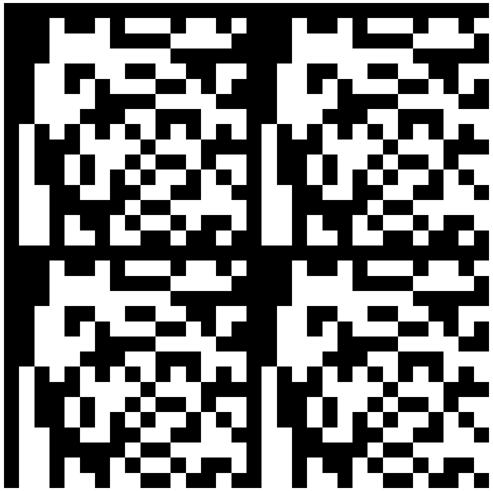
Mask Pattern

- Uniformly Redundant Array (URA)
  Fenimore (~1980)

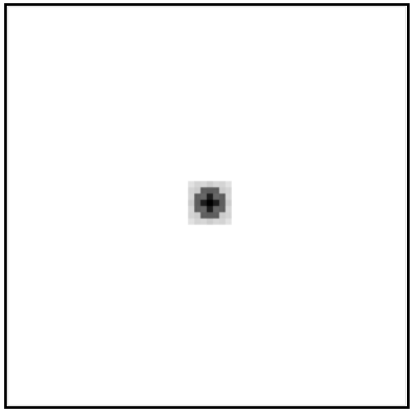
$$(a\,M + b)\cdot M = I$$

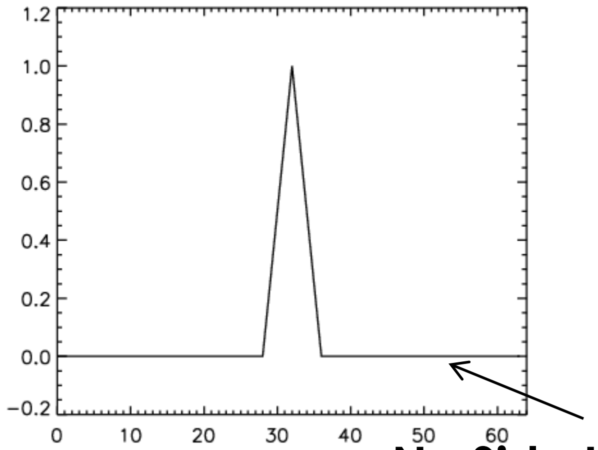
e.g. 2x2 cycle pattern
Detector should sample 1x1 cycle

- No coding noise
- No quantum noise
- limited geometries available
- ghost images
- Often hard to perfect it



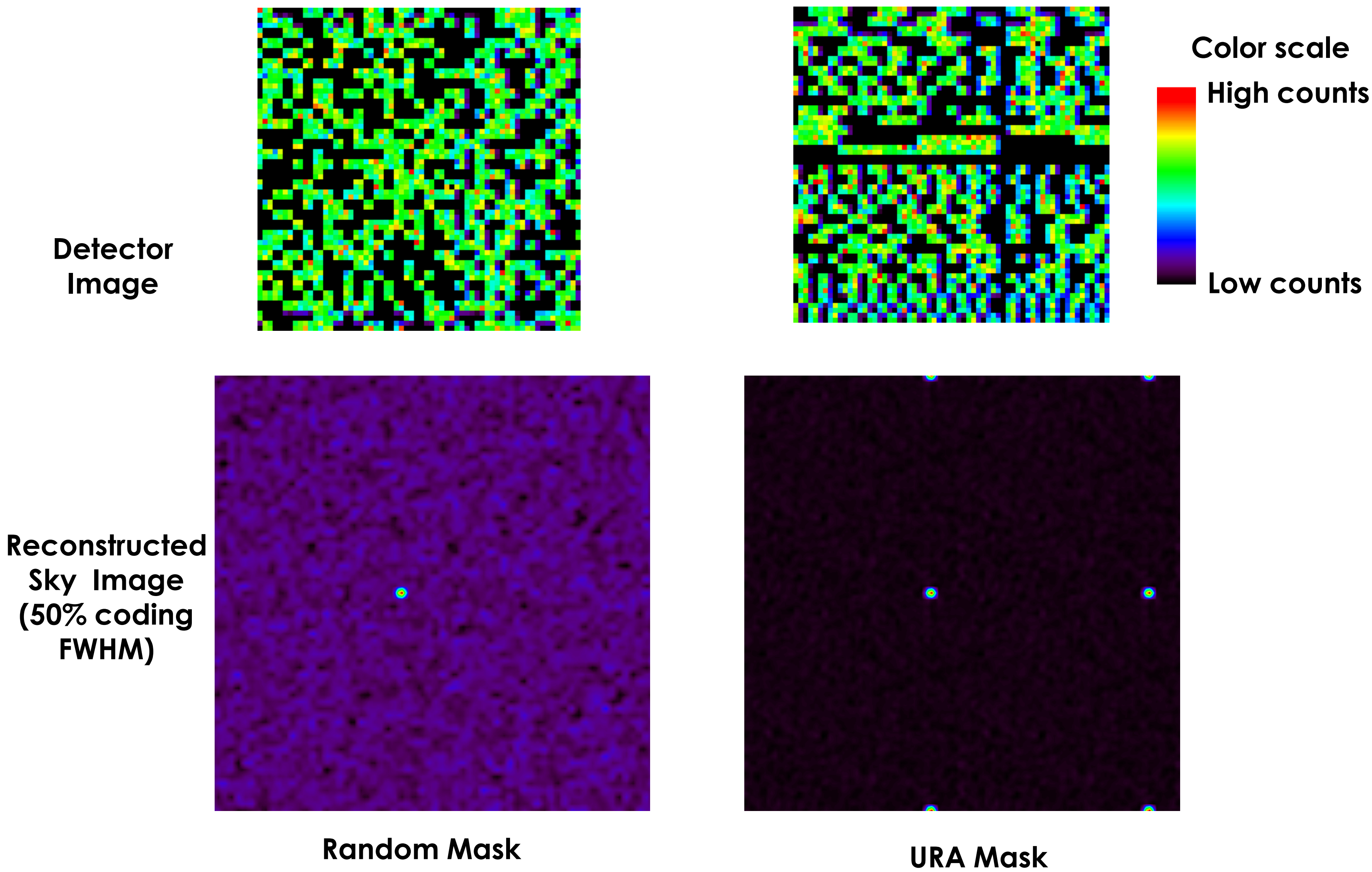
Point Spread Function





No Side Lobe

Random vs URA mask



Gaussian or Normal Distribution

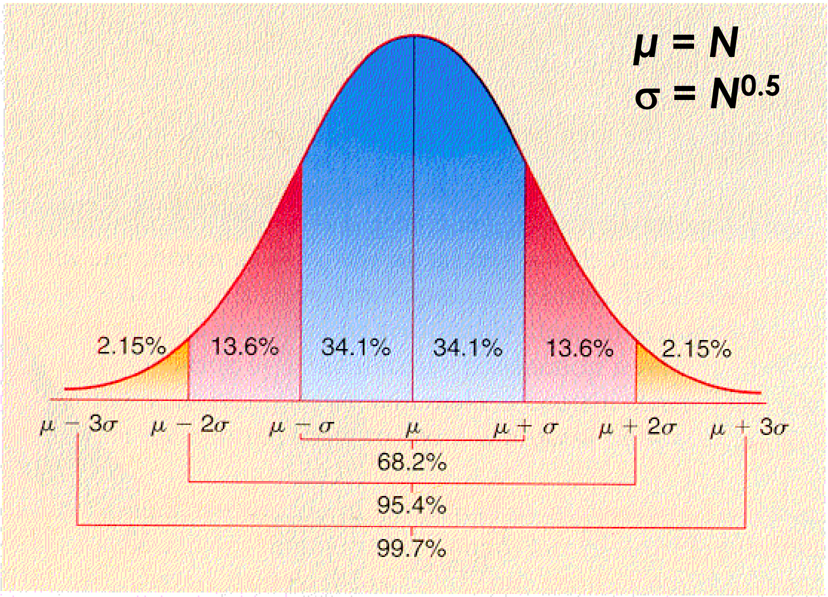
- **When the mean number of expected photon counts is N, its standard deviation (1σ) is $N^{0.5}$.**

- **For 68% of trials, we will get photons in between $N-N^{0.5}$ and $N+N^{0.5}$.**

- **e.g. When $N=25$, its standard deviation is 5. If you repeat the experiments, you should get photons somewhere between 20 and 30 for 68% of the trials.**

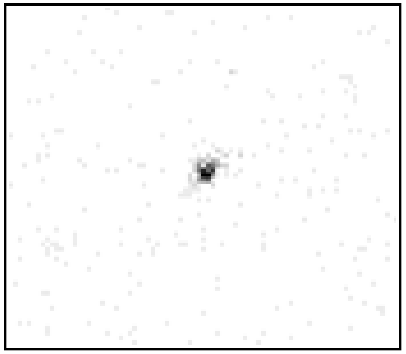
- **$1\sigma = N^{0.5}$ covers 68.2%**
  $2\sigma = 2N^{0.5}$ covers 95.4%
  $3\sigma = 3N^{0.5}$ covers 99.7%
  ….

- **$N^{0.5}$ works for unitless counts**
  e.g. $V = Q/C = eN/C$



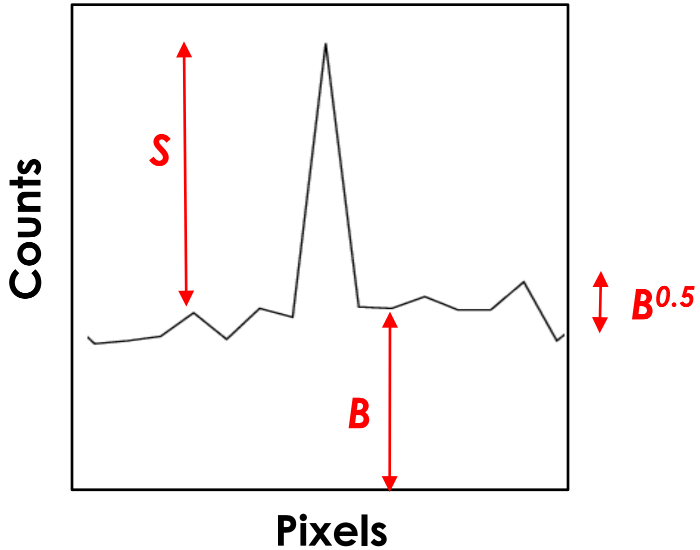
Signal-to-Noise Ratio (SNR) in Focusing Telescopes

- **Quantify the significance of detection**



$$SNR = S/B^{0.5}$$
$$= s\,A_m\,T\,/\,(b\,\Delta A_d\,T)^{0.5}$$
$$= s/b^{0.5}\;\;A_m\,T^{0.5}/\Delta A_d^{0.5}$$

$$SNR \sim s\,A_m$$



S: Total Source Counts
B: Background Counts in PSF

s: Source flux (cts/sec/cm^2)
b: Background rate (cts/sec/cm^2)

A_m: Collecting Area of Mirror
A_d: Effective Area of Detector
ΔA_d: PSF Size << A_d
T: Exposure in sec

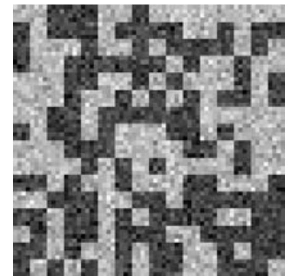
To claim a detection, SNR > ~3 – 5

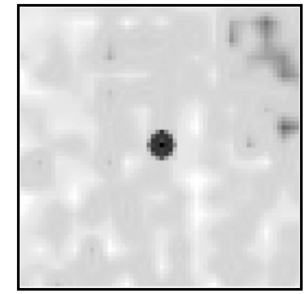
Signal-to-Noise Ratio (SNR) in Coded-Aperture Imaging

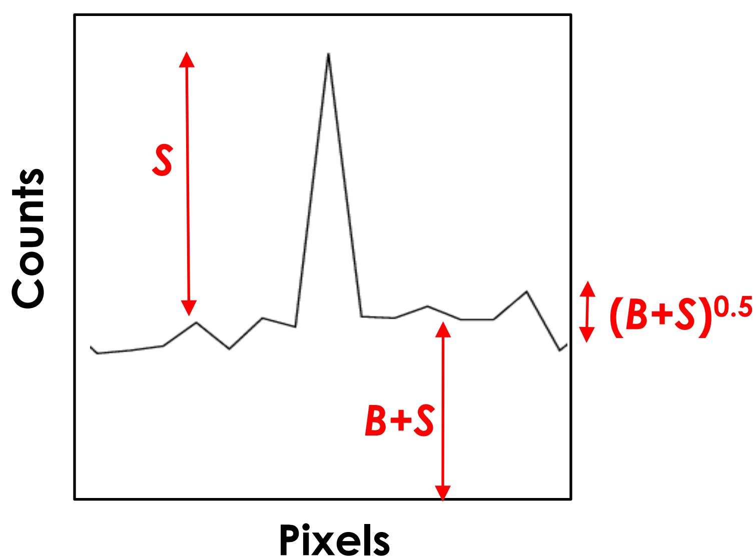
An Ideal Case with URA

$$SNR = S/(B+S)^{0.5\,*}$$
$$= s\,A_d\,T\,/\,((b+s)\,A_d\,T)^{0.5}$$
$$= s\,(A_d\,T)^{0.5}/(b+s)^{0.5}$$

$SNR \sim (s\,A_d)^{0.5}$
even when b=0







S: Total Source Counts
B: Total Background Counts

s: Source flux (cts/sec/cm^2)
b: Background rate (cts/sec/cm^2)

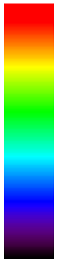
A_d: Effective Area of Detector
T: Exposure in sec

To claim a detection, SNR > ~5 – 7
*Without Imaging factor: 1 – d/(3 m)
SNR drops by 20-30% if d~m.
(Skinner 2008)

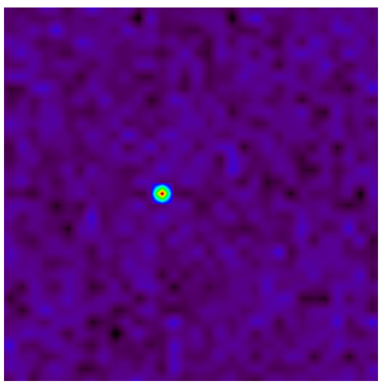
Examples of 10 point sources

Color scale

High counts

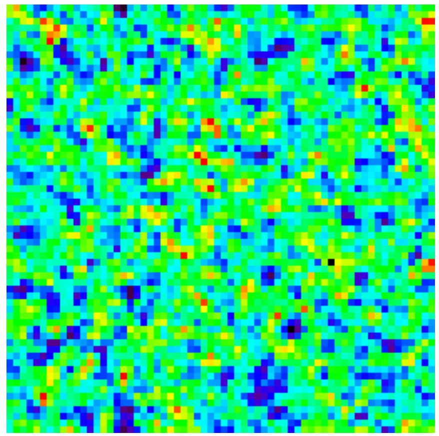


Low counts

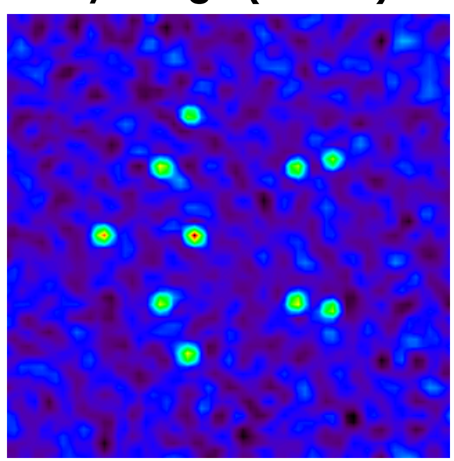


a single source

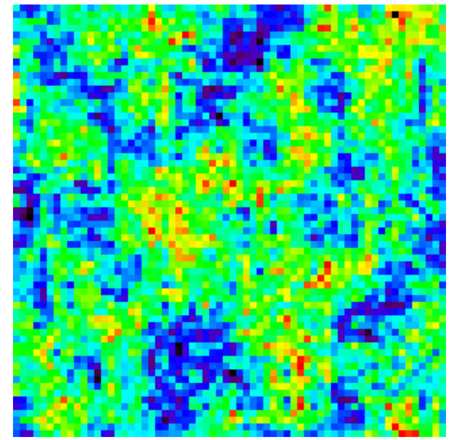
Pixellated Detector with Poisson Noise



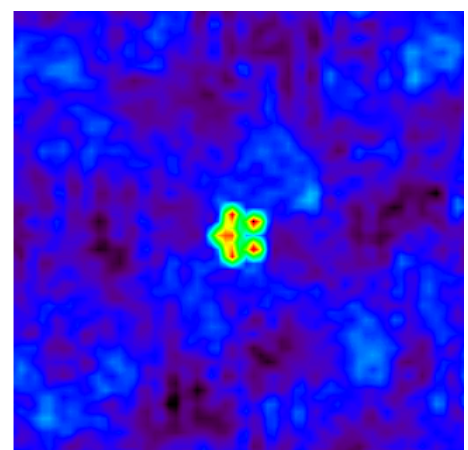
Sky Image (FCFoV)



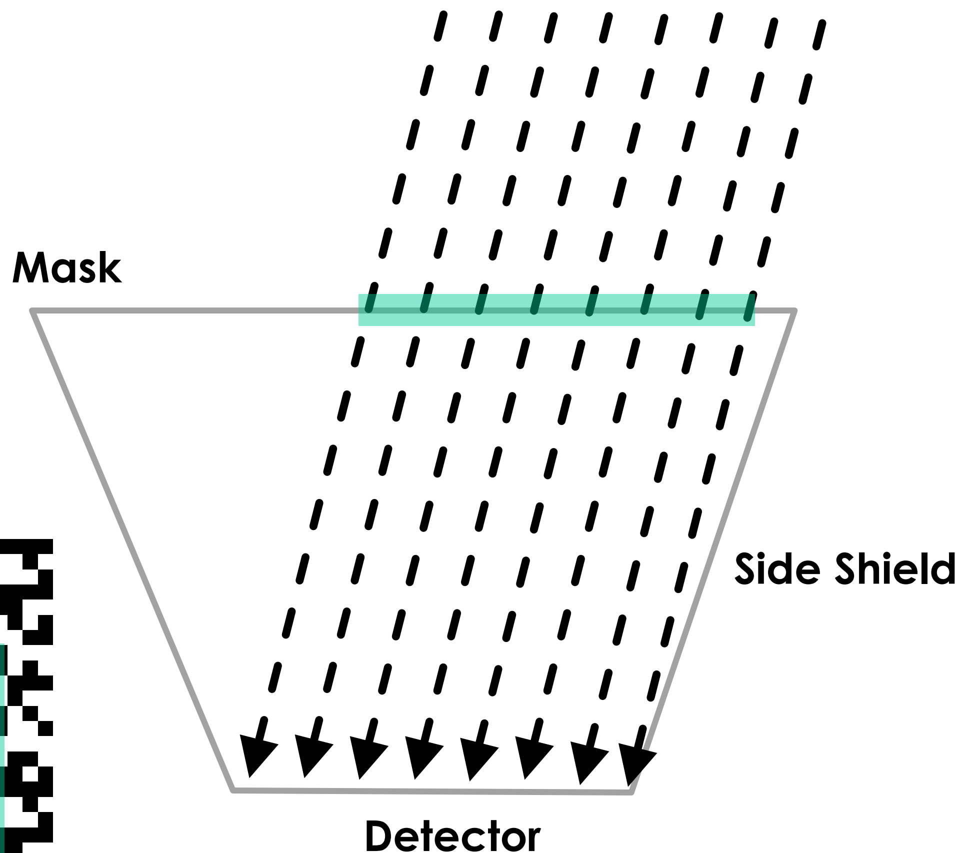
Pixellated Detector with Poisson Noise



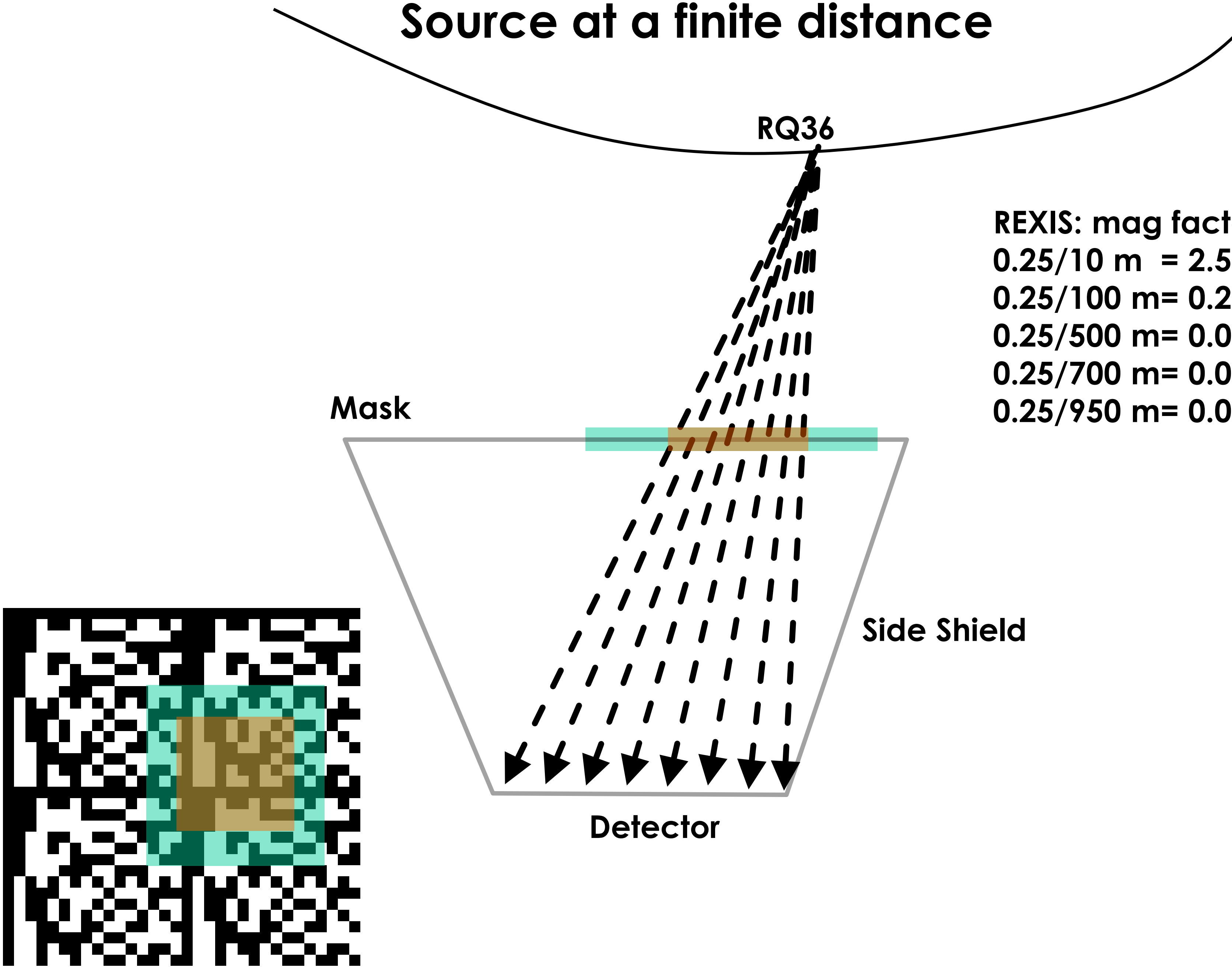
Sky Image (FCFoV)



Source at infinity

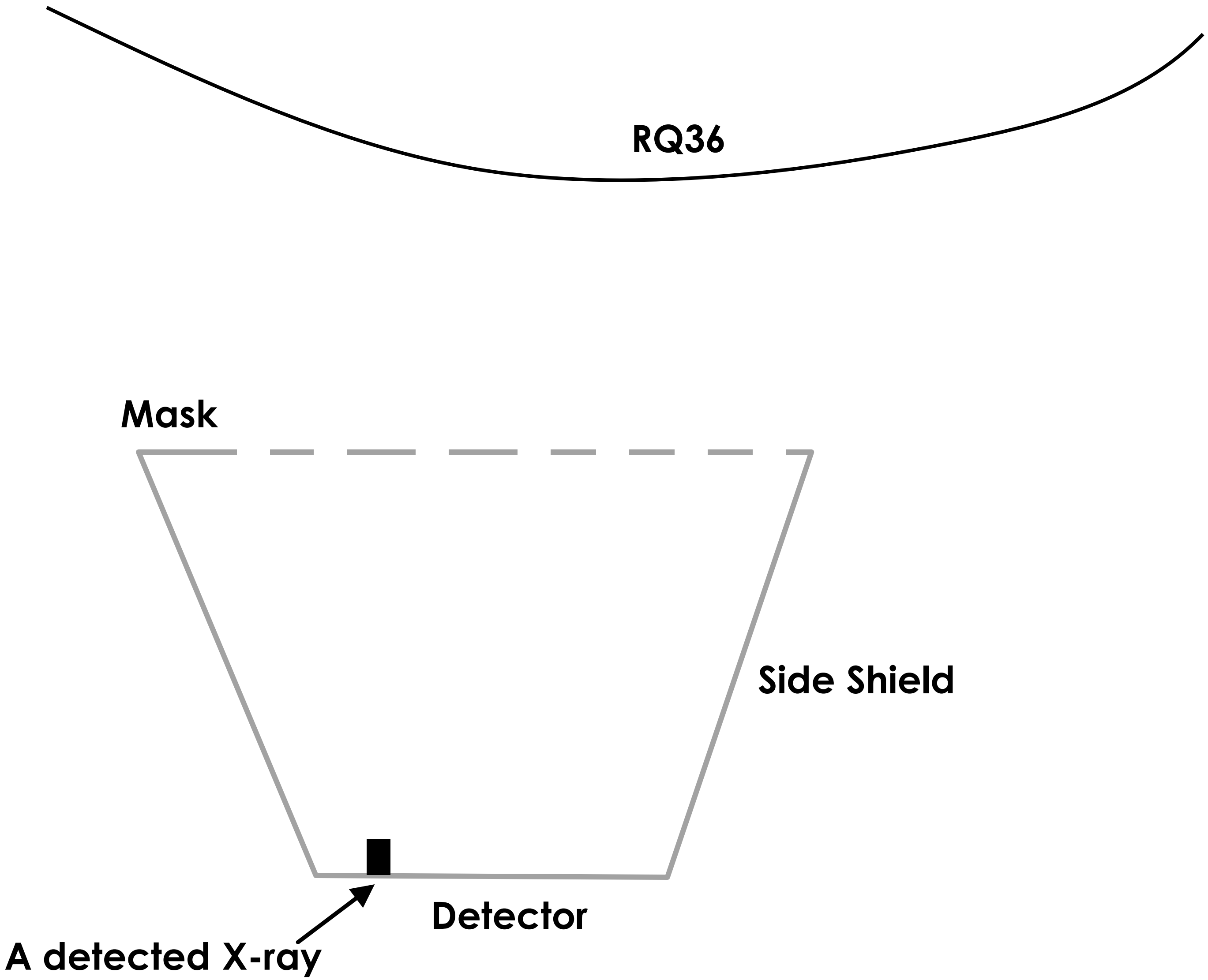


Source at a finite distance

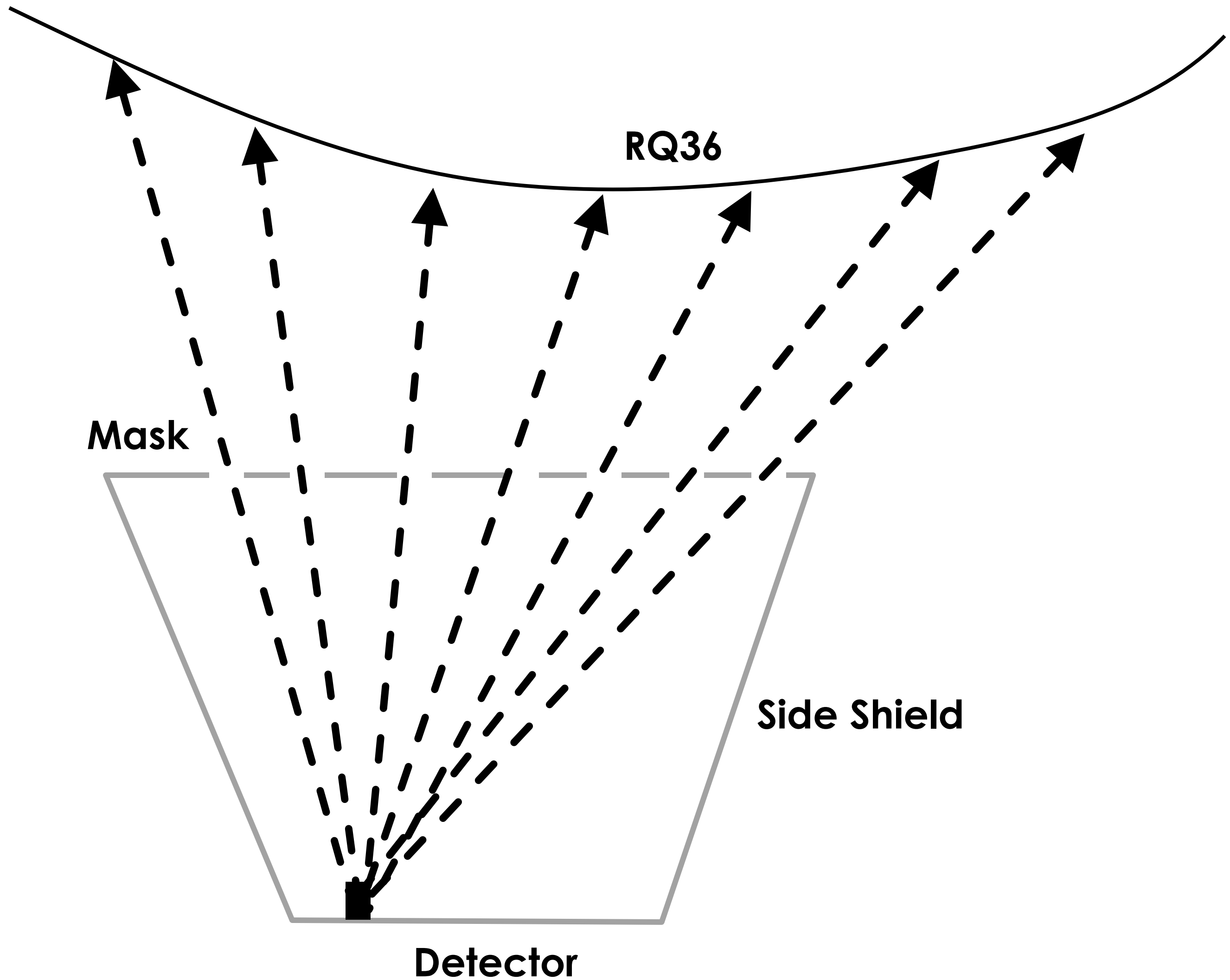


REXIS: mag factor
0.25/10 m = 2.5%
0.25/100 m= 0.25%
0.25/500 m= 0.050%
0.25/700 m= 0.036%
0.25/950 m= 0.026%

Back Projection



Back Projection



Challenges for REXIS

- **Diffuse Sources : redefine SNR**
- **Terminator Orbit**
- **Finite and varying source distances**
- **Scanning Coded-Aperture**
- **Solar flux dependence**
- **Not trivial to handle background subtraction or non-uniformity in the detector**
- **Regolith and surface non-uniformity unrelated atomic element composition**

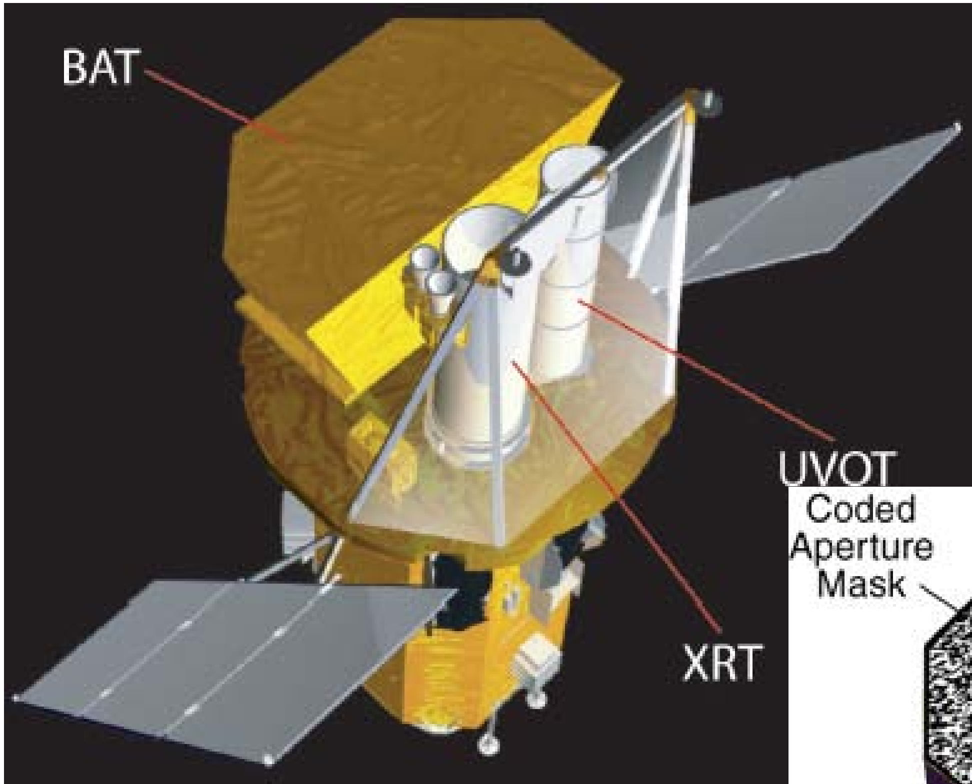
Mask Design for REXIS

- Open Hole Fraction
  - > Impact count rate; we may need >50% for >1keV.
  - > Energy dependent multi open fractional Mask?

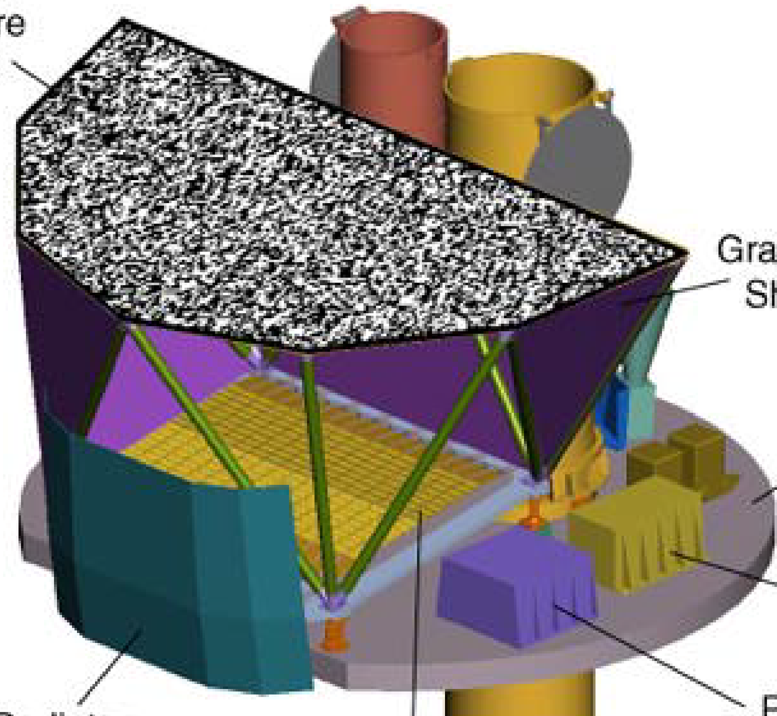
- Mask Pixel Size
  - > Impact Memory Requirement
  - > Multi-scale mask to cover a wide range of blob sizes?

- Mask Pattern (Random vs MURA, 2 Scale Mask)
  - > For (M)URA, allow one-full cycle in the detector
    with magnification factor
  - > Reverse mask pattern on one side for terminator orbits?

Example of Mask Patterns

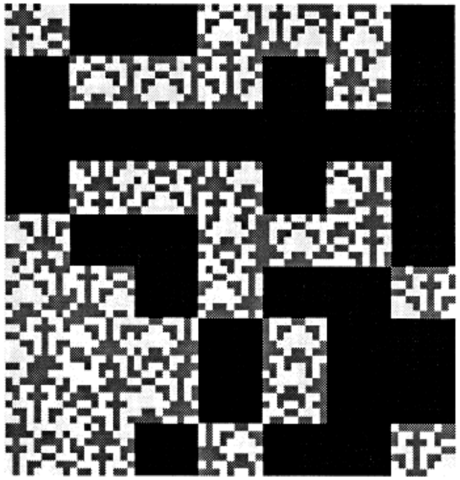


Swift/BAT
2004/11/20 -



Example of Mask Patterns

INTEGRAL/SPI: HURA



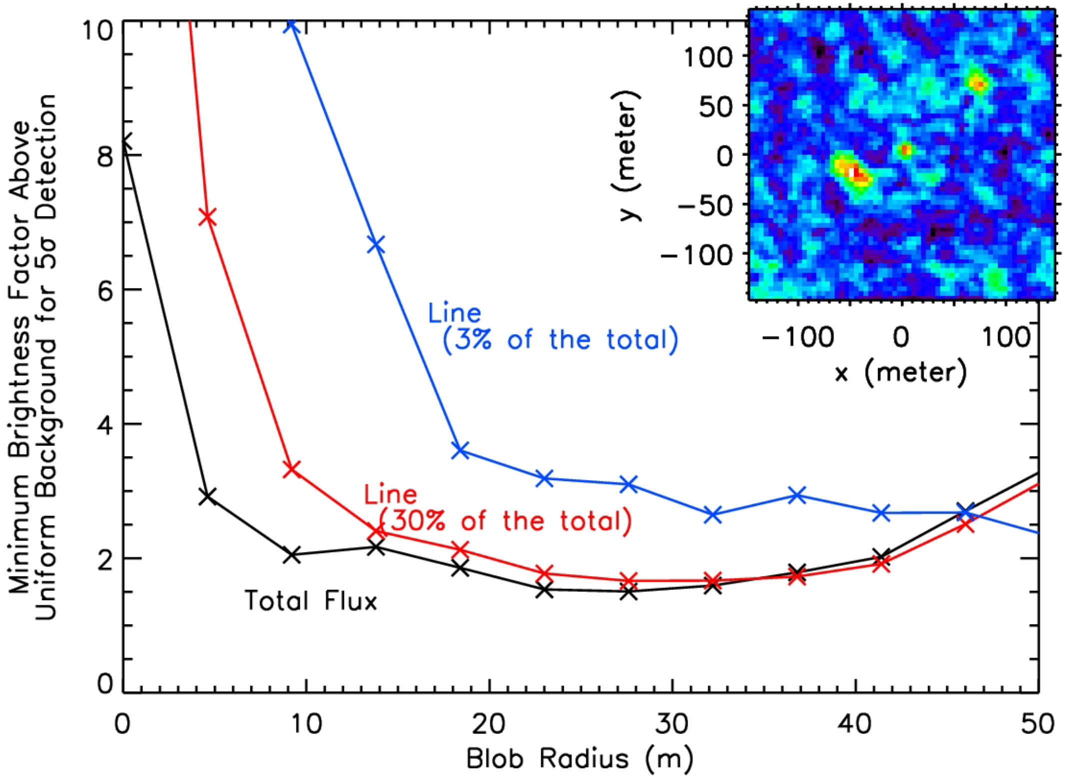
2 scale mask
(Skinner & Grindlay)

Multi Open Fraction Mask for REXIS?

e.g. 20% at 0.5 keV and 50% at 2 keV with multi-layer mask?

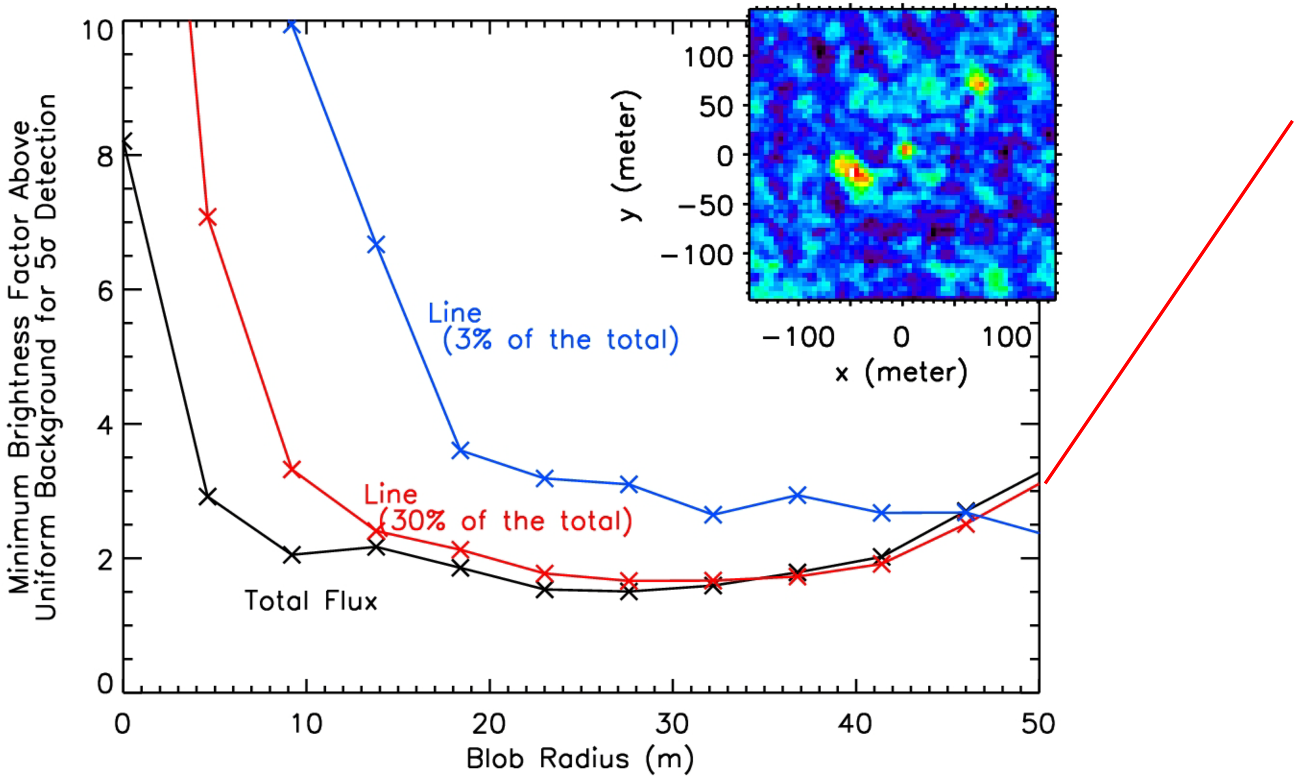
Mask Design for REXIS

- Open Hole Fraction > Impact count rate
- Mask Pixel Size> Impact Memory Requirement
- Mask Pattern (Random vs MURA, 2 Scale Mask)



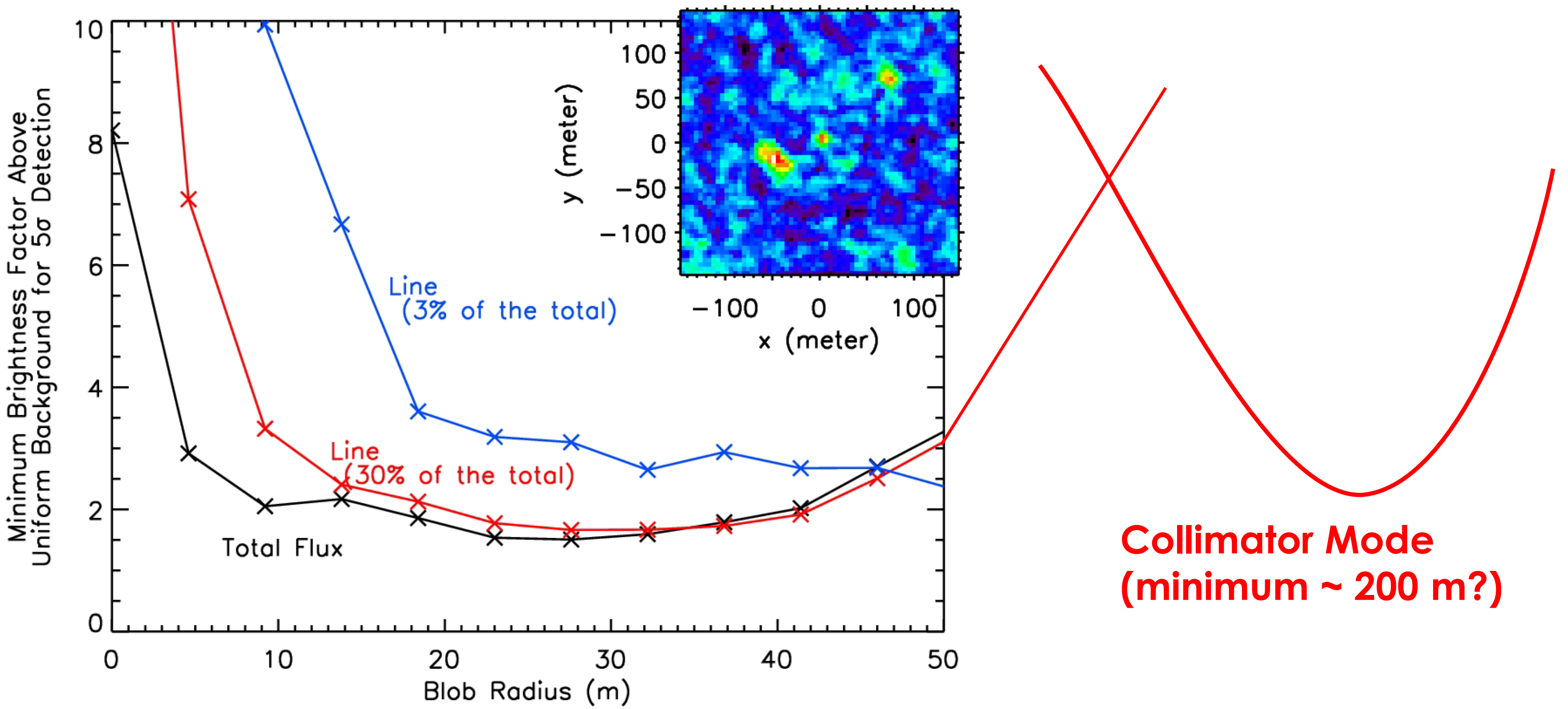
Mask Design for REXIS

- Open Hole Fraction > Impact count rate
- Mask Pixel Size> Impact Memory Requirement
- Mask Pattern (Random vs MURA, 2 Scale Mask)



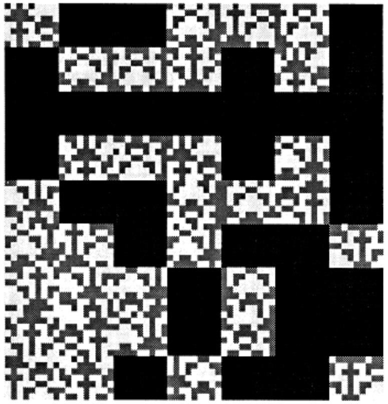
Mask Design for REXIS

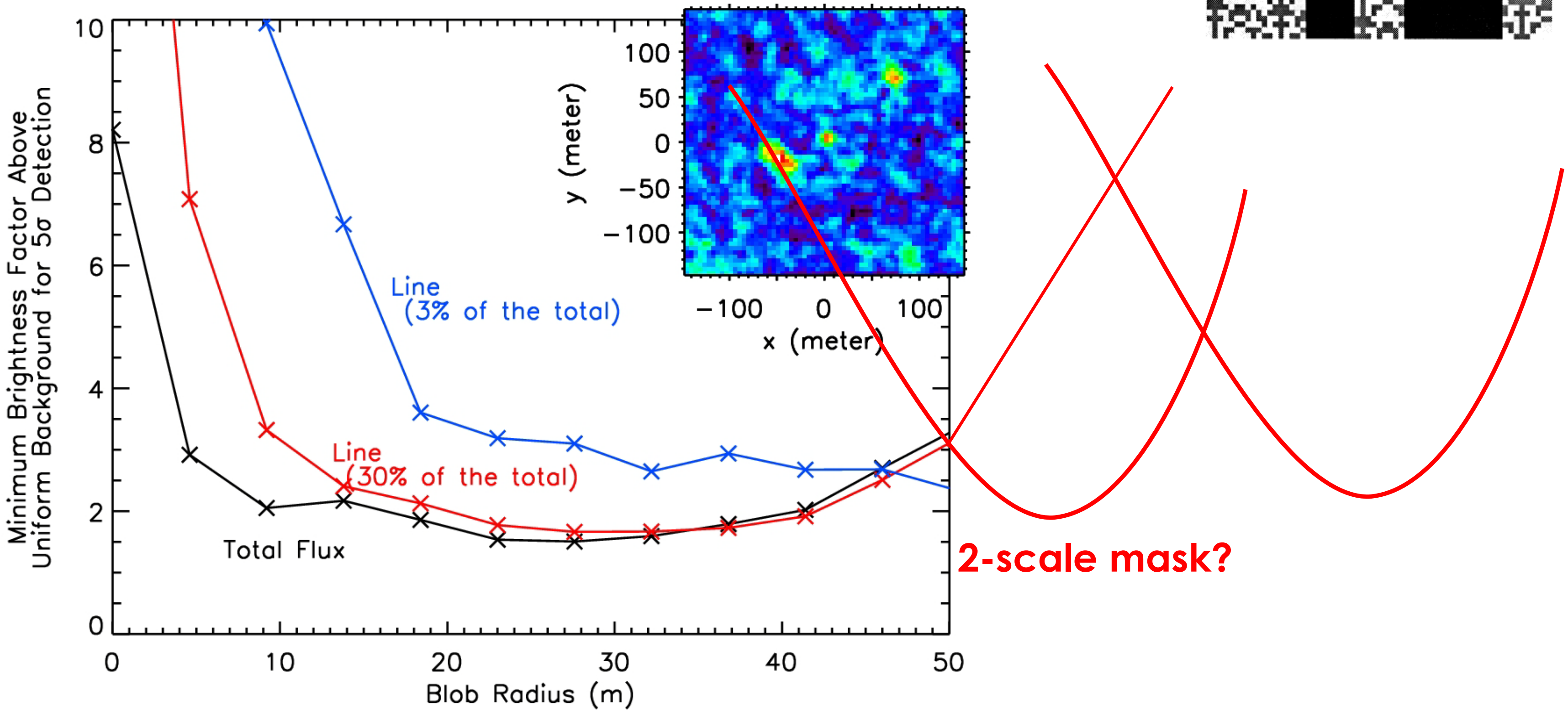
- Open Hole Fraction > Impact count rate
- Mask Pixel Size> Impact Memory Requirement
- Mask Pattern (Random vs MURA, 2 Scale Mask)



Mask Design for REXIS

- Open Hole Fraction > Impact count rate
- Mask Pixel Size> Impact Memory Requirement
- Mask Pattern (Random vs MURA, 2 Scale Mask)



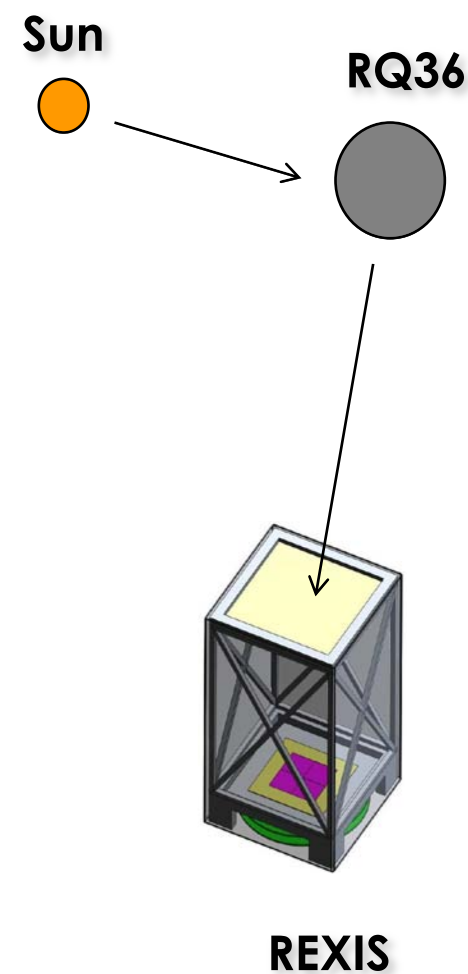


Overview

Solar X-rays
- XRF from RQ36

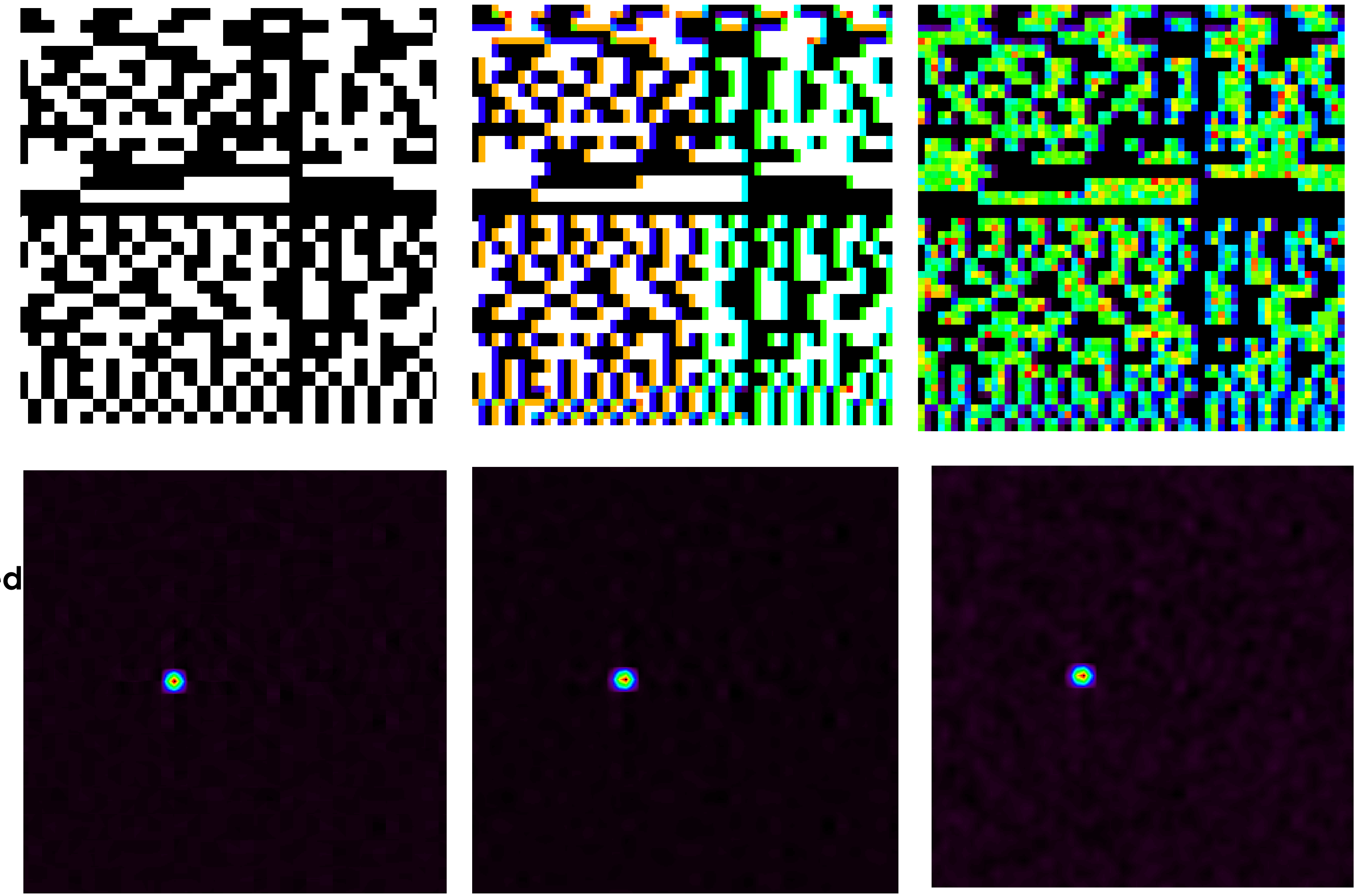
- X-ray Shadow on REXIS CCD by the mask
- Charges collected on CCD
- Amplified and Readout at a fixed cycle
- Series of x, y, E with Time tag

- Detector Image
- Sky Image
- Projection on RQ36 (or Back Projection)
- Map of Atomic Element Composition



Examples of Magnification & Poisson Noise (URA)

	Perfect Detector Without Poisson Noise	Pixellated Detector Without Poisson Noise	Pixellated Detector with Poisson Noise
Detector Image			
Reconstructed Sky Image (FCFoV)			



Examples of Magnification & Poisson Noise (Random Mask)