Center for Astrophysics

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MEMORANDUM

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To: M. Zombeck, S. Murray

From: Almus Kenter

Subject: Degap as a Transformation of Probability Distribution Problem

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1 Introduction

I have been investigating the HRC "degap" procedure from the point of view of transformation of probability distributions. For the sake of this memo I have looked at one "U" coarse position of the HRC-S detector; this was done because the HRC-S detector has a more complicated non-linear degap. Only one coarse position was tested for the sake of simplicity; the general method could and should be applied for each axis and each coarse position separately.

2 Description

Given a probability distribution $P(x)$ on x. If one takes a function of x say $y = y(x)$. The new probability distribution can be derived by the fundamental equation of probability transformations:

$$P(x)dx = P(y)dy \tag{1}$$

Hence the new probability distribution can then be found to be:

$$P(y) = P(x) \times \frac{dx}{dy} \tag{2}$$

(See for example chapter 7 of Numerical Recipes)

This technique is the standard method of producing random numbers which are distributed according to some desired distribution. For example to produce random numbers with an exponential distribution one can start with a random deviate and take the natural logarithm of it.

Choose x so that :

$$P(x)dx = dx \tag{3}$$

if one then examines the distribution of $y = -ln(x)$ one can predict (and observe) that:

$$P(y)dy = e^{-y}dy \tag{4}$$

.

The applicability of this technique to HRC "degap" or flat fielding is therefore readily apparent.

A true flat field is essentially a two dimensional random deviate and each dimension can be considered separately. This hypothesis, of course, assumes uniform illumination.

Consider half of a single coarse position (fine position goes from 0 to 128):

$$P(x)dx = \frac{N}{L}dx \tag{5}$$

ie, the distribution should be flat.

Where N is the number of events in the segment and L is the length of the segment (128 pixels).

The HRC system detects the X-rays and produces an image. Again considering each dimension separately, the resulting projection of the region corresponding to the above "half" coarse position is just the transformed probability distribution.

$$P(y) = \frac{N}{L} \times \frac{dx}{dy} \tag{6}$$

Now the "standard" degap procedure is to take the observed positions, y and transform them to the "true" or corrected positions x by a polynomial transformation. For the HRC-I it is just linear, for the HRC-S it is necessary to use at least a quadratic correction. It is of course possible to use any order polynomial or any function for that matter.

Consider the quadratic "degap"

$$x = ay^2 + by \tag{7}$$

Calculating $\frac{dx}{dy}$ is trivial and and the resulting probability distribution is:

$$P(y) = (2ay + b)\frac{N}{L} \tag{8}$$

$P(y)$ is just the observed projection of the half-coarse position. The shape of the resulting distribution is just a polynomial of order one less than the correction. Fitting $P(y)$ to the observed distribution yields the correction parameters a and b. Furthermore, by examining the shape of the projection within the coarse positions one can "choose" the order of polynomial that would be appropriate.

The simple case of linear (HRC-I) degap is readily apparent: if $a = 0$ then just divide $P(y)$ by $\frac{N}{L}$ and the height of the resulting distribution must equal b; this is of course just reflecting that both distributions must integrate to N. An "idealized" linear degap is presented in Figure 1

For the quadratic degap correction one must fit a straight line to the resulting distribution the slope of which gives $2a$ and the intercept b. This procedure is depicted in Figure 2.

For the HRC-S U projection in Figure 2 there appears to be an inflection point between the center and the gap. This shape would infer that the distribution is of $3d$ order and hence a degap of $4th$ order would perhaps be more appropriate. A 3d order polynomial of form $4ay^3 + 3by^2 + 2cy + d$ was fit to the distribution. The degapped result is presented in Figure 3. The net result is flat apart from some one pixel wide spikes which I believe are due to a binning error in my procedure.

In all cases the range of y in the fit is limited to not include the "zero" values of the actual gap. I have somewhat arbitrarily limited my fine position to be $y < 105$. I believe that this selection allows the fitting algorithm to sufficiently determine the functional form of $P(y)$. Including the actual gap values of $P(y)$ might be including values where P(y) is not defined; the necessity of this is obvious when you look at an idealized linear degap and consider $P(y)$ to be a straight line with 0 slope and the ordinate intercept as the linear correction.

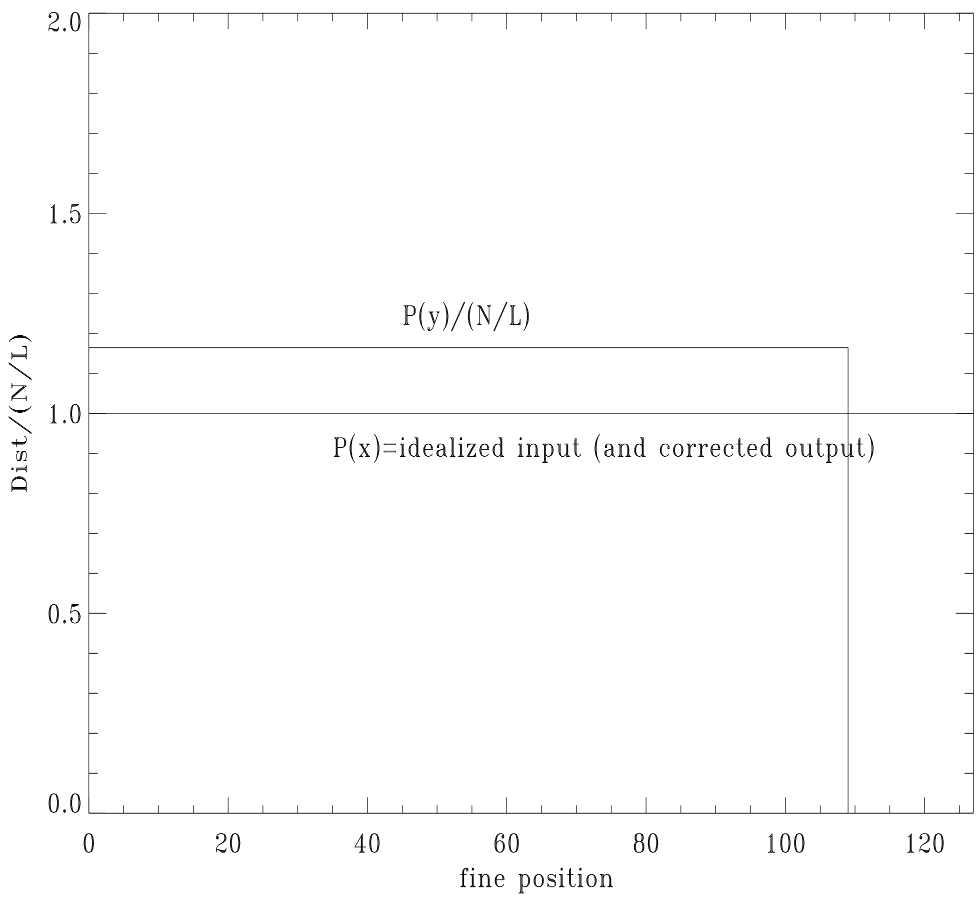

Figure 1: An idealized linear degap. Once normalized (divided by N/L), the height(ordinate intercept) of the undegapped projection equals the linear degap value b. In this case $b = 1.164$.

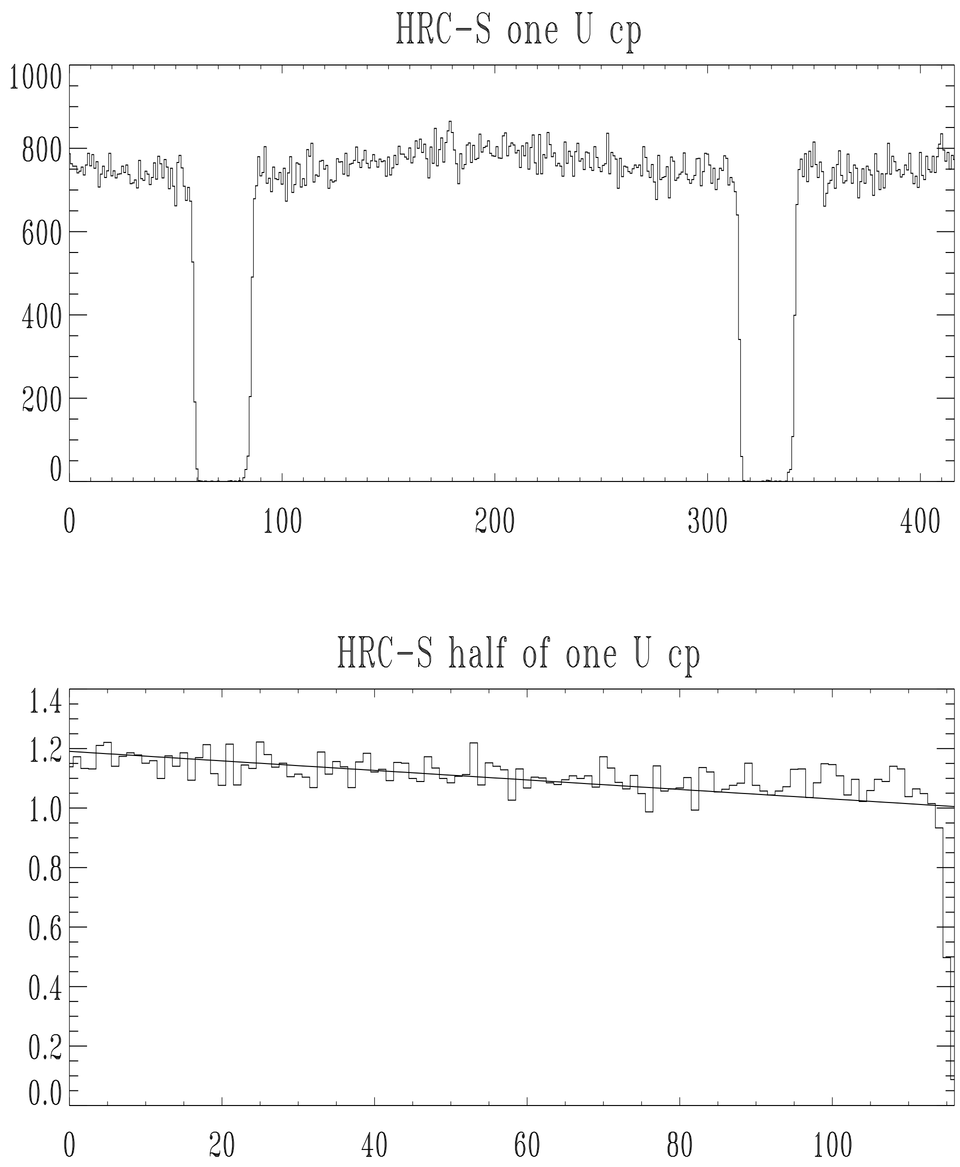

Figure 2: HRC-S U projection and projection and fit. Fit is of form $2ay + b$. The fit is only over half the coarse position; starting from fine position 0. The other half would have an opposite sign slope.

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2a =   -0.00159974
b =       1.19057
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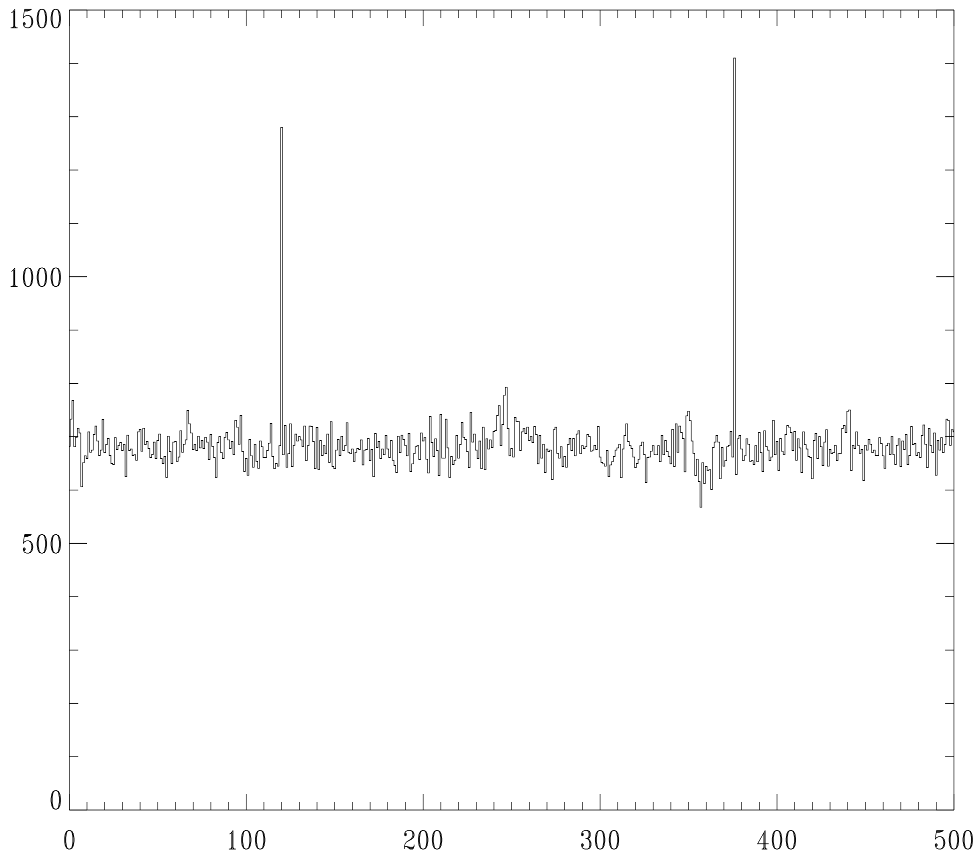

Figure 3: HRC-S U degapped projection. Degap correction is of form $ay^4 + by^3 + cy^2 + dy$. One pixel spikes are believed to be artifacts of binning error in procedure.

3 Conclusion and Caveats

This technique makes it possible to determine the correction parameters by examining the observed projections of HRC flat fields for each coarse position with minimal computation. Images can be binned at various scales to effect various levels of high frequency filtering. Fine scale imaging distortions can be taken care of given a polynomial correction of high enough order.

This method assumes uniform illumination for flat fields and that any structure in the resulting projections of the HRC images is due to spatial distortions only and **not** due to spatial variations in detector QE. Hot spots and dead spots ($QE = \inf$ and $QE = 0$) must be removed or avoided.