

# Incorporating Atomic Data Errors in Stellar DEM Reconstruction

Hosung Kang\*, David A. van Dyk†, Vinay L. Kashyap\*\* and Alanna Connors‡

\**Department of Statistics, Harvard University*

†*Department of Statistics, University of California at Irvine*

\*\**Harvard-Smithsonian Center for Astrophysics*

‡*Eureka Scientific*

## Abstract.

We develop a powerful new method to reconstruct stellar Differential Emission Measures (DEMs), whose Bayesian framework allows us to incorporate atomic and calibration errors as prior information. For instance, known errors in the line locations, as well as missing lines with only minimal information, can be included directly during fitting. Highly correlated systematic errors in the ion balance may be included as well, as a natural sequence during Monte Carlo sampling. Our method uses the statistical framework of data augmentation, where we treat photon counts in each level of the hierarchical structure as missing data. We demonstrate our method by fitting a selected subset of emission lines and continuum in Chandra and EUVE data of Capella to estimate the DEM that best describes the data, and simultaneously determine the element abundances. The Markov Chain Monte Carlo based method also naturally produces error estimates on the fit parameters.<sup>1</sup>

**Keywords:** Differential Emission Measures, Emissivity Matrix, Atomic Data Error, Capella

## INTRODUCTION

The differential emission measures (DEMs) is the distribution of the amount of material in coronae as a function of temperature. DEM summarizes the structure of coronae of stars and provides a powerful tool for understanding the energetics in stellar atmospheres. Hence, it is an important determinant of its physical structure and evolutionary state. Given the emissivities and the observed photon counts of X-ray energies, we aim to reconstruct the marginal distribution of the temperature of the corona.

## SPECTRAL MODEL

We model the intensity of the *ideal* photon counts arriving at energy bin  $i$  without instrumental contamination,  $Y_i$  as independent Poisson variables with intensity

---

<sup>1</sup> The authors gratefully acknowledge funding for this partially provided by NSF Grants DMS-01-04129, DMS-04-38240, and DMS-04-06085 and by NASA Contracts NAS8-39073 and NAS8-03060(Chandra X-ray Center).

$$\lambda_i = \sum_{t=1}^{N_t} \left[ \left\{ \Lambda_{it}^C + \sum_{\text{element } k} \gamma_k \left( \Lambda_{it}^{k,PC} + \Lambda_{it}^{k,L} \right) \right\} \delta_t \right], \quad (1)$$

where  $N_t$  is the number of temperature bins, the  $I \times N_t$  matrix  $\Lambda^C$  represents the continuum emissivity matrix, the  $I \times N_t$  matrix  $\Lambda^{k,L}, \Lambda^{k,PC}$  represents the emission-line emissivity matrix, pseudo-continuum emissivity matrix, respectively, for element  $k$ , (e.g.  $k = \text{Fe, He}$ ),  $\gamma_k$  is the element abundance for element  $k$ , and  $\delta_t$  is the values of the DEM at temperature  $t$ .

Note the similarity in construction here to a spectral response matrix; we treat the emissivity matrix as a redistribution matrix to go from temperature space to wavelength space. The standard instrumental effective area and redistribution matrix are also then applied.

## HIERARCHICAL MISSING DATA STRUCTURING

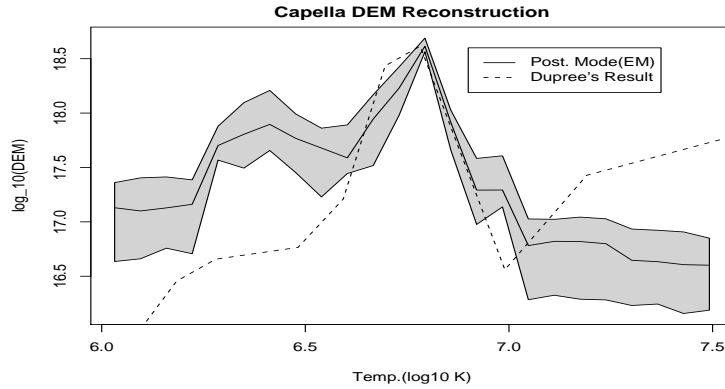
The model (1) has multiple components (e.g., background, continuum and emission lines) and complex structure due to large dimensional matrices. Data augmentation methods such as the EM [1] or DA algorithm [2] make model-fitting simpler especially in highly structured models. Thus, we impute the missing data at each level, treat them as if they were observed, and find the conditional posterior distribution of the next level missing data given the current level. This will eventually lead us to imputing the photon counts emitted from each temperature, where we treat the emissivity matrix as a RMF-like “response matrix” and trace the energy bin counts to the unobserved photon counts in a number of temperature bins. This makes the estimation of DEM straightforward.

## ATOMIC DATA ERRORS

Even the best atomic emissivity databases have missing or misplaced lines and incorrect emissivities. Our method allows prior information on these issues to be directly incorporated into the analysis.

**Error bars on the emissivity matrix:** During each iteration of the MCMC sampling algorithm, we regenerate the emissivity matrix with an error of 10% for H-like and He-like lines and a 20% error on Fe XVII - Fe XX lines. Further corrections that include known differences between the predicted and observed fluxes in the EUV and X-ray regions (e.g., Fe XVIII in X-ray appears to be uniformly underpredicted by 30%; cf. Desai et al. 2004, this conference) can be included in a similar fashion.

**Missing lines:** The two main sources of missing lines in current emissivity database tables are DR lines associated with the weaker resonance lines and lines from high  $n$  ( $n > 5$ ). We plan to include the former by suitably scaling a template derived from, e.g., the satellite lines associated with Fe XVII  $\lambda$  15.02. The magnitude of the latter can be estimated by considering the asymptotic value of the fluxes due to lines from different  $n$  and locating them uniformly between the ionization limit and the last listed line. This has not yet been implemented.



**FIGURE 1.** Error Bars for the Fitted DEMs from Chandra data. The shaded area represents component-wise 95% posterior intervals for the DEM. The DEM reconstructed for the EUVE observation by Dupree et al. [3] is shown. The features in the DEMs are by and large consistent, though our reconstruction shows a significantly larger emission measure below  $\log T=6.5$ .

**Wavelength errors:** Due to incomplete atomic data measurements as well as detector non-linearities, the observed locations of lines does not in general match the theoretical locations. We compensate for this effect by allowing the strongest lines in the spectral region to be shifted during the fit, and then move the remaining weaker lines accordingly.

## RESULTS

The raw spectrum of Capella collected from Chandra HRC-S/LETG data. We computed the maximum a posteriori estimate of the DEM using the EM algorithm starting from a flat DEM. We fit the model via MCMC to compute the posterior mean and component-wise 95% posterior intervals using the same starting values. See Figure 1.

## ACKNOWLEDGMENTS

This work is a product of joint work with the California-Harvard astrostatistics collaboration (CHASC) whose members include J. Chiang, A. Connors, D. van Dyk, D. Esch, P. Freeman, H. Kang, V. L. Kashyap, X.-L. Meng, A. Siemiginowska, E. Surlas, T. Park, Y. Yu, and A. Zezas. We thank Nancy Brickhouse for helpful discussion.

## REFERENCES

1. A. P. Dempster, N. M. Laird, and D. B. Rubin, *Journal of the Royal Statistical Society, Series B, Methodological*, **39**, 1–37 (1977).
2. M. A. Tanner, and W. H. Wong, *Journal of the American Statistical Association*, **82**, 528–550 (1987).
3. A. K. Dupree, N. S. Brickhouse, G. A. Doschek, J. C. Green, and J. C. Raymond, *Astrophysical Journal Letters*, **418**, 115–126 (1993).