To: File

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Subject: Predicted HRC source count rates; ECF (energy flux density to counts conversion factor).

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Introduction

This Mathcad document provides a model of the end-to-end response of the HRC detectors to the observation of celestial x-ray sources. The document is interactive in the sense that all the equations and graphs will recalculate, if the arguments are changed. This document runs under Mathcad 2000. Figure 1 illustrates the detection geometry.

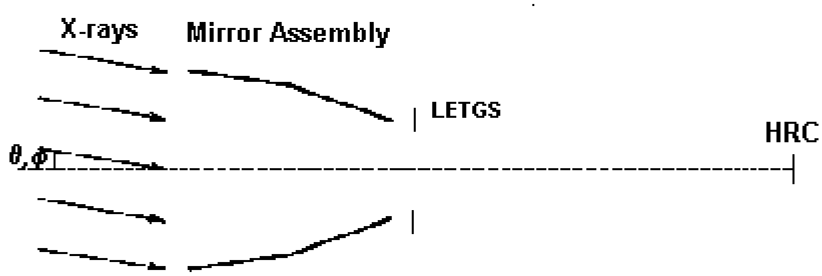


Figure 1. HRC detection geometry.

1. Model for Incident x-rays

The **incident x-rays** from the source are described by

dN(E)/dE, the spectral photon irradiance, in the case of a point source (units: ph s^{-1} keV^{-1} cm^{-2})

$d^2N(E,\theta,\phi)dEd\Omega$, the spectral photon radiance, in the case of an extended source (units: ph s^{-1} keV^{-1} cm^{-2} sr^{-1})

where E is the x-ray energy in keV and θ and ϕ are the field angle and azimuth angle of an emitting element of the extended source, respectively.

1.A Celestial Point Source Model

We model the spectral photon irradiance for a celestial point source by

$$\frac{dN(E)}{dE} = K \cdot \exp\left(-\sigma_e(E) \cdot N_H\right) \cdot f(S,E)$$

where

K = a normalization constant

$\sigma_e(E)$ = photoelectric cross-section per hydrogen atom for absorption of photons of energy E by interstellar medium.

S is a parameter in the spectral shape function f(S,E)

Example spectral shape functions:

Power law

S = n, f(S,E) = E^{-n}

Blackbody

S = T, f(S,E) = $E^2/(\exp(E/kT) - 1)$

Thermal bremsstrahlung

S = T, f(S,E) = $g(T,E)\exp(-E/kT)/E(kT)^{1/2}$

where g(T,E) is the temperature-averaged Gaunt factor

Raymond-Smith thermal plasma

S=T, f(S,E) is given by a table (e.g. from XSPEC)

Additional model spectral shapes can be found in *XSPEC*, *An X-ray Spectral Fitting Package*, *User's Guide for Version*, NASA Goddard Space Flight Center.

1.B. Interstellar absorption

This section provides a model of the net photoelectric absorption cross-section per hydrogen atom as a function of energy for a column of gas with "normal abundances" (see table below). The range of validity is 0.030 - 10.000 keV.The cross-section is given by the following function:

$$\sigma_e(E) = \left(c_0 + c_1 \cdot E + c_2 \cdot E^2\right) \cdot E^{-3} \cdot 10^{-24} \text{cm}^2 \quad \text{(E in keV)}$$

The coefficients c are given by step functions defined by the following vectors, with the break points given by the vector e

$e :=$	$c_0 :=$	$c_1 :=$	$c_2 :=$
.100	17.3	608.1	–2150
.284	34.6	267.9	–476.1
.400	78.1	18.8	4.3
.532	71.4	66.8	–51.4
.707	95.5	145.8	–61.1
.867	308.9	–380.6	294.0
1.303	120.6	169.3	–47.7
1.840	141.3	146.8	–31.5
2.471	202.7	104.7	–17.0
3.210	342.7	18.7	0.0
4.038	352.2	18.7	0.0
7.111	433.9	–2.4	0.75
8.331	629.0	30.9	0.0
10.000	701.2	25.2	0.0

This is to be understood as

c_0 = 17.3 for E < 0.100, c_0 = 34.6 for E < 0.284 but >= 0.100, etc.

The break at 0.100 keV is introduced to give an adequate fit. The other breaks are at the elemental absorption edges.

The model is from Morrison & McCammon, Ap. J., **270**, 119, 1983.

The coefficients can then be written in the following functional forms:

$$C_0(E) := \text{if}\left(E < e_1, c_{0_1}, \text{if}\left(E < e_2, c_{0_2}, \text{if}\left(E < e_3, c_{0_3}, \text{if}\left(E < e_4, c_{0_4}, \text{if}\left(E < e_5, c_{0_5}, \text{if}\left(E < e_6, c_{0_6}, \text{if}\left(E < e_7, c_{0_7}, \text{if}\left(E <\right.\right.\right.\right.\right.\right.\right.\right.$$

$$C_1(E) := \text{if}\left(E < e_1, c_{1_1}, \text{if}\left(E < e_2, c_{1_2}, \text{if}\left(E < e_3, c_{1_3}, \text{if}\left(E < e_4, c_{1_4}, \text{if}\left(E < e_5, c_{1_5}, \text{if}\left(E < e_6, c_{1_6}, \text{if}\left(E < e_7, c_{1_7}, \text{if}\left(E <\right.\right.\right.\right.\right.\right.\right.\right.$$

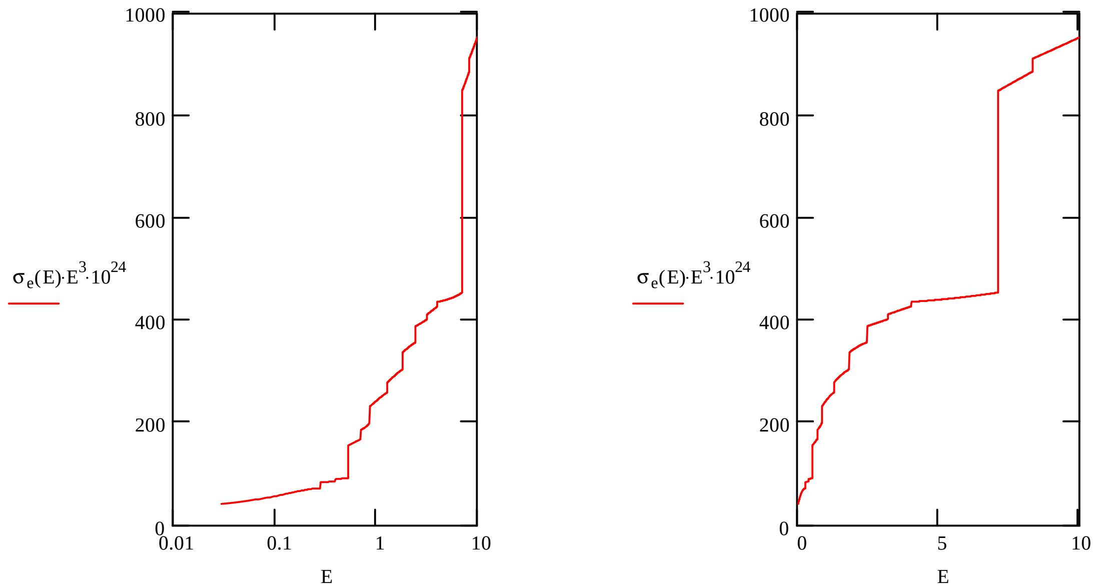
$$C_2(E) := \text{if}\left(E < e_1, c_{2_1}, \text{if}\left(E < e_2, c_{2_2}, \text{if}\left(E < e_3, c_{2_3}, \text{if}\left(E < e_4, c_{2_4}, \text{if}\left(E < e_5, c_{2_5}, \text{if}\left(E < e_6, c_{2_6}, \text{if}\left(E < e_7, c_{2_7}, \text{if}\left(E <\right.\right.\right.\right.\right.\right.\right.\right.$$

We can now define the cross-section function:

$$\sigma_e(E) := \left(C_0(E) + C_1(E)\cdot E + C_2(E)\cdot E^2\right)\cdot E^{-3}\cdot 10^{-24}$$

Plotting σ scaled by E^3:

$$E := 0.030, 0.035 .. 10$$



Net photoelectric absorption cross-section per hydrogen atom as a function of energy, scaled by $(E/1\text{keV})^3$ and in units of 10^{-24} cm^2.

The assumed elemental abundances (Log_{10}) relative to hydrogen (12.00) are:

H:12.00, **He**:11.00, **C**:8.65, **N**:7.96,**O**:8.87, **Ne**:8.14, **Na**:6.32, **Mg**:7.60, **Al**:6.49, **Si**:7.57, **S**:7.28, **Cl**:5.28, **Ar**:6.58, **Ca**:6.35, **Cr**:5.69, **Fe**:7.52, **Ni**:6.26

1.C. Source models

1.C.1. Power Law

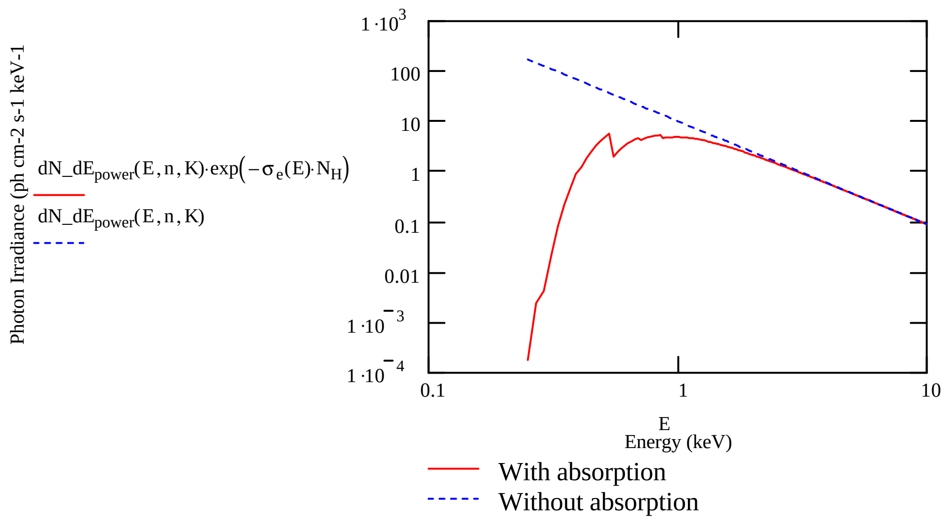
$$dN\_dE_{power}(E, n, K) := K \cdot E^{-n}$$ photons cm^{-2} s^{-1} keV^{-1}

A good example of a power law spectrum is the Crab Nebula with

$K := 10$ $n := 2.05$ $N_H := 3 \cdot 10^{21}$ cm^{-2} (valid for 0.1 - 100 keV)

Plotting the spectrum from 0.25 keV to 10 keV:

$E := 0.25, 0.27 .. 10$



Crab Nebula Spectrum

As a further example, we can calculate the total luminosity of the Crab Nebula in the 0.1 - 100 keV band:

Distance to the Crab: $D_{Crab} := 2200$

$$Lum_{Crab} := 1.60 \cdot 10^{-9} \cdot 4 \cdot \pi \cdot \left(D_{Crab} \cdot 3.09 \cdot 10^{18}\right)^2 \cdot \int_{.1}^{100} E \cdot dN\_dE_{power}(E, n, K)\, dE$$ (no absorption)

$Lum_{Crab} = 6.09 \times 10^{37}$ erg s^{-1}

1.C.2. Blackbody

$$dN\_dE_{BB}(E,kT,K) := K \cdot \frac{E^2}{\left(\exp\left(\frac{E}{kT}\right) - 1\right)}$$ photons cm^{-2} s^{-1} keV^{-1}

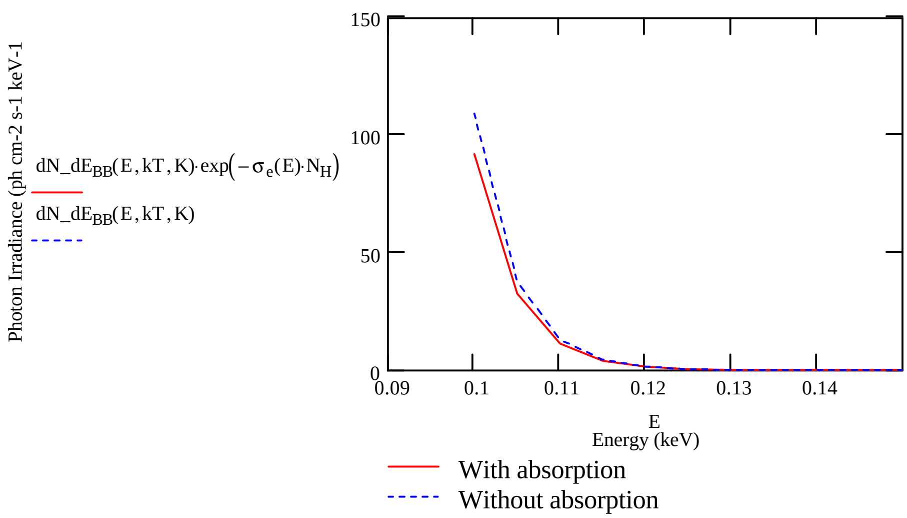
where the temperature kT is given in keV.

A good example of a blackbody spectrum is the white dwarf HZ43 with

$K := 1.3 \cdot 10^{14}$ $\quad kT := \frac{50000}{11.6 \cdot 10^{6}}$ keV $\quad N_H := 3 \cdot 10^{18}$ cm^{-2}

Plotting the spectrum from 0.1 keV to 0.15 keV:

$E := 0.1, 0.105 .. 0.15$



HZ43 Spectrum

1.C.3. Thermal Bremsstrahlung

Definition of temperature averaged Gaunt factor:

$$\Gamma := \exp(0.577)$$

$$g(kT, E) := if\left[\frac{kT}{E} < 0.01, \frac{\sqrt{3}}{\pi} \cdot \ln\left(\frac{4 \cdot kT}{\Gamma \cdot E}\right), \left(\frac{E}{kT}\right)^{-0.4}\right]$$

$$dN\_dE_{thermal}(E, kT, K) := K \cdot \frac{\exp\left(-\frac{E}{kT}\right)}{E \cdot \sqrt{kT}} \cdot g(kT, E)$$ photons cm^{-2} s^{-1} keV^{-1}

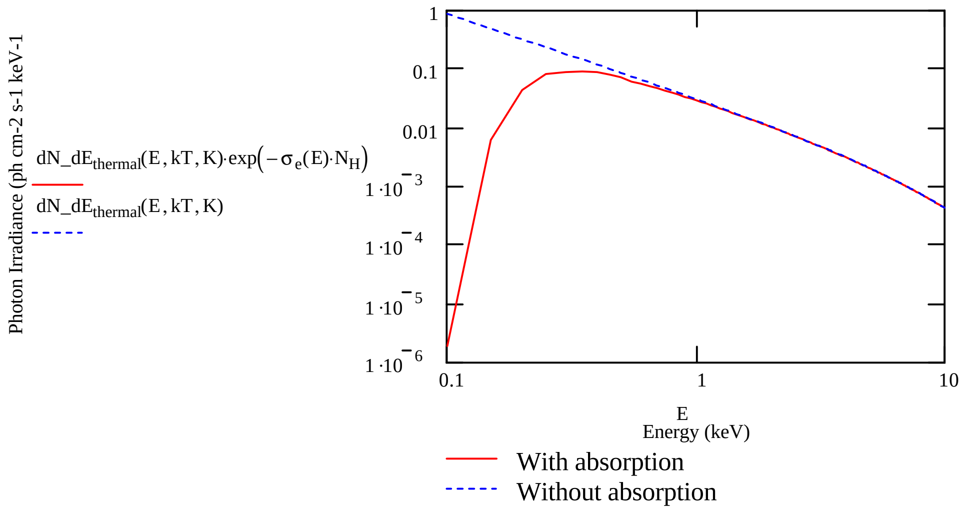
where the temperature kT is given in keV.

A good example of a thermal bremsstrahlung spectrum is the continuum from the central region of the Coma cluster of galaxies with

$K := 4.3 \cdot 10^{-2}$ $kT := 8.5$ keV $N_H := 2.3 \cdot 10^{20}$ cm^{-2}

Plotting the spectrum from 0.05 keV to 0.2 keV:

$$E := 0.1, 0.15 .. 10.0$$



Coma Central Region Spectrum

1.C.4. Raymond-Smith thermal plasma

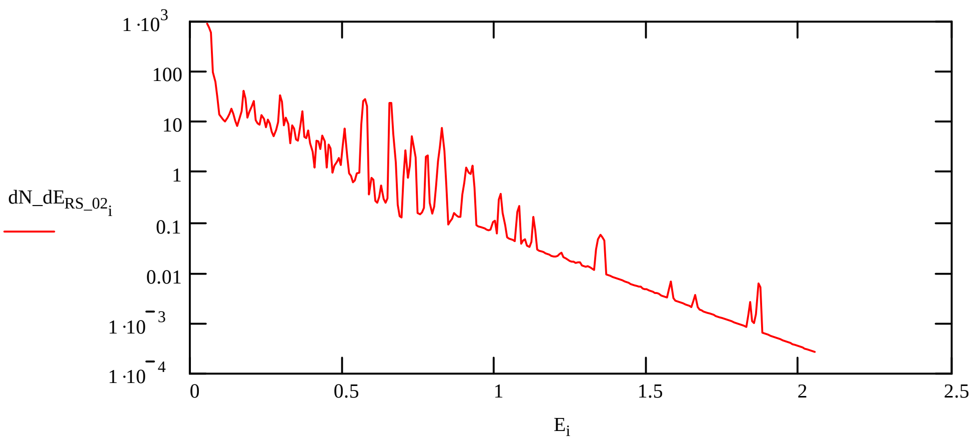
Import photon spectrum from XSPEC:

RS_02 := D:\..\rs.0.2.prn   kT=0.2 keV   bin width= 3.32502 eV

$E := RS\_02^{\langle 0 \rangle}$ $dN\_dE_{RS\_02} := RS\_02^{\langle 2 \rangle}$

Plot the spectrum:

$i := 0..300$



2.A HRC-S effective area model (central T)

Import model:

$hrcsea :=$

D:\..\hrcsea.prn

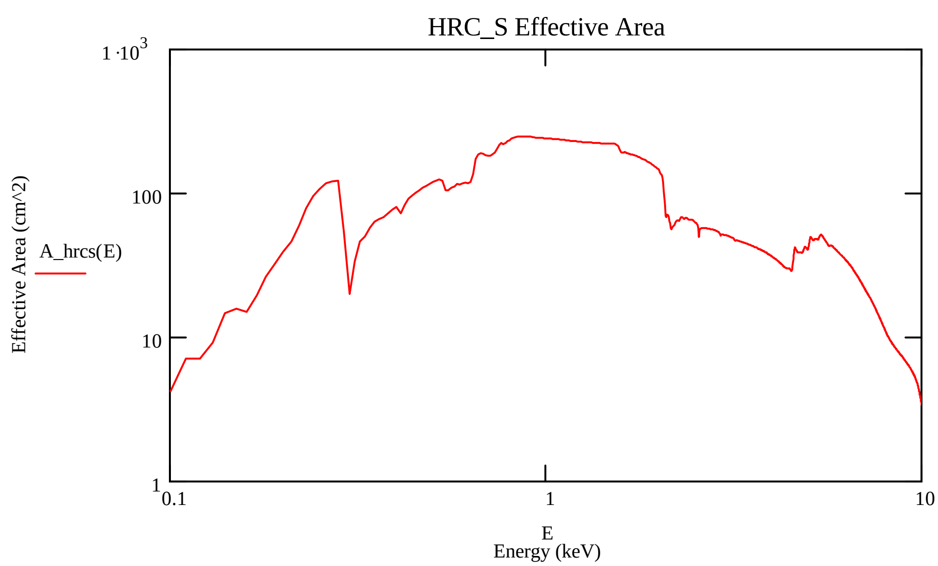
$Energy := hrcsea^{\langle 0 \rangle}$ $\quad Energy := \frac{Energy}{1000}$ $\quad Area\_hrcs := hrcsea^{\langle 4 \rangle}$

Create effective area function through linear interpolation:

$A\_hrcs(E) := linterp(Energy, Area\_hrcs, E)$

Plot the effective area model:

$E := 0.1, 0.11 .. 10.0$



2.B HRC-I effective area model

Import model:

hrciea :=

D:\..\hrciea.prn

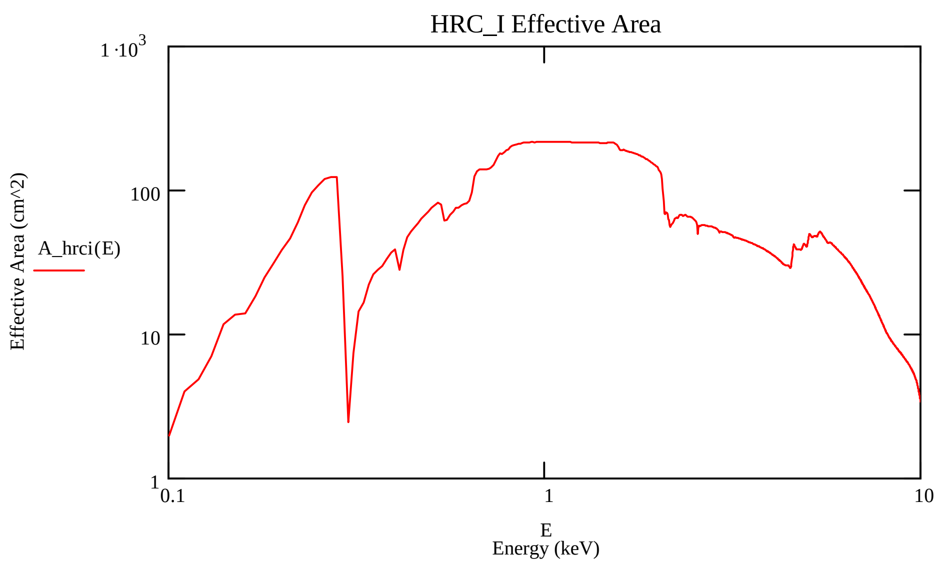
$Energy := hrciea^{\langle 0 \rangle}$ $Energy := \frac{Energy}{1000}$ $Area\_hrci := hrciea^{\langle 4 \rangle}$

Create effective area function through linear interpolation:

$A\_hrci(E) := linterp(Energy, Area\_hrci, E)$

Plot the effective area model:

$E := 0.1, 0.11 .. 10.0$



3. Predicted HRC point source count rates

The predicted HRC count rates depend on the assumed incident spectrum, which is not well known for many sources. In order to facilitate count rate predictions we will provide the standard energy-to-counts conversion factor (ECF) for several spectral models (see sec. 1) and various spectral parameters.

3.A. HRC-S

The HRC-S count rate from a point source is the convolution of the source spectrum (sec. 1) with the HRC-S effective area (sec. 2):

$$CR = \int_0^{\infty} \frac{dN(E)}{dE} \cdot A\_hrcs(E)\, dE \qquad \text{ct s}^{-1}$$

It is useful to calculate the unabsorbed energy flux density (in units of 10^{-10} erg s^{-1} cm^{-2}) in the AXAF energy band (0.1 - 10.0 keV) at the entrance aperture of the mirror assembly:

$$\Phi = 16 \cdot \int_{0.1}^{10.0} E \cdot \frac{dN_{unabs}}{dE}\, dE \qquad 10^{-10} \text{ erg s}^{-1} \text{ cm}^{-2}$$

We can then calculate the energy-to-counts conversion factor ECF by dividing CR by Φ:

$$ECF = \frac{\int_0^{\infty} \frac{dN(E)}{dE} \cdot A\_hrcs(E)\, dE}{16 \cdot \int_{0.1}^{10.0} E \cdot \frac{dN_{unabs}}{dE}\, dE}$$

ECF is independent of the normalization factor K (sec. 1) of the source spectrum and can be used to predict HRC-S count rates from a knowledge of the spectral shape and the incident unabsorbed energy flux density obtained from a model of the source or from previous observations. K can be set equal to 1 in the model expressions for spectra and the limits on the integral in the numerator can be set equal to 0.07 and 10.0 keV, the UV/Ion shield and mirror cutoffs, respectively.

ECF is used as follows:

1. Based upon prior knowledge or upon a model calculate the expected source flux density Φ (in units of 10^{-10} erg cm^{-2} s^{-1}) at the AXAF aperture and in the AXAF passband 0.07 **-** 10.0 keV assuming **no** interstellar absorption.

2. Determine the expected interstellar absorption column density.

3. Calculate ECF using the assumed values for N_H and the values of the spectral parameter (n or kT).

4. Multiply Φ by ECF to obtain the expected HRC-S count rate.

3.A.1 Power law spectrum

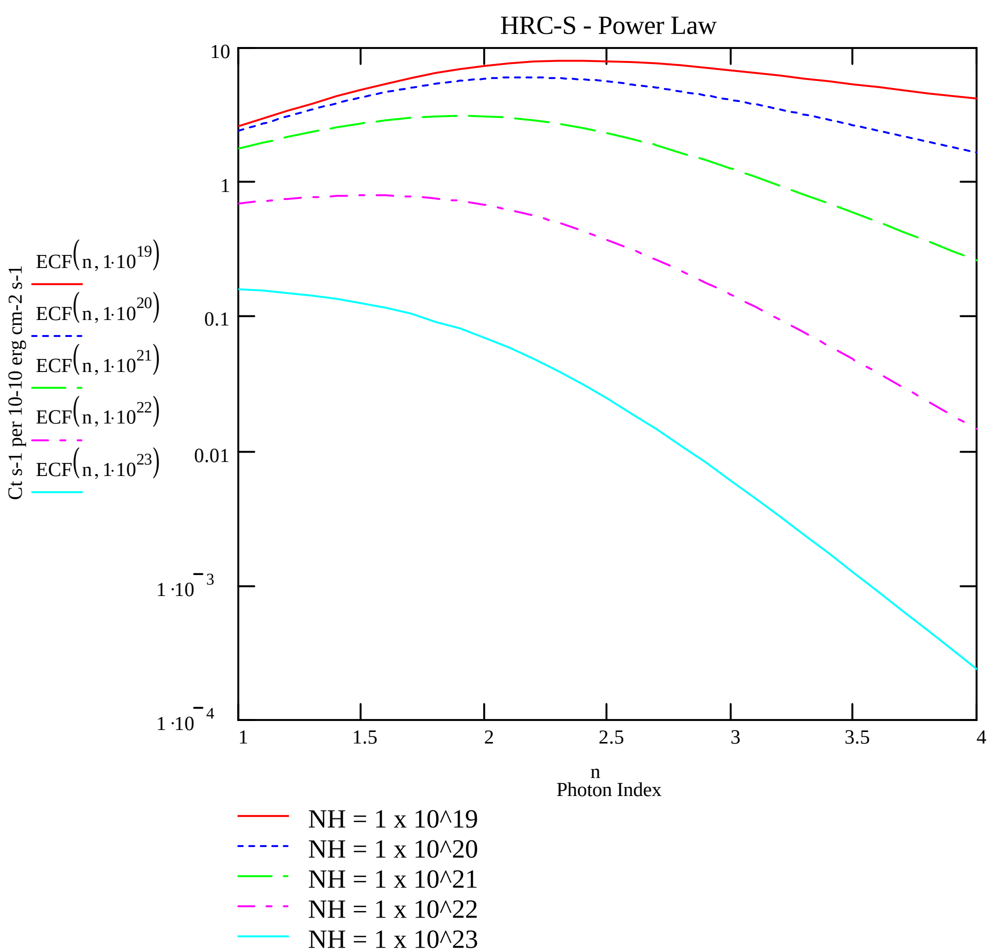
The spectral shape function is given by a power law (sec. 1.C.1):

$$dN\_dE(E,n,K) := dN\_dE_{power}(E,n,K) \qquad K := 1$$

$$ECF(n,N_H) := \frac{\int_{0.07}^{10.0} dN\_dE(E,n,K)\cdot \exp(-\sigma_e(E)\cdot N_H)\cdot A\_hrcs(E)\, dE}{16\cdot \int_{0.07}^{10.0} E\cdot dN\_dE(E,n,K)\, dE}$$

Plotting ECF:

$$n := 1, 1.1 .. 4$$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

$i := 0..30$ $\qquad$ $j := 0..8$

$n_i := 0.1 \cdot i + 1$ photon index

$ecf\_S\_pl_{i,j} := ECF\left(n_i, NH_j\right)$ $\qquad$ HRC-S power-law

$ecf\_S\_pl := augment(n, ecf\_S\_pl)$

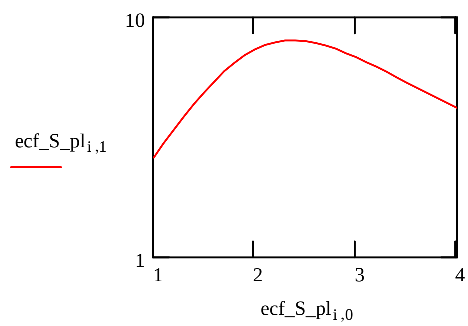
$rows(ecf\_S\_pl) = 31$ $\qquad$ $cols(ecf\_S\_pl) = 10$

Export the array:

ecf_S_pl.prn

ecf_S_pl

Plot ecf as a check:



3.A.2 Blackbody spectrum

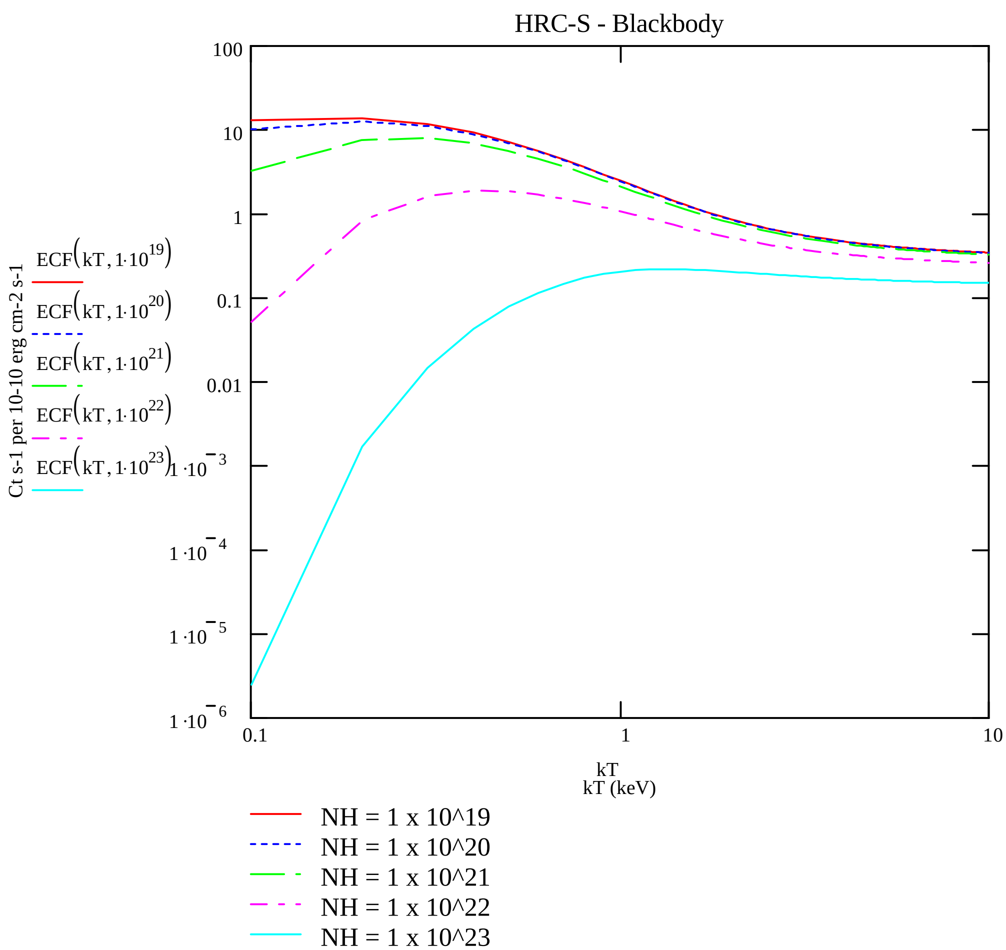
The spectral shape function is given by a blackbody law (sec. 1.C.2):

$$dN\_dE(E, kT, K) := dN\_dE_{BB}(E, kT, K) \qquad K := 1$$

$$ECF(kT, N_H) := \frac{\int_{0.07}^{10.0} dN\_dE(E, kT, K) \cdot \exp(-\sigma_e(E) \cdot N_H) \cdot A\_hrcs(E)\, dE}{16 \cdot \int_{0.07}^{10.0} E \cdot dN\_dE(E, kT, K)\, dE}$$

Plotting ECF:

$$kT := 0.1, 0.2 .. 10$$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

$i := 0..99$ $\quad\quad$ $j := 0..8$

$kT_i := 0.1 \cdot i + .1$ temperature

$ecf\_S\_BB_{i,j} := ECF\left(kT_i, NH_j\right)$ HRC-S blackbody

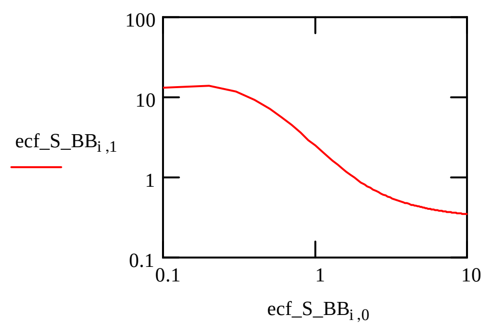
$ecf\_S\_BB := augment(kT, ecf\_S\_BB)$

$rows(ecf\_S\_BB) = 100$ $\quad\quad$ $cols(ecf\_S\_BB) = 10$

Plot ecf as a check:

ecf_S_BB.prn

ecf_S_BB



3.A.3 Thermal bremsstrahlung spectrum

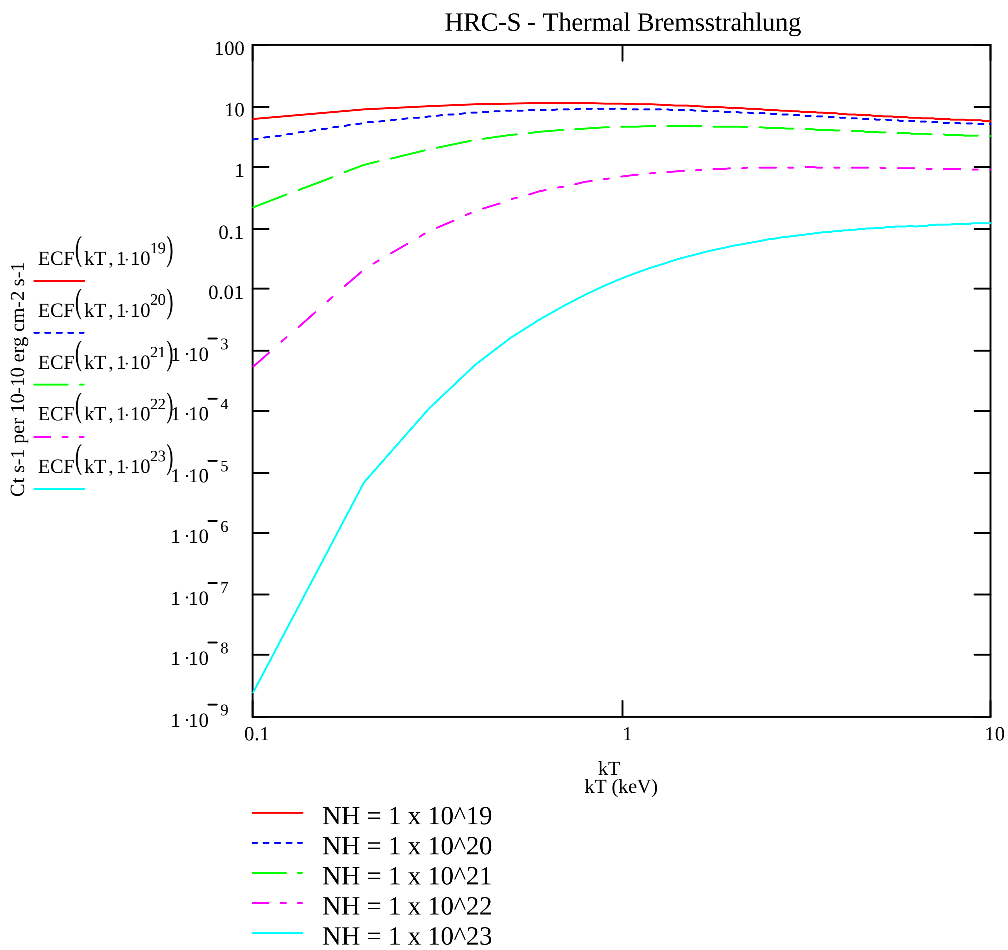
The spectral shape function is given by a blackbody law (sec. 1.C.3):

$$dN\_dE(E,kT,K) := dN\_dE_{thermal}(E,kT,K) \qquad K := 1$$

$$ECF\left(kT,N_H\right) := \frac{\displaystyle\int_{0.07}^{10.0} dN\_dE(E,kT,K)\cdot \exp\left(-\sigma_e(E)\cdot N_H\right)\cdot A\_hrcs(E)\, dE}{16\cdot \displaystyle\int_{0.07}^{10.0} E\cdot dN\_dE(E,kT,K)\, dE}$$

Plotting ECF:

$$kT := 0.1, 0.2 .. 10$$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

$i := 0..99$ $j := 0..8$

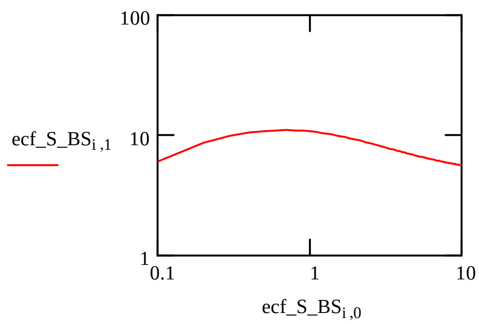
$kT_i := 0.1 \cdot i + .1$ temperature

$ecf\_S\_BS_{i,j} := ECF(kT_i, NH_j)$ HRC-S thermal bremsstrahlung

$ecf\_S\_BS := augment(kT, ecf\_S\_BS)$

$rows(ecf\_S\_BS) = 100$ $cols(ecf\_S\_BS) = 10$

Plot ecf as a check:



3.A.4 Raymond-Smith thermal plasma spectrum

The spectral shape function is given by a set of tables (sec. 1.C.4):

Import photon spectrum tables (source: XSPEC):

RS_02 := D:\..\rs.0.2.prn   kT=0.2 keV   bin width= 3.32502 eV

$E := RS\_02^{\langle 0 \rangle}$ $dN\_dE_{RS\_02} := RS\_02^{\langle 2 \rangle}$

RS_04 := D:\..\rs.0.4.prn   kT=0.4 keV   bin width= 3.32502 eV

$dN\_dE_{RS\_04} := RS\_04^{\langle 2 \rangle}$

RS_06 := D:\..\rs.0.6.prn   kT=0.6 keV   bin width= 3.32502 eV

$dN\_dE_{RS\_06} := RS\_06^{\langle 2 \rangle}$

RS_08 := D:\..\rs.0.8.prn   kT=0.8 keV   bin width= 3.32502 eV

$dN\_dE_{RS\_08} := RS\_08^{\langle 2 \rangle}$

$RS\_1 :=$ D:\..\rs.1.0.prn   kT=1.0 keV   bin width= 3.32502 eV

$$dN\_dE_{RS\_1} := RS\_1^{\langle 2 \rangle}$$

$RS\_15 :=$ D:\..\rs.1.5.prn   kT=1.5 keV   bin width= 3.32502 eV

$$dN\_dE_{RS\_1.5} := RS\_15^{\langle 2 \rangle}$$

$RS\_2 :=$ D:\..\rs.2.0.prn   kT=2.0 keV   bin width= 3.32502 eV

$$dN\_dE_{RS\_2} := RS\_2^{\langle 2 \rangle}$$

$RS\_3 :=$ D:\..\rs.3.0.prn   kT=3.0 keV   bin width= 3.32502 eV

$$dN\_dE_{RS\_3} := RS\_3^{\langle 2 \rangle}$$

RS_5 :=

D:\..\rs.5.0.prn kT=5.0 keV bin width= 3.32502 eV

$dN\_dE_{RS\_5} := RS\_5^{\langle 2 \rangle}$

RS_7 :=

D:\..\rs.7.0.prn kT=7.0 keV bin width= 3.32502 eV

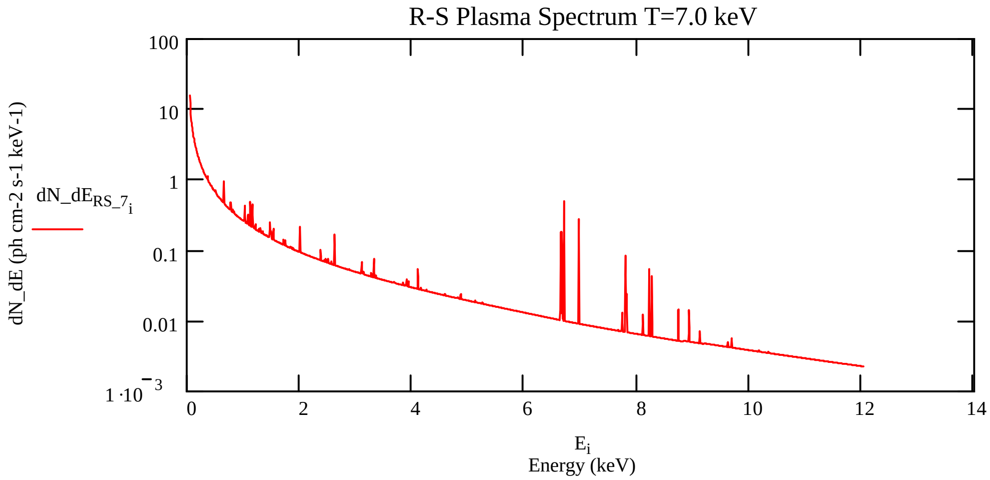
$dN\_dE_{RS\_7} := RS\_7^{\langle 2 \rangle}$

RS_10 :=

D:\..\rs.10.0.prn kT=10.0 keV bin width= 3.32502 eV

$dN\_dE_{RS\_10} := RS\_10^{\langle 2 \rangle}$

As sanity check, plot the 7.0 keV spectrum: $i := 0 .. 1800$ $\text{length}(E) = 3000$



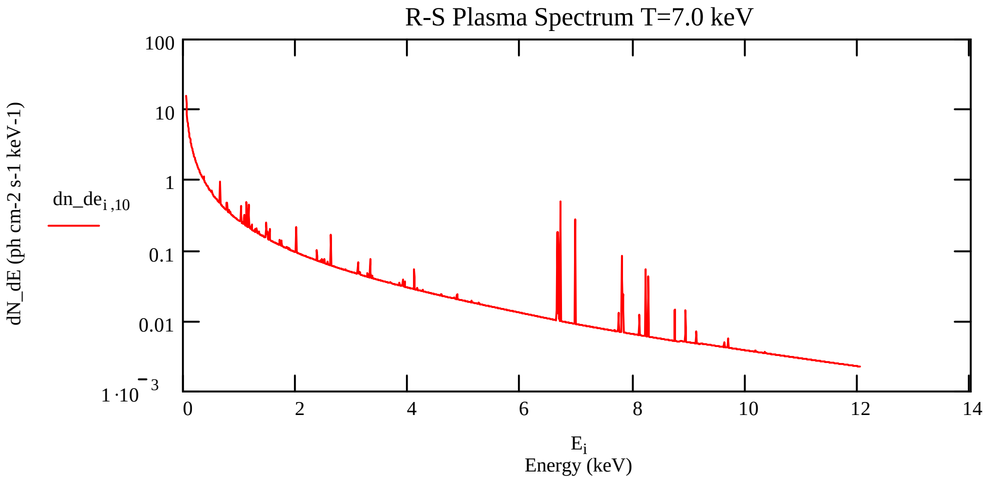
Create a spectrum matrix where the first column is the vector of energies and the remaining 11 columns are the spectra for the T=0.2, ..., 10.0 keV:

$dn\_de := augment\left(RS\_02^{\langle 0\rangle}, RS\_02^{\langle 2\rangle}, RS\_04^{\langle 2\rangle}, RS\_06^{\langle 2\rangle}, RS\_08^{\langle 2\rangle}, RS\_1^{\langle 2\rangle}, RS\_15^{\langle 2\rangle}, RS\_2^{\langle 2\rangle}, RS\_3^{\langle 2\rangle}, RS\_5^{\langle\right.$

$E_i := dn\_de_{i,0}$

As sanity check, plot the 7.0 keV spectrum:

$i := 0..1800$



Create a temperature vector:

$$T := \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 1.0 \\ 1.5 \\ 2.0 \\ 3.0 \\ 5.0 \\ 7.0 \\ 10.0 \end{pmatrix}$$

Create an N_H vector:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

For the Raymond-Smith spectra, ECF will be calculated by evaluating summations instead of integrals.

Energy limits: $E_3 = 0.073$ $E_{1495} = 9.995$

$i := 1..11$

$$ECF0_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e(E_j) \cdot NH_0\right) \cdot A\_hrcs(E_j)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_0 = 1 \times 10^{19}$

$$ECF1_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e(E_j) \cdot NH_1\right) \cdot A\_hrcs(E_j)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_1 = 1 \times 10^{20}$

$$ECF2_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e(E_j) \cdot NH_2\right) \cdot A\_hrcs(E_j)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_2 = 1 \times 10^{21}$

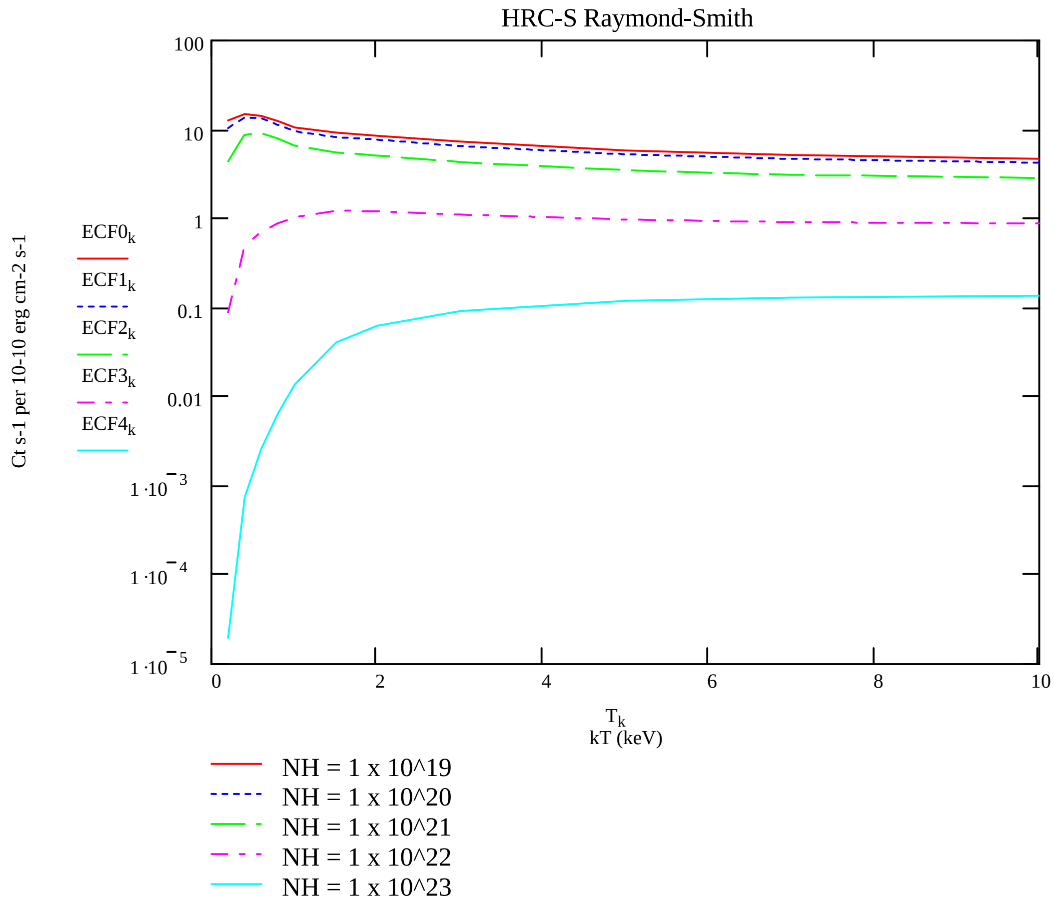
$$ECF3_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e(E_j) \cdot NH_3\right) \cdot A\_hrcs(E_j)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_3 = 1 \times 10^{22}$

$$ECF4_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e(E_j) \cdot NH_4\right) \cdot A\_hrcs(E_j)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_4 = 1 \times 10^{23}$

Plot the ECF's vs T: $k := 0 .. 10$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

Create a temperature vector:

$$kT := \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 1.0 \\ 1.5 \\ 2.0 \\ 3.0 \\ 5.0 \\ 7.0 \\ 10.0 \end{pmatrix}$$

For the Raymond-Smith spectra, ECF will be calculated by evaluating summations instead of integrals.

Energy limits: $E_3 = 0.073$ $E_{1495} = 9.995$ $\text{rows}(dn\_de) = 3 \times 10^3$

$i := 0 .. 10$ $j := 0 .. 8$ $\text{cols}(dn\_de) = 12$

$$ecf\_S\_RS_{i,j} := \frac{\left(\sum_{k=3}^{1495} dn\_de_{k,i+1} \cdot \exp\left(-\sigma_e(E_k) \cdot NH_j\right) \cdot A\_hrcs(E_k) \right)}{16 \cdot \sum_{k=3}^{1495} E_k \cdot dn\_de_{k,i+1}}$$

$\text{rows}(ecf\_S\_RS) = 11$ $\text{cols}(ecf\_S\_RS) = 9$ $\text{rows}(kT) = 11$ $\text{cols}(kT) = 1$

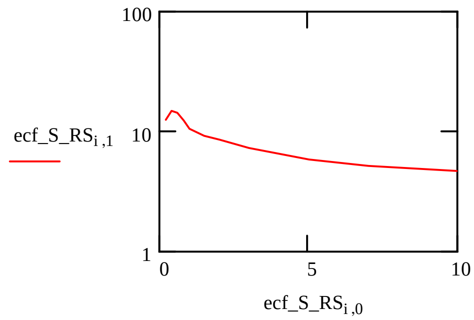
ecf_S_RS := augment(kT, ecf_S_RS)

rows(ecf_S_RS) = 11 cols(ecf_S_RS) = 10

Plot ecf as a check:

ecf_S_RS.prn

ecf_S_RS



ecf_S_RS =

	0	1	2	3	4	5	6
0	0.2	12.467	11.799	10.24	7.778	4.361	1.389
1	0.4	14.876	14.511	13.639	12.025	8.704	4.033
2	0.6	14.194	13.852	13.17	11.933	9.098	4.652
3	0.8	12.281	11.882	11.229	10.193	7.924	4.36
4	1	10.524	10.098	9.441	8.488	6.632	3.896
5	1.5	9.176	8.836	8.183	7.174	5.503	3.412
6	2	8.497	8.209	7.596	6.615	5.031	3.126
7	3	7.259	7.025	6.499	5.646	4.283	2.68
8	5	5.808	5.629	5.217	4.544	3.468	2.209
9	7	5.132	4.977	4.62	4.035	3.096	1.995
10	10	4.653	4.516	4.2	3.68	2.84	1.851

3.B. HRC-I

The HRC-I count rate from a point source is the convolution of the source spectrum (sec. 1) with the HRC-I effective area (sec. 2):

$$CR = \int_0^{\infty} \frac{dN(E)}{dE} \cdot A\_hrci(E)\, dE \quad ct\ s^{-1}$$

ECF is calculated as in 3.B.

3.B.1 Power law spectrum

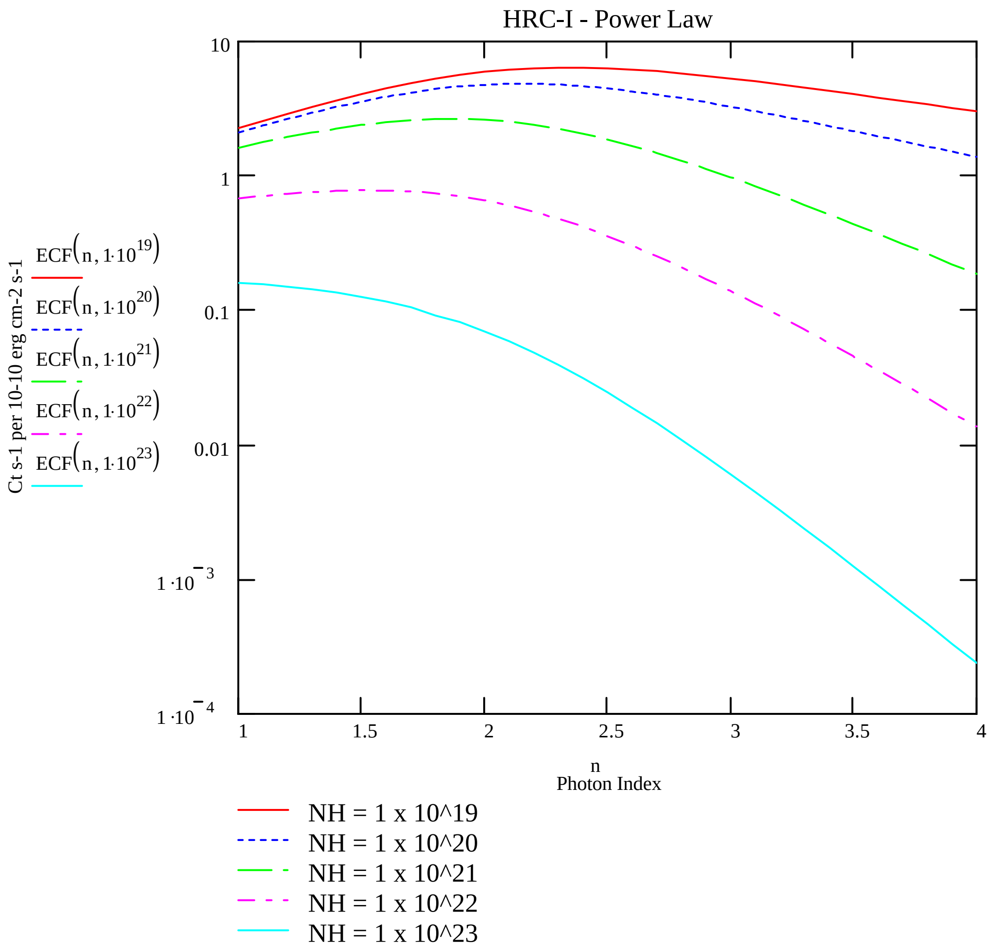
The spectral shape function is given by a power law (sec. 1.C.1):

$$dN\_dE(E,n,K) := dN\_dE_{power}(E,n,K) \qquad K := 1$$

$$ECF(n, N_H) := \frac{\int_{0.07}^{10.0} dN\_dE(E,n,K) \cdot \exp(-\sigma_e(E) \cdot N_H) \cdot A\_hrci(E)\, dE}{16 \cdot \int_{0.07}^{10.0} E \cdot dN\_dE(E,n,K)\, dE}$$

Plotting ECF:

$$n := 1, 1.1 .. 4$$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

$i := 0..30$ $j := 0..8$

$n_i := 0.1 \cdot i + 1$ photon index)

$ecf\_I\_pl_{i,j} := ECF(n_i, NH_j)$ HRC-I power-law

$ecf\_I\_pl := augment(n, ecf\_I\_pl)$

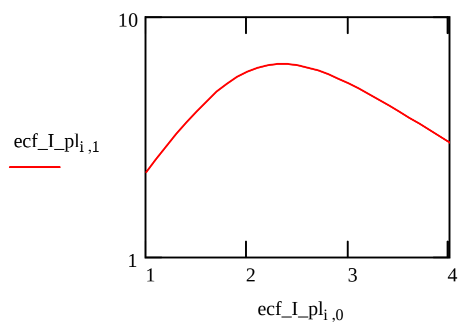
$rows(ecf\_I\_pl) = 31$ $cols(ecf\_I\_pl) = 10$

Export the array:

ecf_I_pl.prn

ecf_I_pl

Plot ecf as a check:



3.B.2 Blackbody spectrum

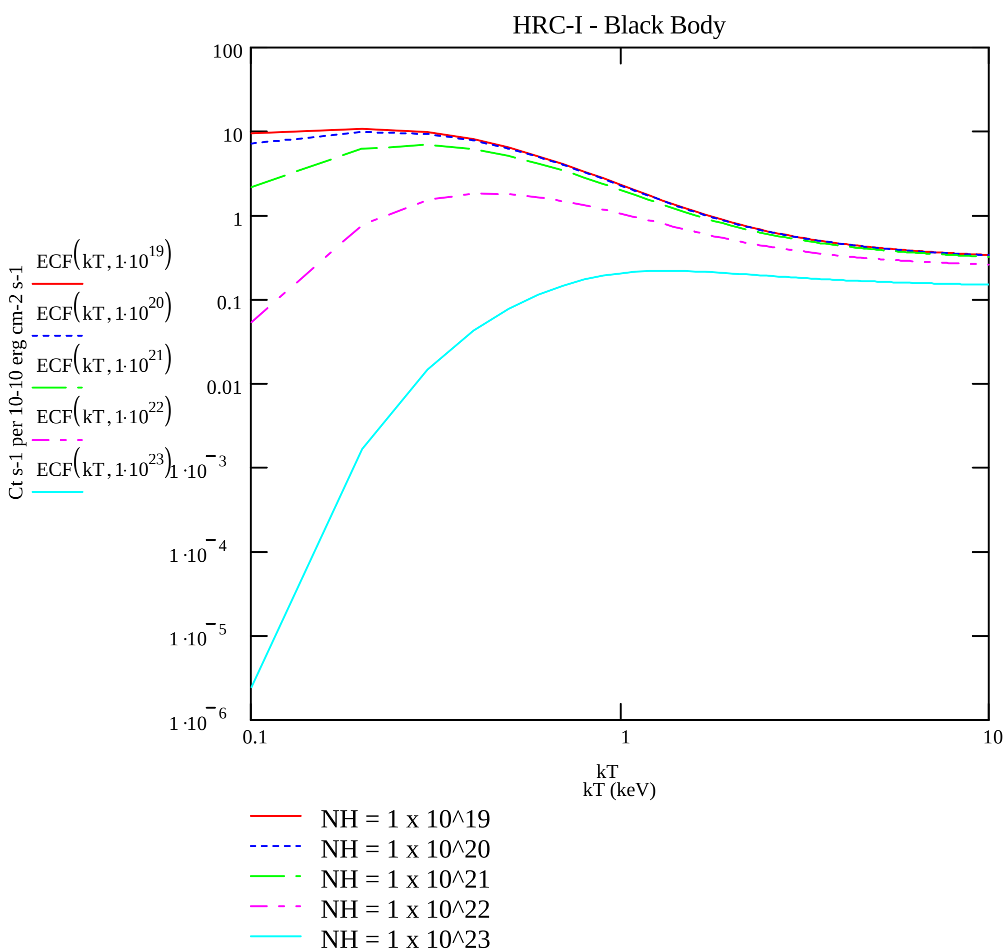
The spectral shape function is given by a blackbody law (sec. 1.C.2):

$$dN\_dE(E,kT,K) := dN\_dE_{BB}(E,kT,K) \qquad K := 1$$

$$ECF(kT,N_H) := \frac{\int_{0.07}^{10.0} dN\_dE(E,kT,K)\cdot \exp(-\sigma_e(E)\cdot N_H)\cdot A\_hrci(E)\,dE}{16\cdot\int_{0.07}^{10.0} E\cdot dN\_dE(E,kT,K)\,dE}$$

Plotting ECF:

$$kT := 0.1, 0.2 .. 10$$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

$i := 0..99$ $\quad$ $j := 0..8$

$kT_i := 0.1 \cdot i + .1$ temperature

$ecf\_I\_BB_{i,j} := ECF(kT_i, NH_j)$ HRC-I blackbody

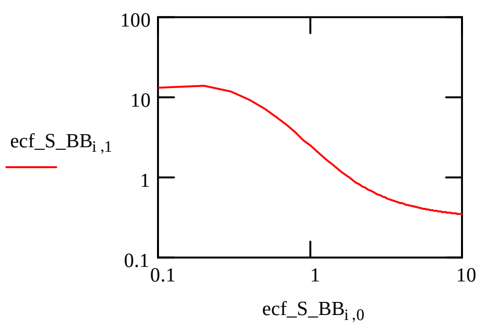
$ecf\_I\_BB := augment(kT, ecf\_I\_BB)$

$rows(ecf\_I\_BB) = 100$ $\quad$ $cols(ecf\_I\_BB) = 10$

Plot ecf as a check:

ecf_I_BB.prn

ecf_I_BB



3.B.3 Thermal bremsstrahlung spectrum

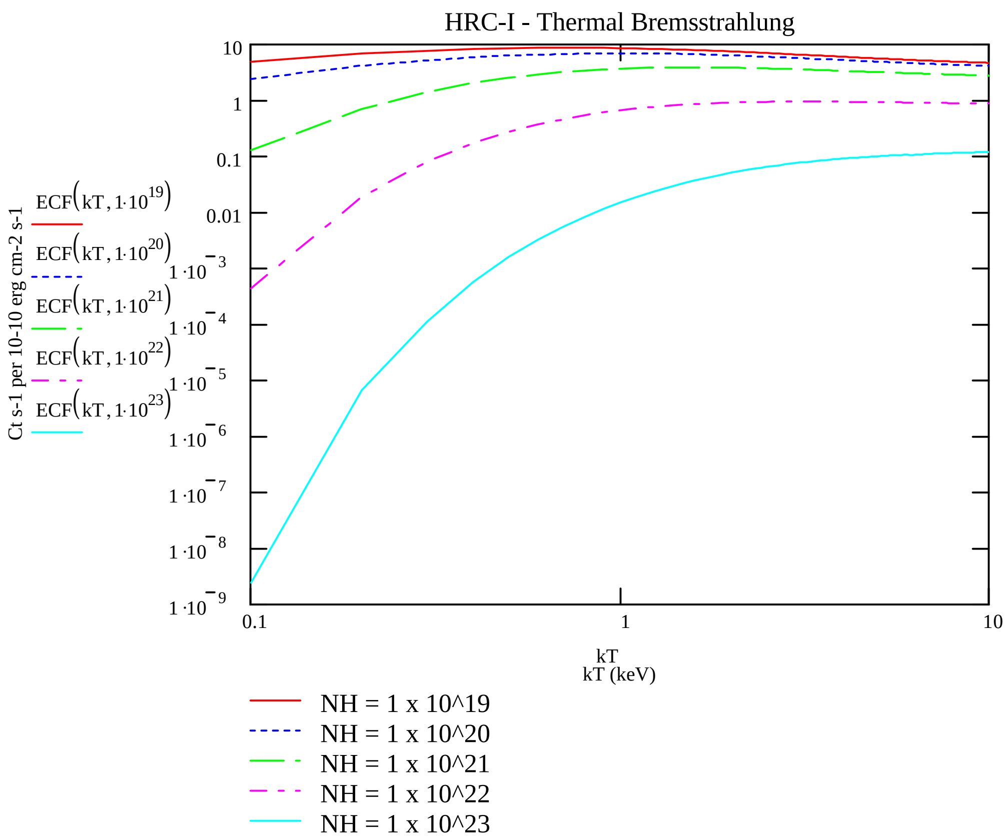
The spectral shape function is given by a blackbody law (sec. 1.C.3):

$$dN\_dE(E,kT,K) := dN\_dE_{thermal}(E,kT,K) \quad K := 1$$

$$ECF\left(kT,N_H\right) := \frac{\int_{0.07}^{10.0} dN\_dE(E,kT,K)\cdot \exp\left(-\sigma_e(E)\cdot N_H\right)\cdot A\_hrci(E)\,dE}{16\cdot\int_{0.07}^{10.0} E\cdot dN\_dE(E,kT,K)\,dE}$$

Plotting ECF:

$kT := 0.1, 0.2 .. 10$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

$i := 0..99$ $\quad$ $j := 0..8$

$kT_i := 0.1 \cdot i + .1$ temperature

$ecf\_I\_BS_{i,j} := ECF(kT_i, NH_j)$ HRC-I thermal bremsstrahlung

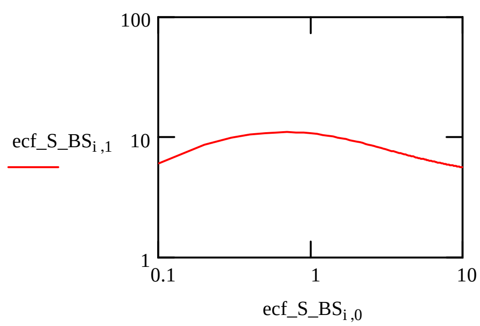
$ecf\_I\_BS := augment(kT, ecf\_I\_BS)$

$rows(ecf\_I\_BS) = 100$ $\quad$ $cols(ecf\_I\_BS) = 10$

Plot ecf as a check:

ecf_I_BS.prn

ecf_I_BS



3.B.4 Raymond-Smith spectrum

For the Raymond-Smith spectra, ECF will be calculated by evaluating summations instead of integrals.

Energy limits: $E_3 = 0.073$ $E_{1495} = 9.995$

$i := 1 .. 11$

$$ECF0_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e\left(E_j\right) \cdot NH_0\right) \cdot A\_hrci\left(E_j\right)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_0 = 1 \times 10^{19}$

$$ECF1_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e\left(E_j\right) \cdot NH_1\right) \cdot A\_hrci\left(E_j\right)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_1 = 3 \times 10^{19}$

$$ECF2_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e\left(E_j\right) \cdot NH_2\right) \cdot A\_hrci\left(E_j\right)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for $NH_2 = 1 \times 10^{20}$

$$ECF3_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e(E_j) \cdot NH_3\right) \cdot A\_hrci(E_j)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for

$NH_3 = 3 \times 10^{20}$

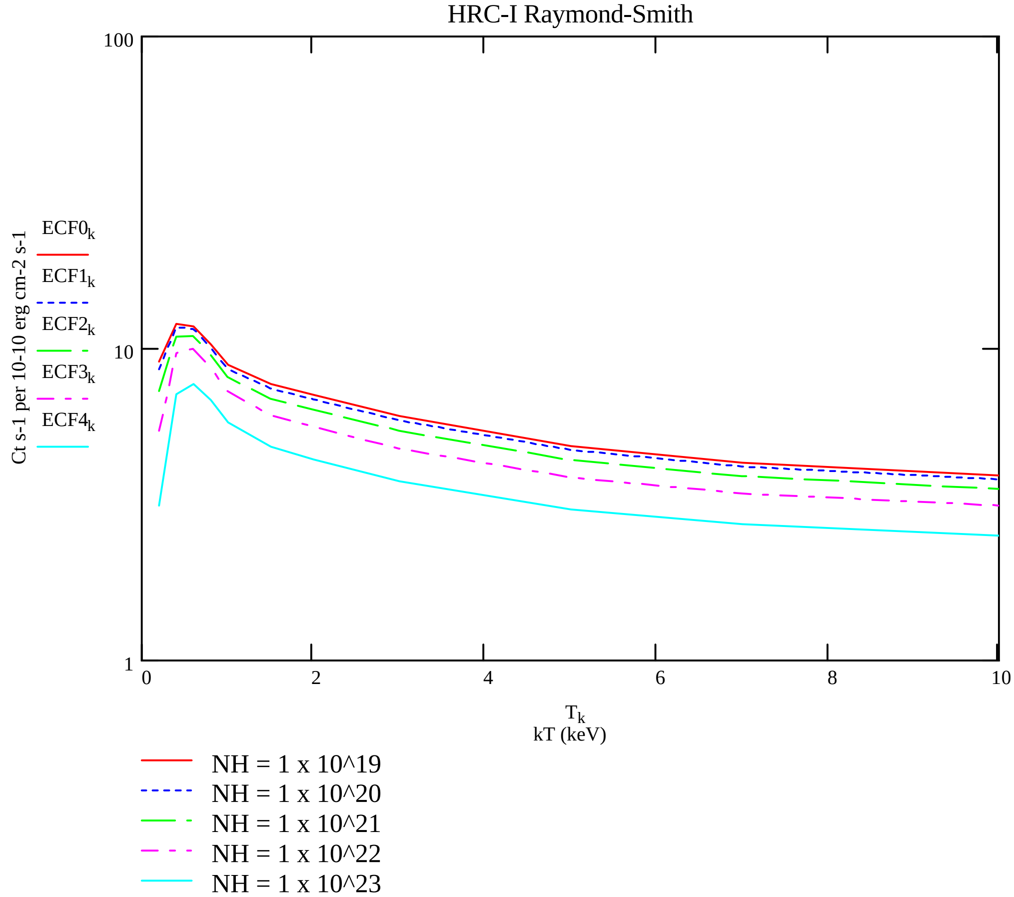
$$ECF4_{i-1} := \frac{\left(\sum_{j=3}^{1495} dn\_de_{j,i} \cdot \exp\left(-\sigma_e(E_j) \cdot NH_4\right) \cdot A\_hrci(E_j)\right)}{16 \cdot \sum_{j=3}^{1495} E_j \cdot dn\_de_{j,i}}$$

for

$NH_4 = 1 \times 10^{21}$

Plot the ECF's vs T:

$k := 0 .. 10$



Column density matrix:

$$NH := \begin{pmatrix} 1 \cdot 10^{19} \\ 3 \cdot 10^{19} \\ 1 \cdot 10^{20} \\ 3 \cdot 10^{20} \\ 1 \cdot 10^{21} \\ 3 \cdot 10^{21} \\ 1 \cdot 10^{22} \\ 3 \cdot 10^{22} \\ 1 \cdot 10^{23} \end{pmatrix}$$

Create a temperature vector:

$$kT := \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 1.0 \\ 1.5 \\ 2.0 \\ 3.0 \\ 5.0 \\ 7.0 \\ 10.0 \end{pmatrix}$$

For the Raymond-Smith spectra, ECF will be calculated by evaluating summations instead of integrals.

Energy limits: $E_3 = 0.073$ $E_{1495} = 9.995$ $\text{rows}(dn\_de) = 3 \times 10^3$

$i := 0..10$ $j := 0..8$ $\text{cols}(dn\_de) = 12$

$$ecf\_I\_RS_{i,j} := \frac{\left(\sum_{k=3}^{1495} dn\_de_{k,i+1} \cdot \exp\left(-\sigma_e(E_k) \cdot NH_j\right) \cdot A\_hrci(E_k)\right)}{16 \cdot \sum_{k=3}^{1495} E_k \cdot dn\_de_{k,i+1}}$$

$\text{rows}(ecf\_I\_RS) = 11$ $\text{cols}(ecf\_I\_RS) = 9$ $\text{rows}(kT) = 11$ $\text{cols}(kT) = 1$

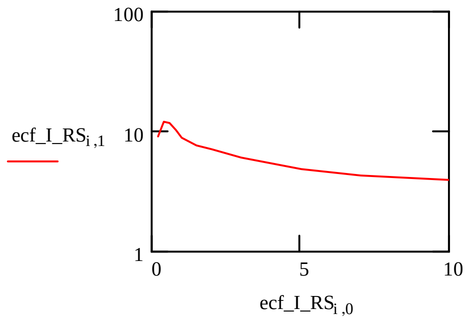
$ecf\_I\_RS := augment(kT, ecf\_I\_RS)$

$rows(ecf\_I\_RS) = 11$ $cols(ecf\_I\_RS) = 10$

Plot ecf as a check:

ecf_I_RS.prn

ecf_I_RS



$$e_8, {c_0}_8, \mathrm{if}\left(E < e_9, {c_0}_9, \mathrm{if}\left(E < e_{10}, {c_0}_{10}, \mathrm{if}\left(E < e_{11}, {c_0}_{11}, \mathrm{if}\left(E < e_{12}, {c_0}_{12}, \mathrm{if}\left(E < e_{13}, {c_0}_{13}, {c_0}_{14}\right)\right)\right)\right)\right)))))))))$$

$$e_8, {c_1}_8, \mathrm{if}\left(E < e_9, {c_1}_9, \mathrm{if}\left(E < e_{10}, {c_1}_{10}, \mathrm{if}\left(E < e_{11}, {c_1}_{11}, \mathrm{if}\left(E < e_{12}, {c_1}_{12}, \mathrm{if}\left(E < e_{13}, {c_1}_{13}, {c_1}_{14}\right)\right)\right)\right)\right)))))))))$$

$$e_8, {c_2}_8, \mathrm{if}\left(E < e_9, {c_2}_9, \mathrm{if}\left(E < e_{10}, {c_2}_{10}, \mathrm{if}\left(E < e_{11}, {c_2}_{11}, \mathrm{if}\left(E < e_{12}, {c_2}_{12}, \mathrm{if}\left(E < e_{13}, {c_2}_{13}, {c_2}_{14}\right)\right)\right)\right)\right)))))))))$$

$^{2\rangle}, RS\_7^{\langle 2\rangle}, RS\_10^{\langle 2\rangle})$