Predicting a Multi-Peak Solar Cycle Using a Multi-Stage Analysis

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JSM 2024: Aug 7, 2024

Leveraging 3 centuries of sunspot data: Predicting A Double Peaked Solar Cycle 25

(presented by Vinay Kashyap)

(Center for Astrophysics | Harvard & Smithsonian)

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A message from David...

Sorry that I am not present to deliver this talk! I am grateful to Vinay for agreeing to give it on my behalf; any mistakes are mine. I welcome any follow-up questions (david_stenning@sfu.ca), and I hope to see you next year at JSM 2025!

Baby Violet, born July 5, 2024, with her very tired dad (l) and beginning training for JSM 2044 (r).

Acknowledgements

Collaborators:

- **Youwei Yan (M.Sc. Statistics; Simon Fraser University)**
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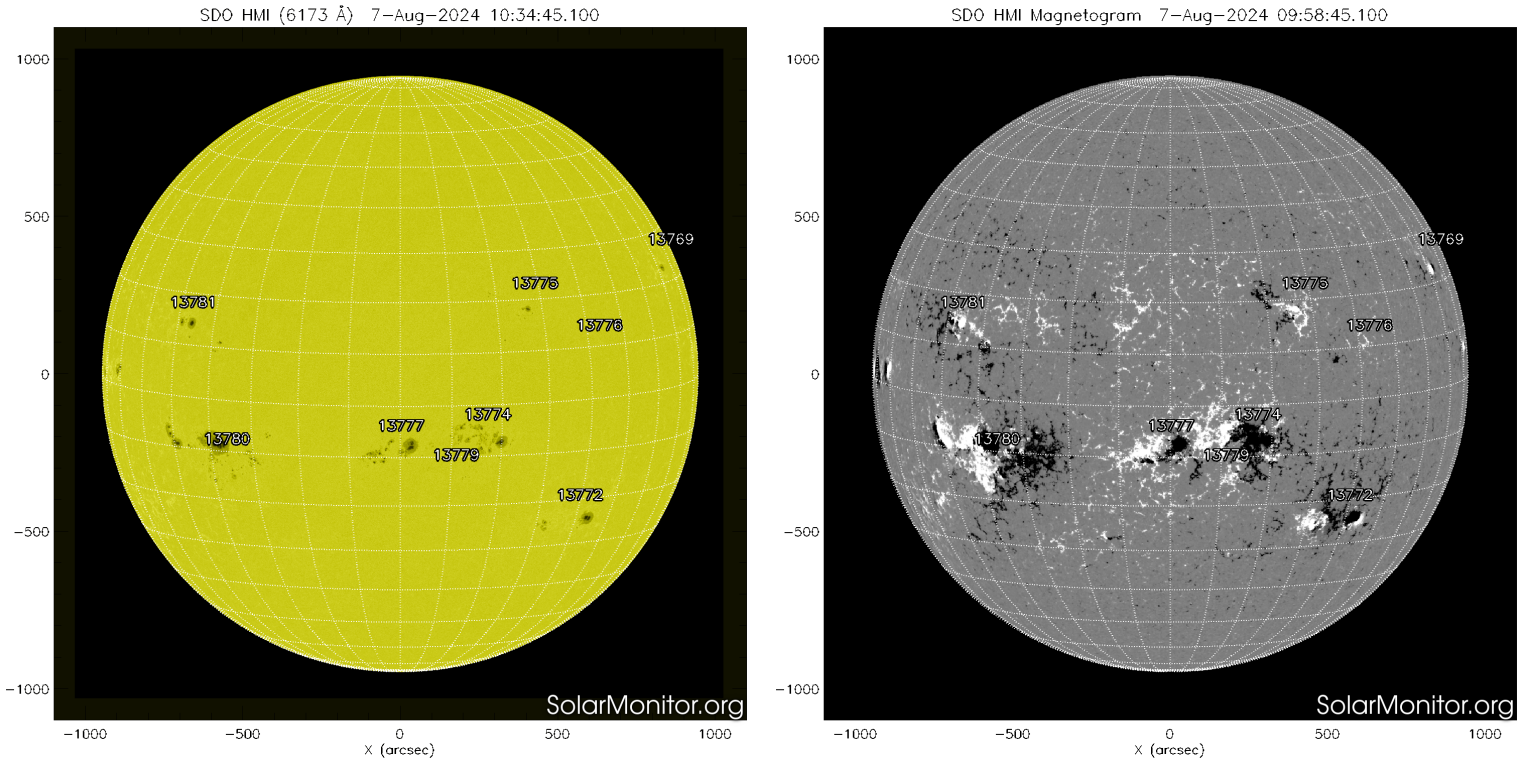
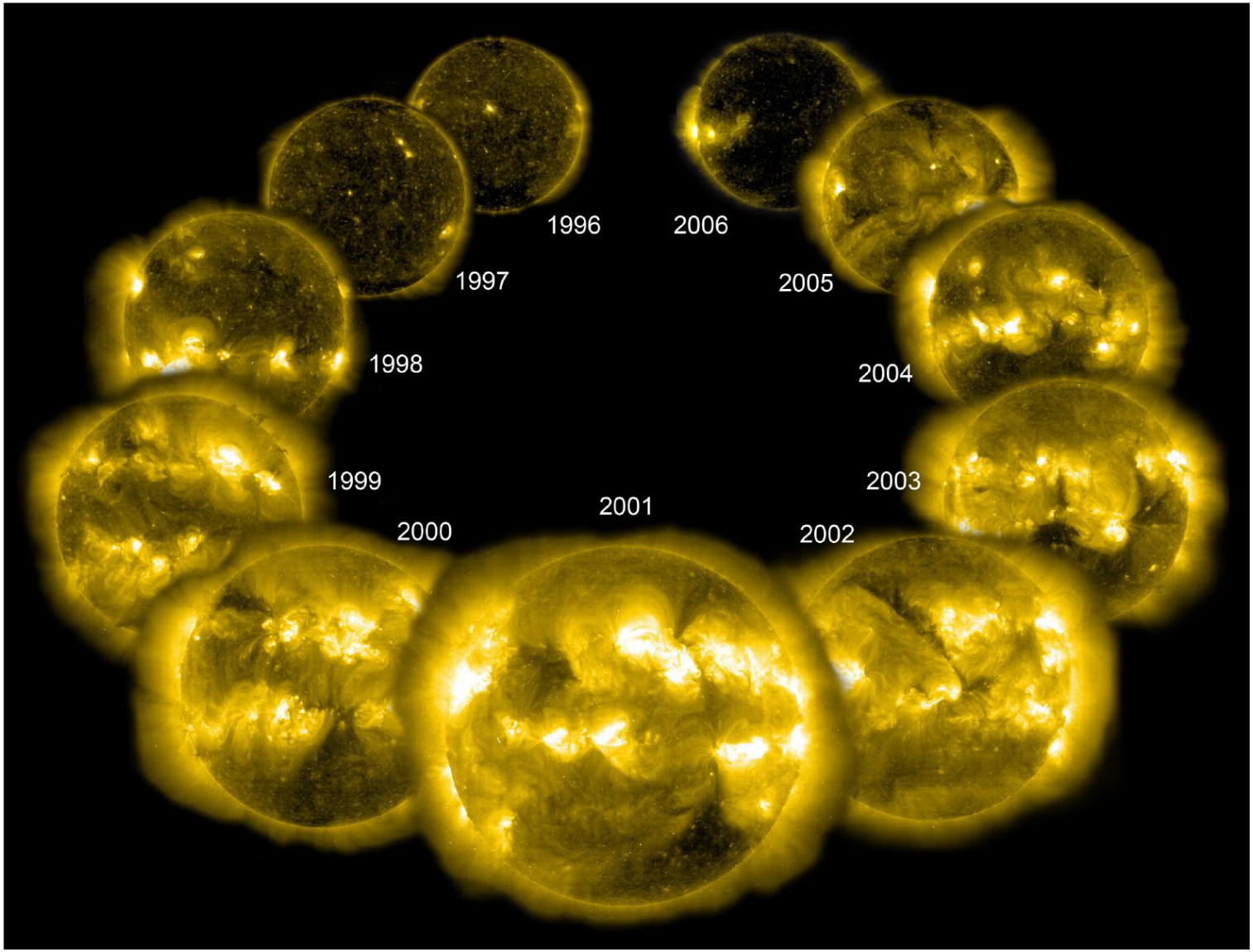
Sun, spots, and activity



The Sun as on Aug 7, 2024, c.10am UT, SDO/HMI 6173 Å and magnetogram via SolarMonitor.org

The Sun is magnetically active, and strong magnetic fields emerge from the surface in sunspots, in areas called Active Regions. Active Regions are sites of energetic phenomena, like flares and coronal mass ejections.

Activity on the Sun varies cyclically with an 11 year period



Solar Cycle 23 as seen by the *Yohkoh* Solar X-ray Telescope

Solar Activity and Space Weather

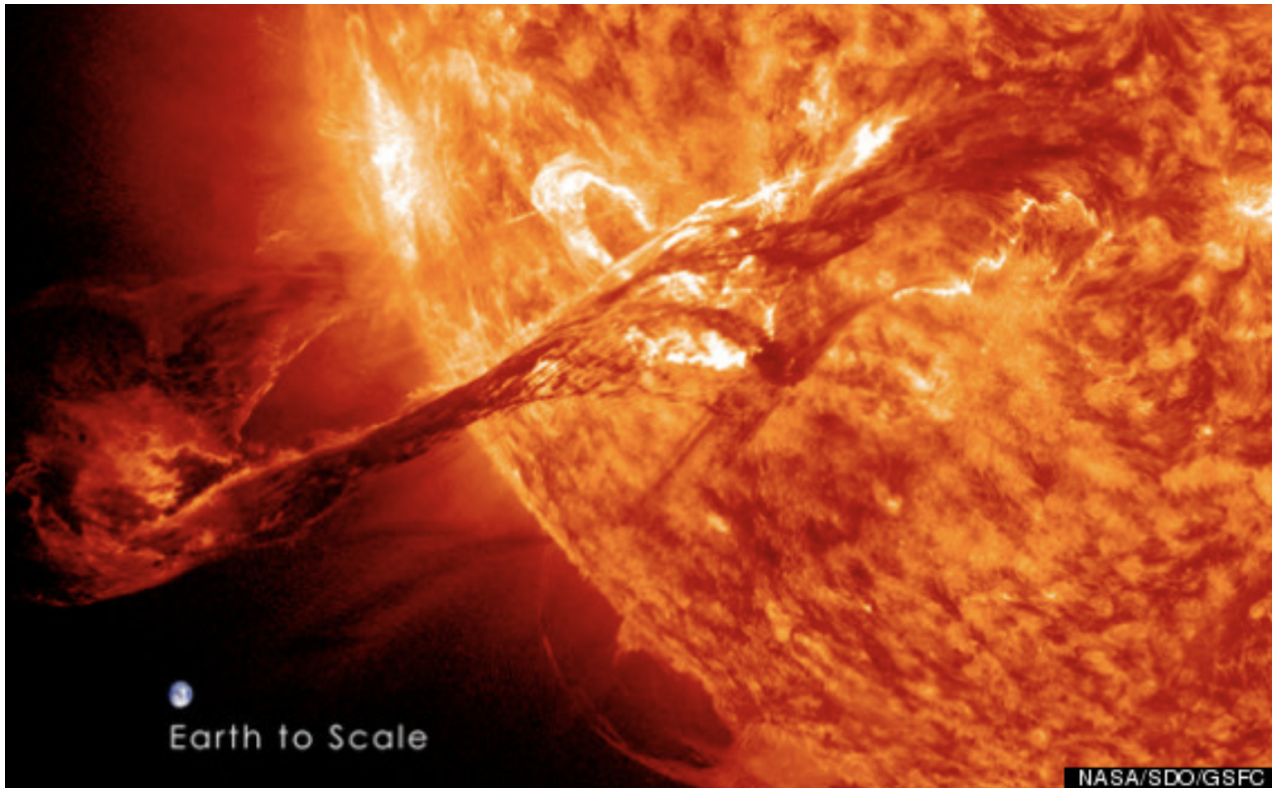
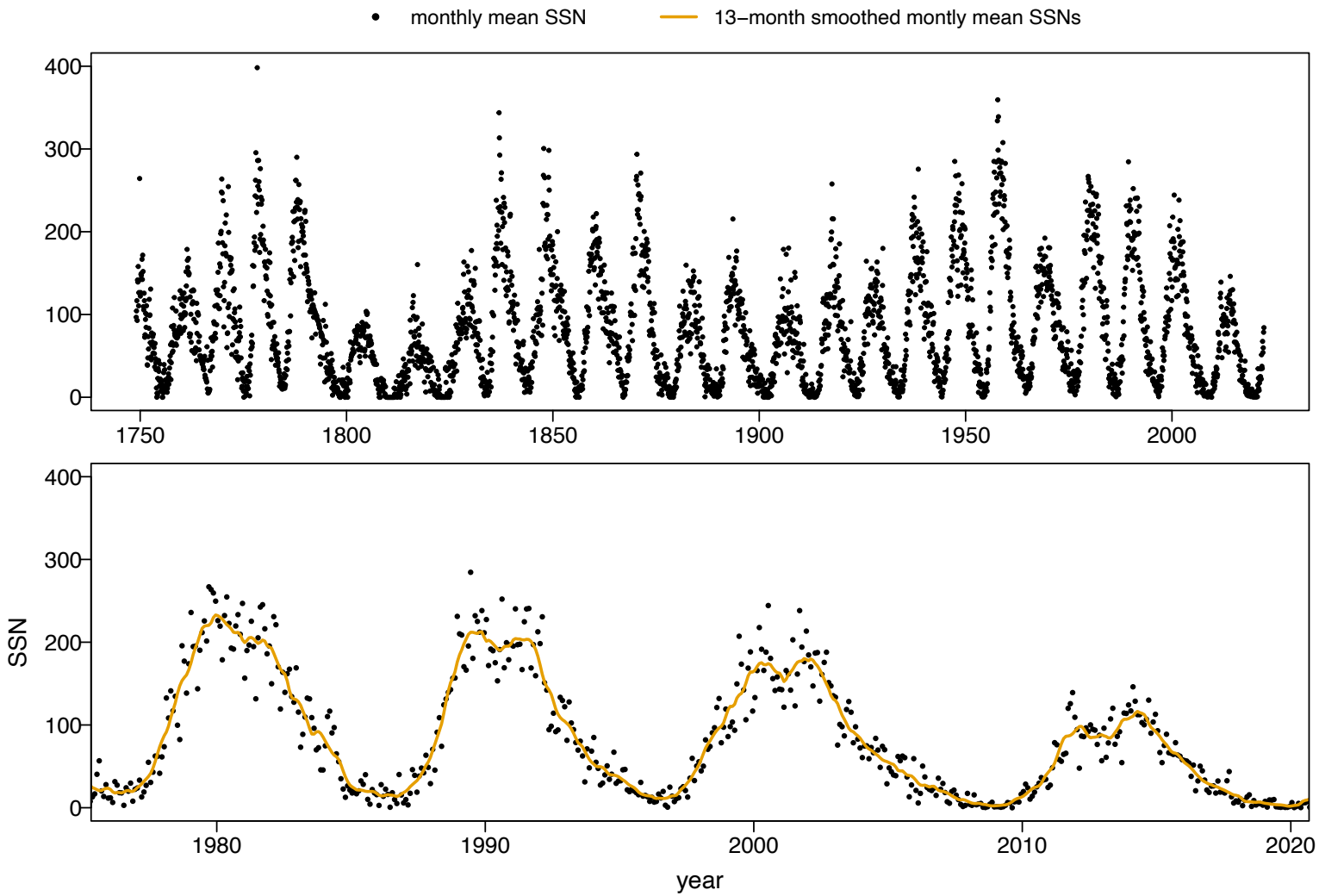


Image Credit: NASA Solar Dynamics Observatory - Goddard Space Flight Center

- Wolf (1852) discovered a correlation between SSNs and geomagnetic activity on Earth (e.g., aurorae).
- Energetic events such as solar flares occur more frequently at cycle peak.
- Predicting the timing and morphology of upcoming cycles helps construct physical models of the solar dynamo that powers the activity.

The longest continuous astronomical dataset in existence

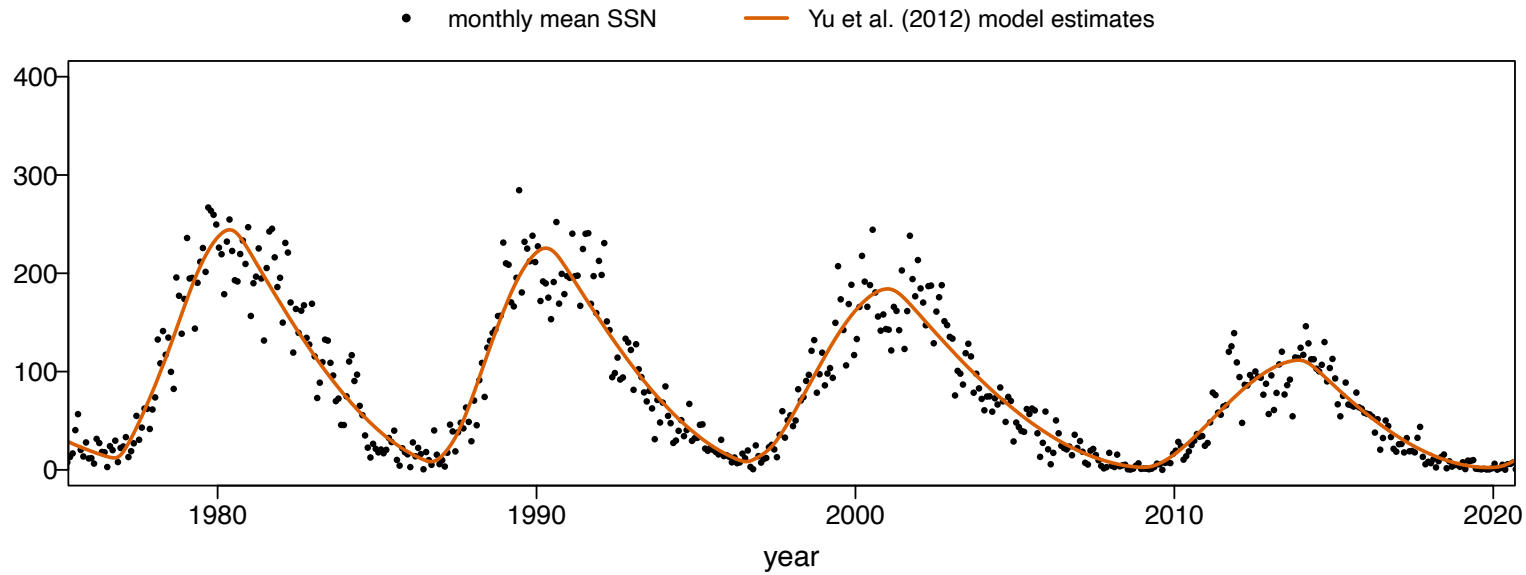


- *Solar activity* is defined by observable proxy variables such as sunspot numbers (SSNs).
- SSNs and other proxies follow a $\approx$ 11-year cyclic pattern: the *solar cycle*.

Solar Cycle Prediction

- Accurately predicting the solar cycle remains a challenge.
- **Yu et al. (2012)** developed a two-level model that relatively accurately predicts the sunspot cycle and provides uncertainty quantification.
- This model, however, assumes the solar cycles are single-peaked.
  - Multiple peaks in sunspot cycles became more pronounced after a major revision of the SSN series in 2015.

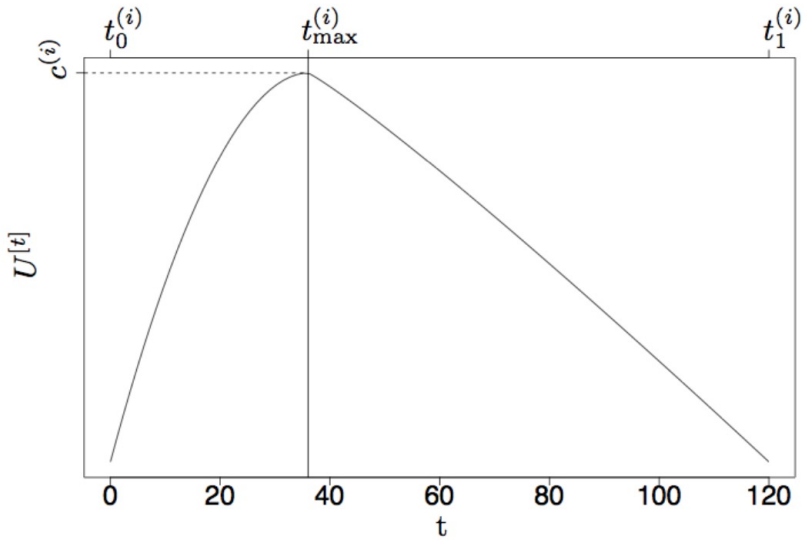
Goal: Extend the two-level model to accommodate the potential multi-peak structure in the solar cycle.



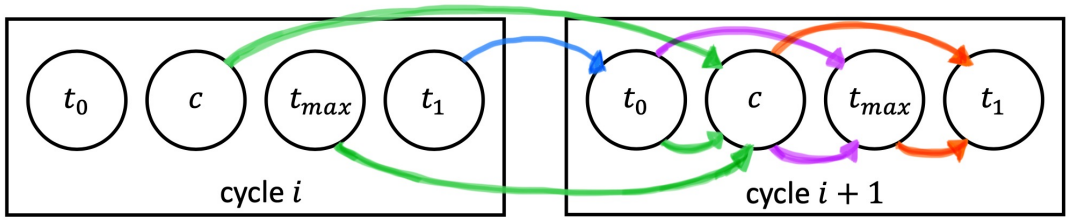
The Yu et al. (2012) Model (Two-Level Model)

- A Bayesian two-level model that fits the **monthly mean SSNs**.
- The first level parameterizes the shape of a sunspot cycle into a rising phase and a declining phase.
- The second level describes the evolution of the solar cycle through a set of correlations between the cycle parameters.

Level One: Modeling cycle i with SSNs



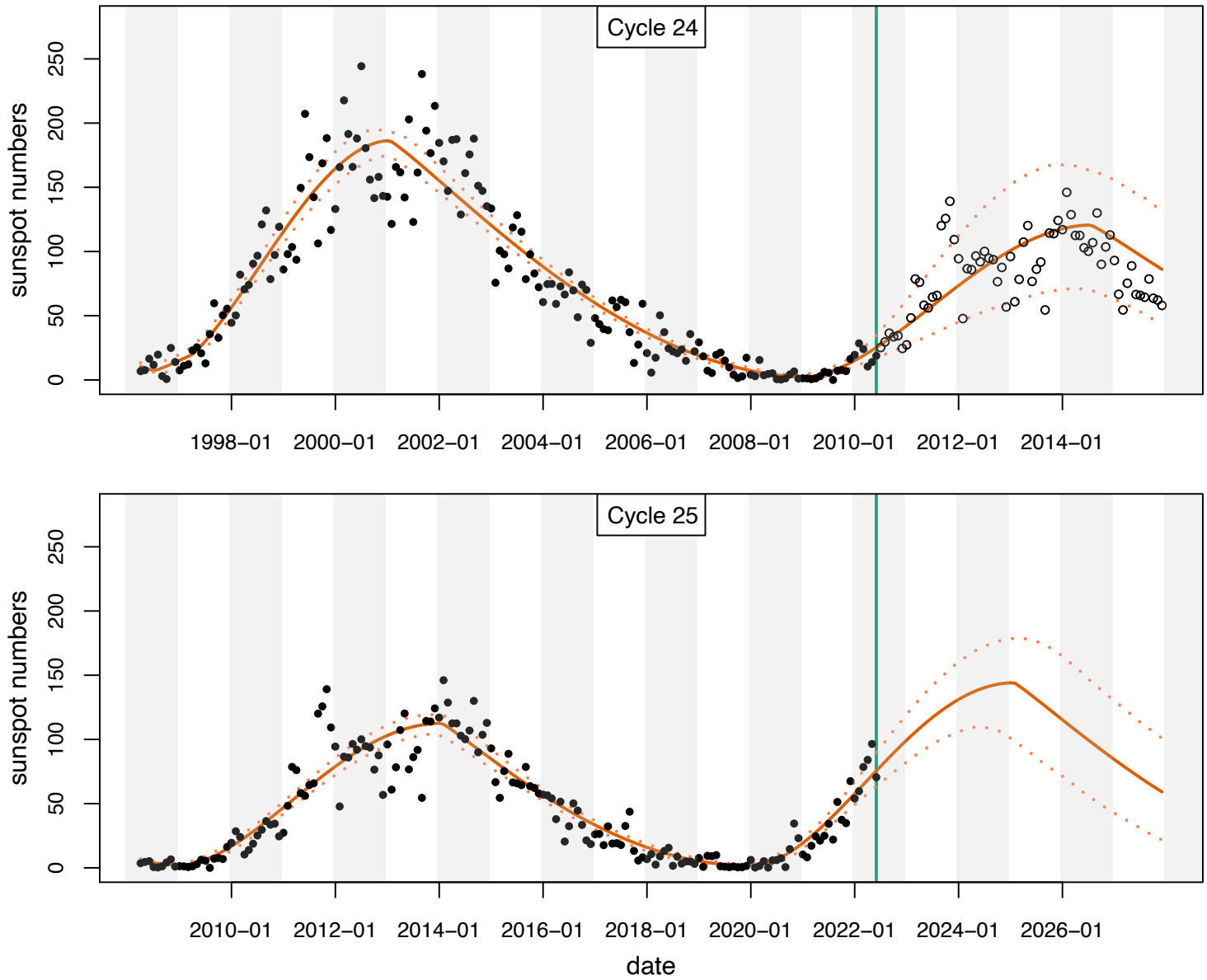
Level Two: Relationships between consecutive cycles



For cycle i:

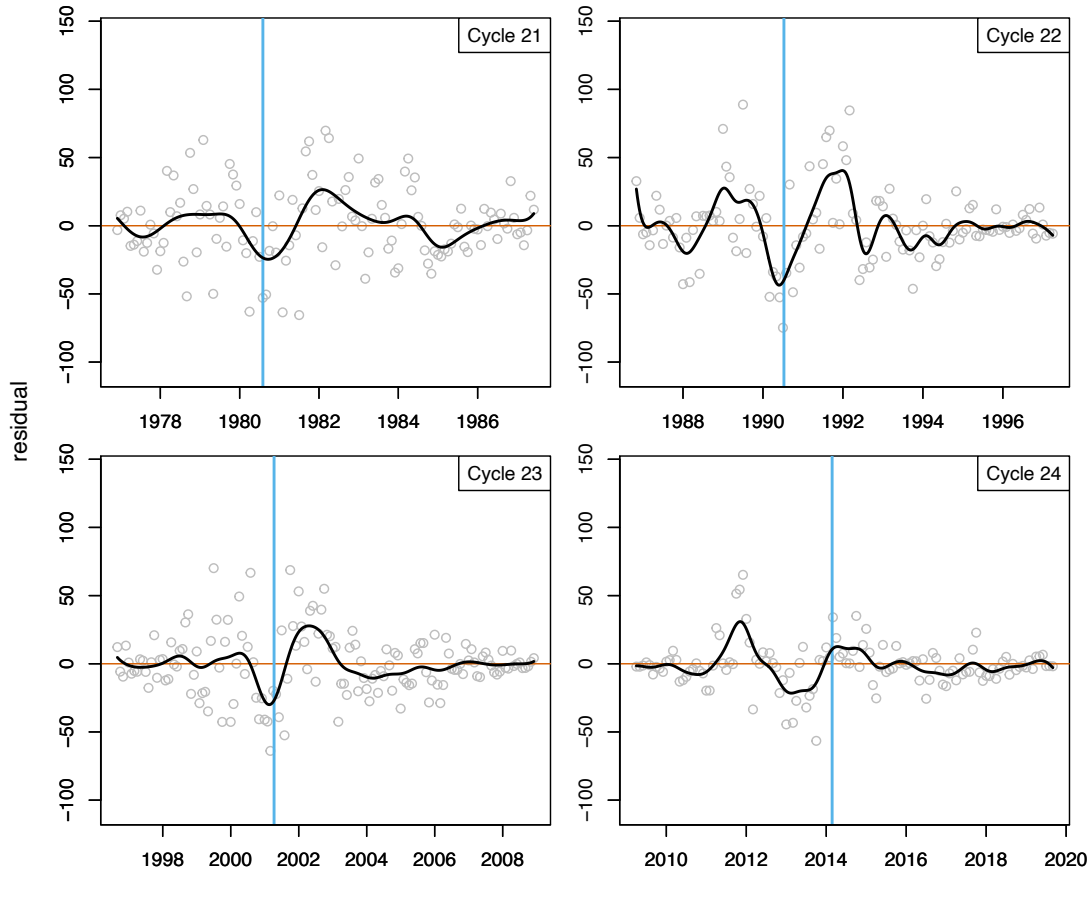
- $t_0^{(i)}$ is the *start time*.
- $t_{\max}^{(i)}$ is the *time of cycle maximum*.
- $t_1^{(i)}$ is the *end time*.
- $c^{(i)}$ is the *amplitude*.
- $U^{[t]}$ is the "average solar activity level" at time t.

Forecast and Hindcast: Two-Level Model



Accounting for Discrepancy

The potentially flawed single-peak assumption leads to discrepancy between the two-level model estimates and the observed SSNs.



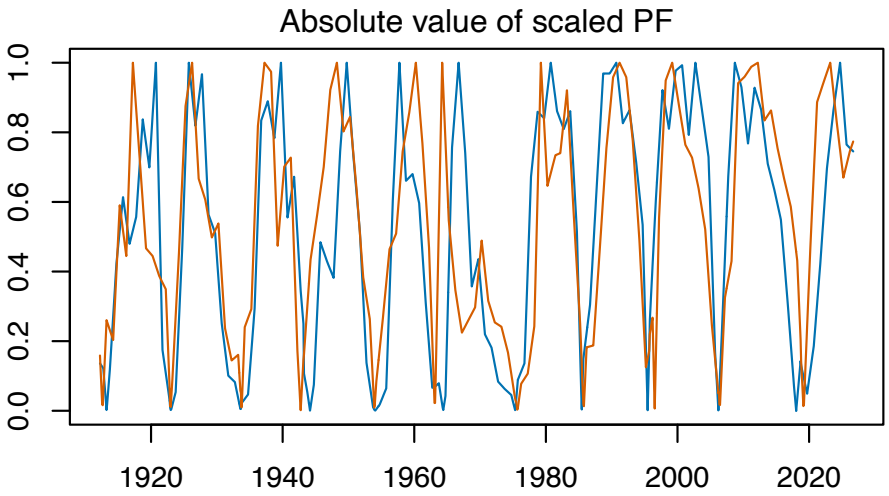
Residuals of SSN from Two-level unimodal model (open circles), local polynomial smoothed (black curves). These are fit using Gaussian Process (GP) regression (see next).

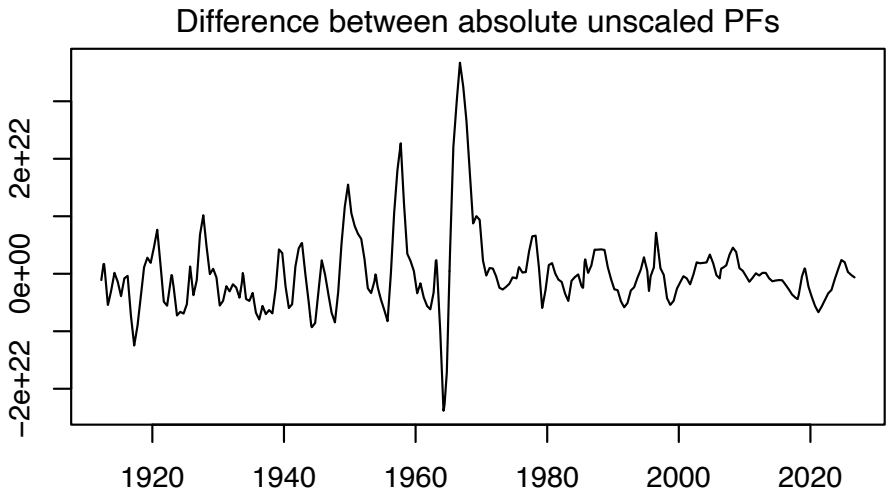
Input Variables for GP

1. **Time in cycle**: A variable representing the time within a cycle, using posterior means from the two-level model fit.

2,3 **Scaled polar fluxes**: Absolute values of the polar fluxes, northern and southern, scaled by their cycle maxima to emphasize within-cycle fluctuations.

4. **Difference in polar fluxes**: The differences between the absolute unscaled northern and southern polar fluxes.



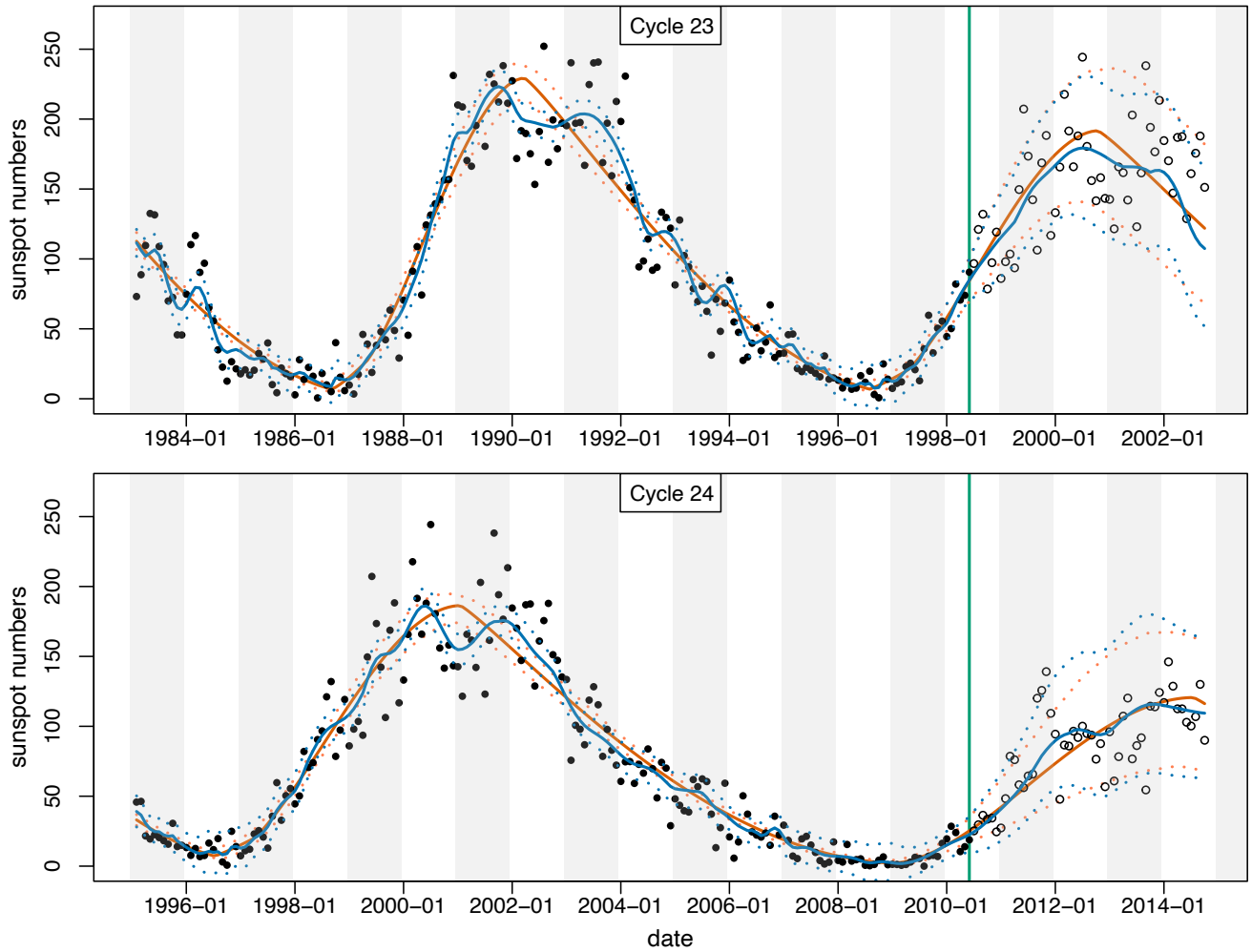


Multi-Stage (Modular) Model Fitting

- We first fit the two-level model to generate posterior samples of the expected SSNs and residuals.
- 100 sets of residuals are randomly selected and a GP is fit to each set.
- For each GP, 2000 posterior samples of the expected residuals are generated and added back to the corresponding SSN estimates to form new estimates.
- In total, 100×2000 simulations of expected SSNs are generated. A fitted mean curve is obtained by computing the pointwise averages; pointwise quantiles form the confidence bands.

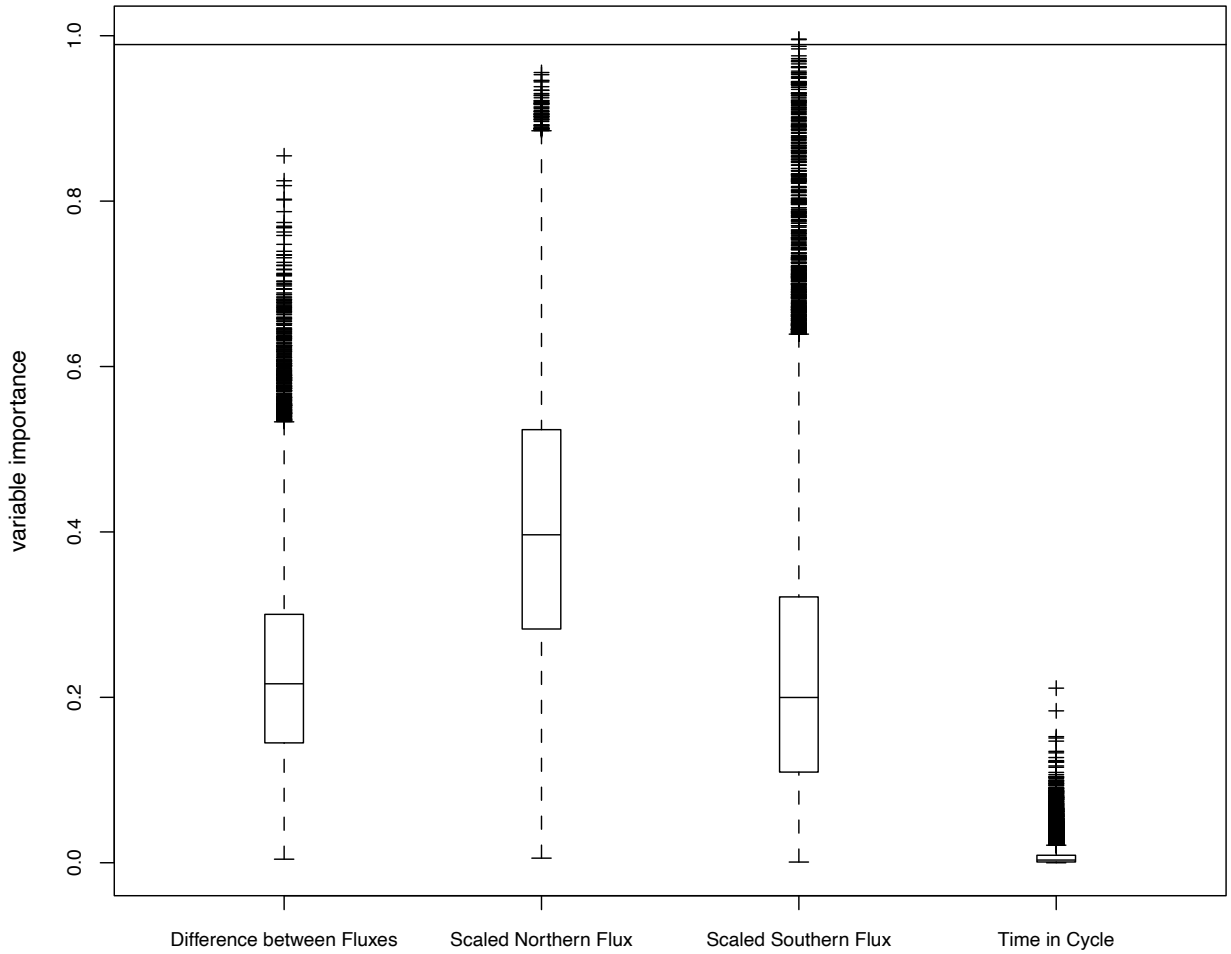
Results - Hindcasts and Skill Scores

- Improvement in predictive performance is checked via skill score[1] and BIC.



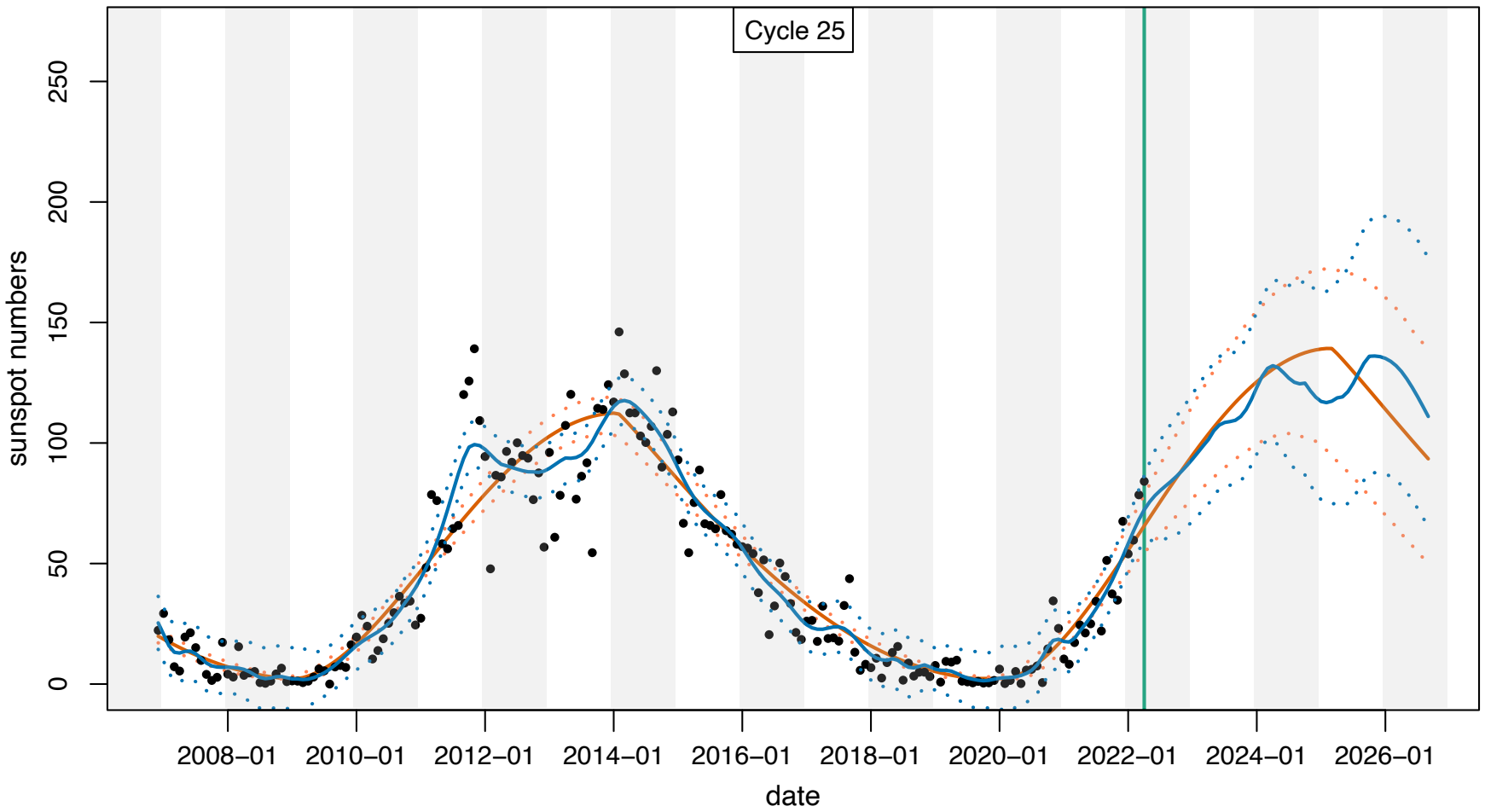
[1] $1 - \mathrm{MSE}\,(\text{forecast, observed})/\mathrm{MSE}\,(\text{reference, observed})$. Cycle 23: SS = 0.098; Cycle 24: SS = 0.193

Results - Reference Distribution Variable Selection



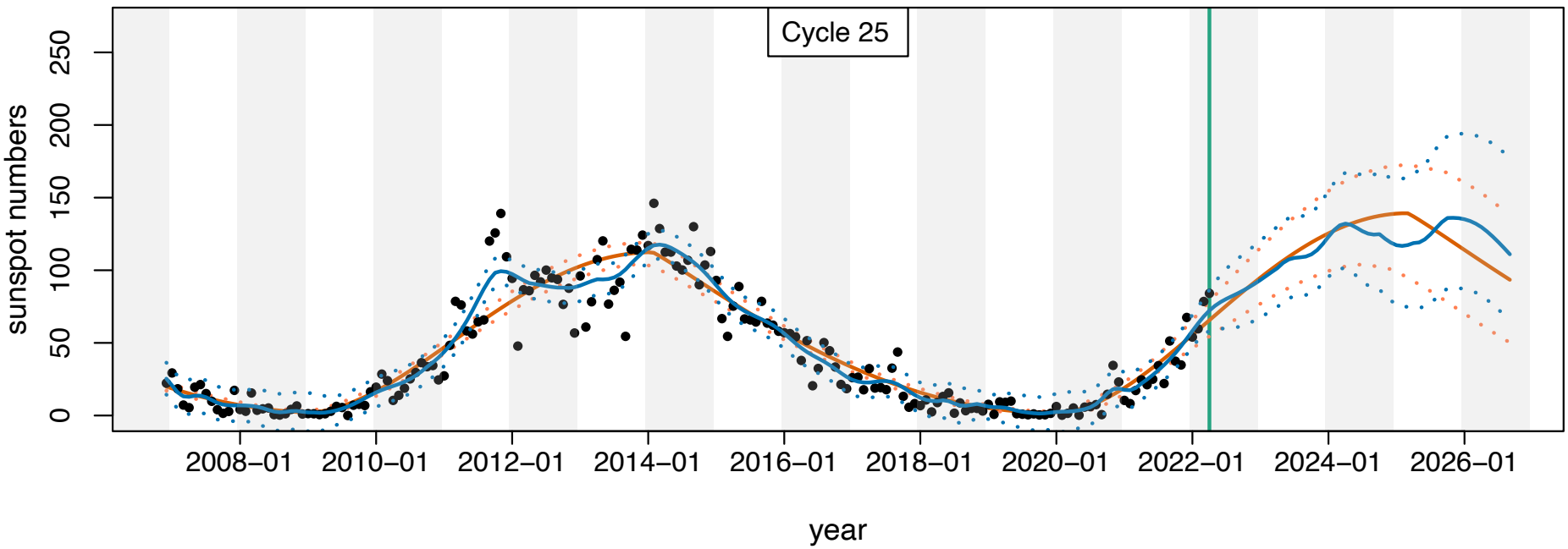
- We carry out *reference distribution variable selection* (Linkletter et al., 2006) to assess the significance of the GP inputs via considering an uncorrelated, inert variable.
- We fit the GP model with the inert variable as an additional input and obtain the distribution of a measure of importance. The 10th percentile of this *reference distribution* is used as a cutoff for selecting important variables.

Results - Cycle 25 Prediction



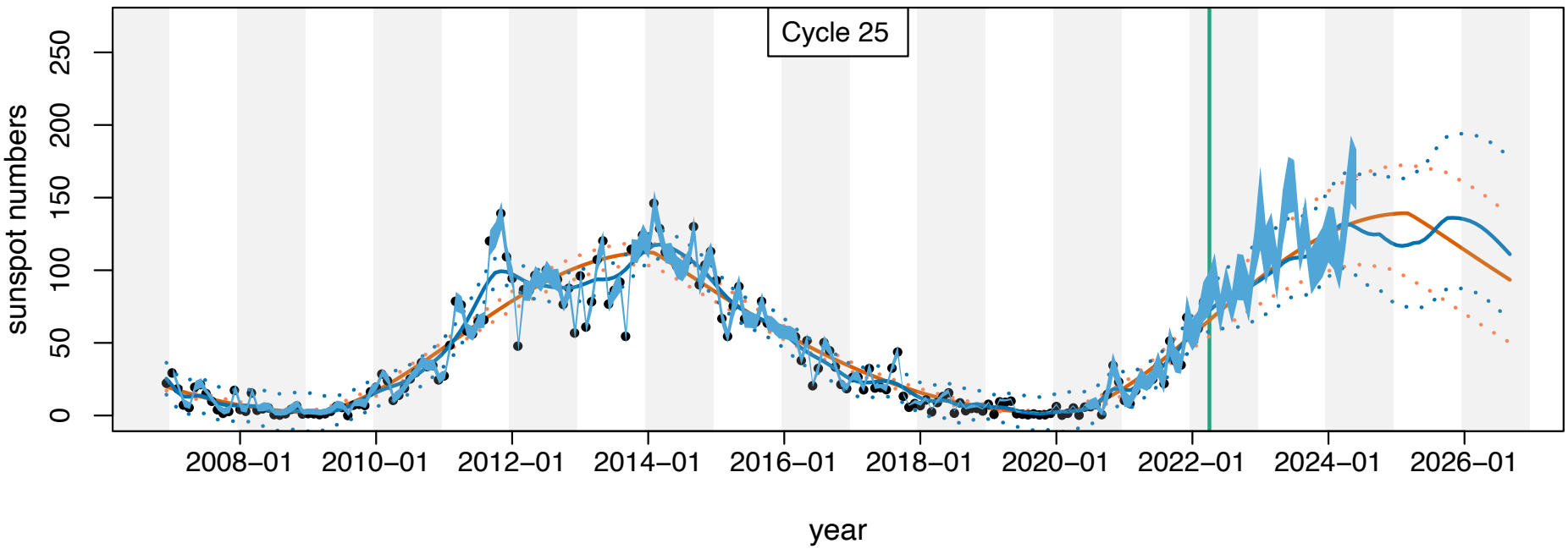
- Solid/dotted orange curves are the prediction/uncertainty interval under the two-level model, which has a single-peaked assumption for the cycle morphology.
- Solid/dotted blue curves are the prediction/uncertainty interval under our expanded model, which anticipates a more complicated morphology.

Results - Cycle 25 Prediction



- Prediction into Cycle 25 (two-level model; expanded model)

Results - Cycle 25 Prediction



- Prediction into Cycle 25 (two-level model; expanded model) actual (observed)

Discussion

1. The multiple peaks predicted by the model appear to capture real features of the observed data. This is the only empirical (non-Physics based) model that has matched observations.
2. Using the polar flux data reduces the amount of data which can be used to fit the GP models and limits the window in which the predictions can be made
3. isotropic covariance model undercovers variance at cycle peak, overcovers at cycle minimum; using asymmetric/non-symmetric covariance might better capture variations
4. the two-level model and the GP model are fitted separately instead of jointly, *however* the benefit of adopting a modular fitting approach is that the Yu et al. (2012) model used during the first stage can be replaced by (almost) any other model. This can be helpful for integrating polar flux data into some existing models without changing their formulation
5. include more proxies (like F10.7 radio flux, flare numbers, cosmic ray flux, etc.) as inputs to GP

For further reading...

1. Yu, Y., van Dyk, D. A., Kashyap, V. L., & Young, C. A. 2012, *Solar Physics*, 281, 847, doi: 10.1007/s11207-012-0090-x
2. Linkletter, C., Bingham, D., Hengartner, N., Higdon, D., & Ye, K. Q. 2006, *Technometrics*, 48, 478
3. Yan, Y., Stenning, D. C., Kashyap, V. L., & Yu, Y. 2021, *Research Notes of the AAS*, 5, 192
4. Yan, Y., Stenning, D. C., Kashyap, V. L., Bingham, D., Nandy, D. & Yu, Y. *In preparation*

Polar Flux Measurements

- It's known that the magnitude of the Sun's polar flux at a cycle minimum is positively correlated with the amplitude of the following sunspot cycle.
- Polar fluxes, shifted forward by 5 yrs 1 mo (empirically chosen to maximize correlation with the SSNs), are incorporated into the GP model as inputs.