# Bounding a good region

Xufei Wang

February 7, 2017

# Introduction

Suppose a continuous random variable $X$ follows power law distribution when $X > x_{min}$ and $\alpha > 1$, then we have

$$f(x) = Cx^{-\alpha}, \quad x \in (x_{min}, \infty).$$

Let $Y = \log X$ and $y_{min} = \log x_{min}$, then the conditional distribution of $Y - y_{min} | Y > y_{min}$ follows $\text{Exp}(\alpha - 1)$.

# Different estimators

## Hill estimator

Suppose we have observation $X_{(1)} \geq X_{(k)} \geq X_{(n)}$, then

$$H_{k,n} = 1 + \frac{1}{\frac{1}{k}\sum_{i=1}^{k}(y - y_{min})}.$$

## QQ estimator

We have

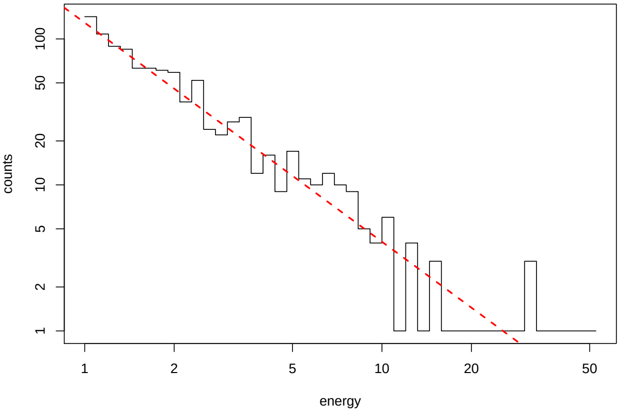
$$1 - F(y|y > y_{min}) = e^{-(\alpha-1)(y-y_{min})}$$

so we can get an estimator for $\frac{1}{1-\alpha}$ if we perform linear regression for

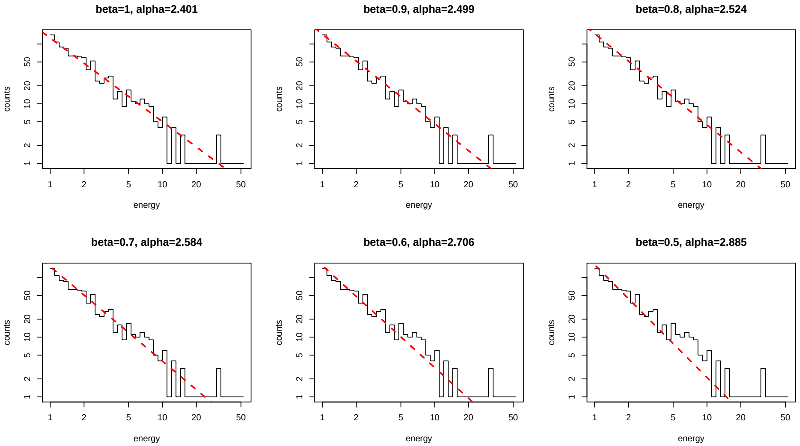
$$\left[\log\left(\frac{i}{k+1}\right), Y_{(i)}\right], \quad 1 \leq i \leq k.$$

# An example

Suppose $X_1, \ldots, X_{1000}$ follows power law distribution with $\alpha = 2.5$, by Hill estimator we get $\alpha = 2.522$ and by QQ estimator we get $\alpha = 2.599$. If we plot the log-log plot of the histogram of $X$, with the true density function.

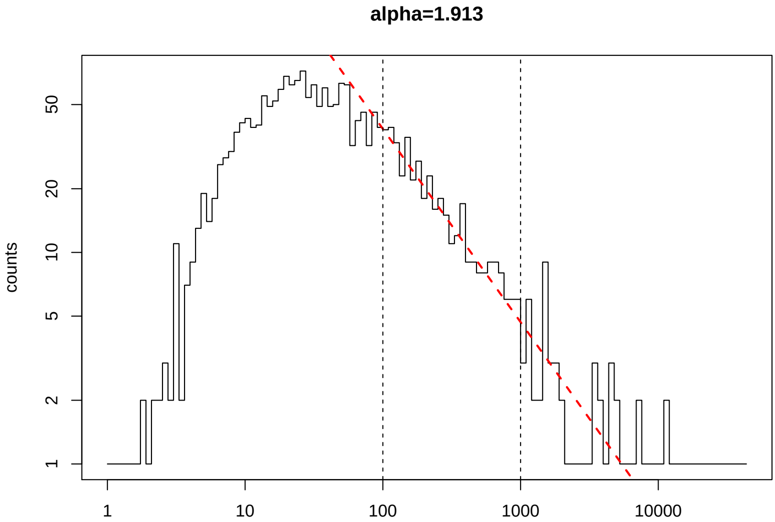


# weighted regression



# real data

By Hill estimator we get $\alpha = 1.977$ and by QQ estimator we get $\alpha = 1.951$.



# weighted regression