Time Delay Estimation: Microlensing

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Stat310

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Introduction

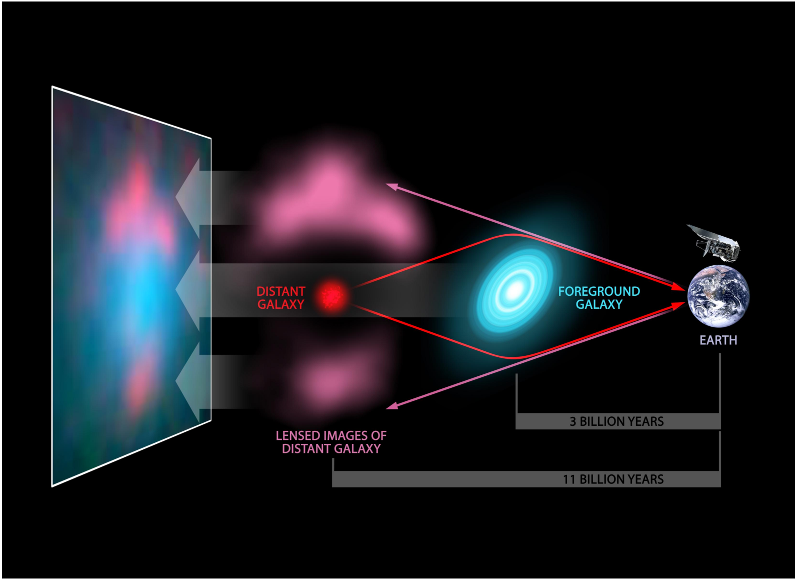
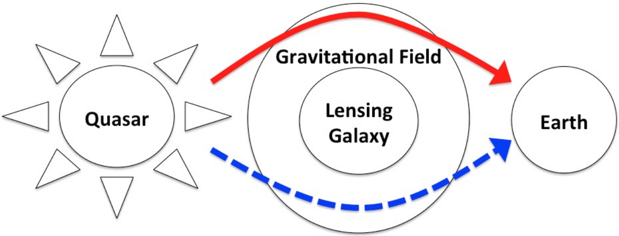


Image Credit: NASA/JPL-Caltech

Introduction



Light rays are bent by a strong gravitational field of a lensing galaxy.

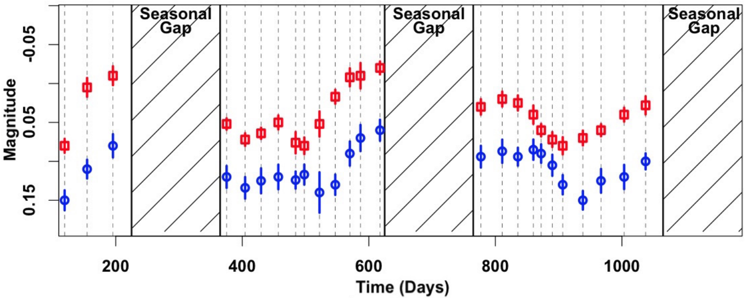
- Each route has different length.
- Difference between arrival times of light rays $\rightarrow$ time delay

Why time delay?

- Hubble constant, H_0, expansion rate of the Universe
- Equation of state of dark energy, e.g., accelerating Universe

Data

Doubly-lensed quasar

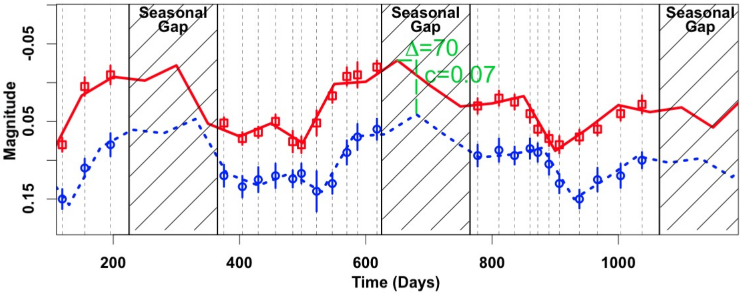


Data comprise of two time series with measurement errors.

- Observation times $\mathbf{t}' \equiv \{t_1, t_2, \ldots, t_n\}$
- Observed magnitudes $\mathbf{x(t)}' \equiv \{x(t_1), x(t_2), \ldots, x(t_n)\}$, and $\mathbf{y(t)}$
- SD of measurement errors $\boldsymbol{\delta}\mathbf{(t)}' \equiv \{\delta(t_1), \delta(t_2), \ldots, \delta(t_n)\}$ and $\boldsymbol{\eta}\mathbf{(t)}$

Our job is to estimate time delay (shift in x-axis) between two time series.

State-space model



- Assumption 1: $\exists$ latent light curves representing the unobserved true magnitudes in continuous time (red and blue dashed curves).

  $\mathbf{X(t)} = (X(t_1), X(t_2), \ldots, X(t_n))^\top$ and $\mathbf{Y(t)}$, values on curves at $\mathbf{t}$

- Assumption 2 (Curve Shifting): $\mathbf{Y(t)} = \mathbf{X}(\mathbf{t} - \Delta) + c$

Probability Distributions

Observed data: Independent Gaussian measurement errors

- $x(t_j) \mid X(t_j) \sim \mathrm{N}[X(t_j),\ \delta^2(t_j)]$
- $y(t_j) \mid Y(t_j) \sim \mathrm{N}[Y(t_j),\ \eta^2(t_j)]$

  $y(t_j) \mid X(t_j - \Delta), \Delta, c \sim \mathrm{N}[X(t_j - \Delta) + c,\ \eta^2(t_j)].$

Latent data: Ornstein-Uhlenbeck process for $\mathbf{X(\cdot)}$

- Kelly et al. (2009), Kozlowski et al. (2010), MacLeod et al. (2010), Zu et al. (2013) have supported the O-U.
- $dX(t) = -\frac{1}{\tau}\big(X(t) - \mu\big)dt + \sigma dB(t)$
- Solution: Sampling distribution on $\mathbf{X(t)}$ via Markovian property:

  $X(t_j) \mid X(t_{j-1}), \mu, \sigma, \tau \sim \mathrm{N}\left[\mu + B_j\big(X(t_{j-1}) - \mu\big),\ \frac{\tau\sigma^2}{2}(1 - B_j^2)\right]$

Bayesian and Profile Likelihood Methods

Bayesian method

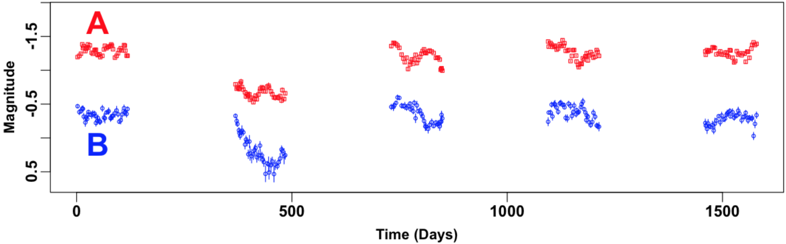
- Prior distributions for the model parameters; $\Delta, c, \mu, \sigma^2, \tau$
- Metropolis-Hastings within Gibbs sampler
- Pros: Complete investigation on all the model parameters
- Cons: Computationally expensive implementation

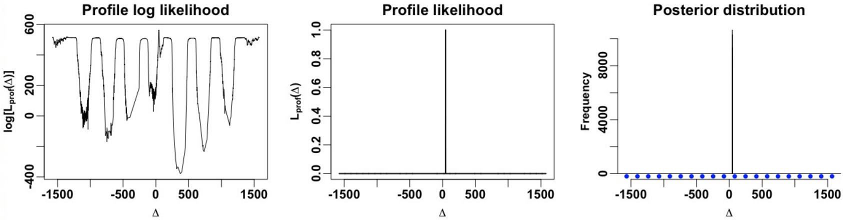
Profile likelihood method

- $L_{prof}(\Delta) \equiv \max_{c,\mu,\sigma^2,\tau} L(\Delta, c, \mu, \sigma^2, \tau)$
- $p(\Delta | D_{obs}) \approx \frac{(2\pi)^2}{u_2 - u_1} L_{prof}(\Delta) \propto L_{prof}(\Delta)$
- Pros: Simple and fast implementation
- Cons: The time delay only

Example 1: Simulated Data

Simulated data of doubly-lensed quasar





Example: Simulated Data (cont.)

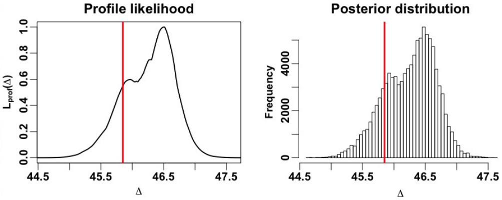
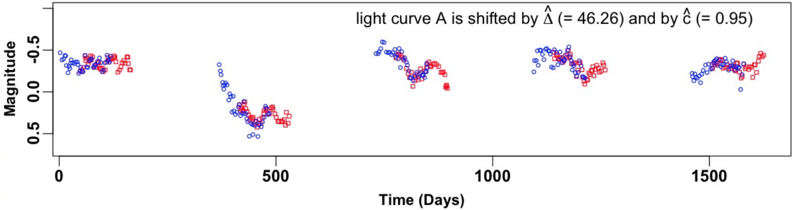
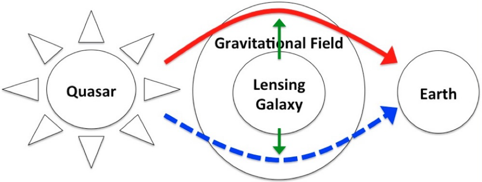


Table 1 : Estimation summary for Δ

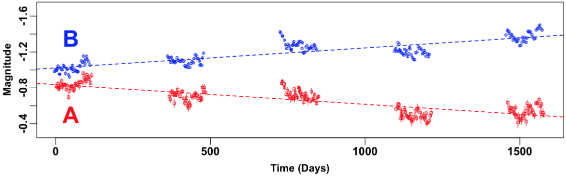
Method	Truth	Post. Mean	Post. Mode	Post. SD
Bayesian	45.85	46.26	N/A	0.41
Profile likelihood		46.26	46.51	0.40



Microlensing

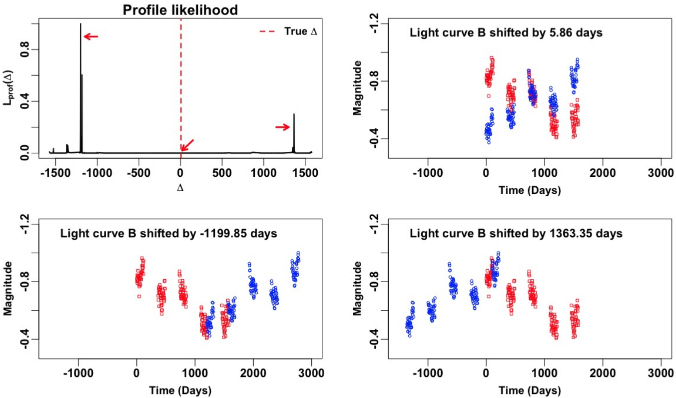


- Microlensing effect occurs when stars inside the lensing galaxy introduce independent flickering noises into the paths of light (Tewes, Courbin and Meylan, 2012).
- If timescale of microlensing is larger than that of quasar variability, light curves can have different long-term trends, e.g., polynomial.



Modes near edges: A sign for microlensing

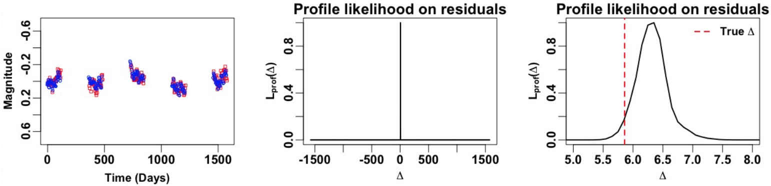
- Curve shifting assumption does not hold because one of the latent curves is no longer a shifted version of the other.



- A small overlap between two light curves → the only similar fluctuation patterns detectable by shifting one of the light curves → several modes near margins of the entire range of Δ.

Time delay estimation with microlensing

- One way to reduce the microlensing effect is to remove the long-term trend by fitting a regression on each light curve, treating residuals as observed light curves (Courbin et al., 2011).
- The intrinsic quasar variability remains even after removing the independent extrinsic variability.



- Posterior mean (6.32) catches the blinded true time delay (5.86) within 1.7 posterior standard deviations (SD=0.27).
- Cons: Ignoring uncertainties in estimating regression coefficients.

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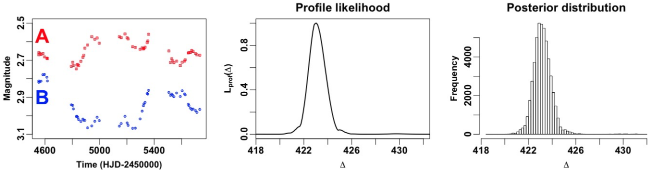
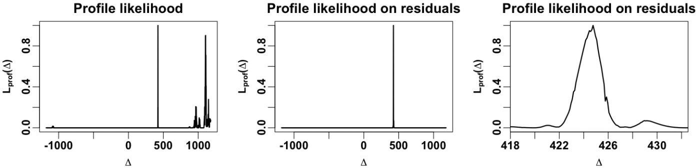
- Analysis on a limited range, [300, 600], based on previous analyses.



- Analysis on the entire range (the mode is shifted to the right a little).



Discussion

- Next: I am incorporating the regression into the model and Integrating out all the mean parameters.
- Next Next: We will consistently analyze quadruply-lensed data in one model.