Separating image structures via graph-based seeded region growing

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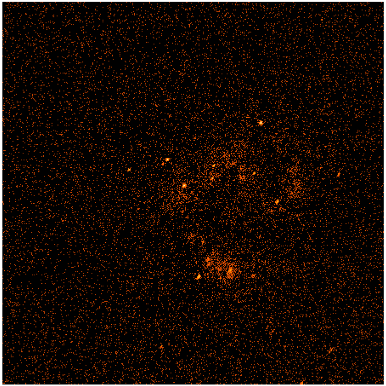
Data description

- X-ray observatory data: spatial coordinates and energy of photons detected.
- Binning the data gives us an X-ray image.



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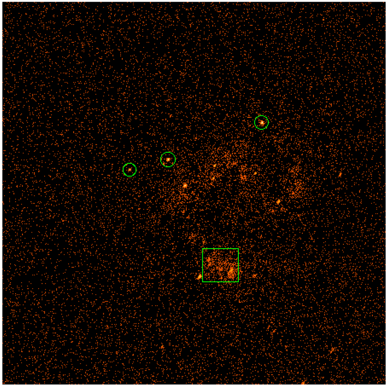

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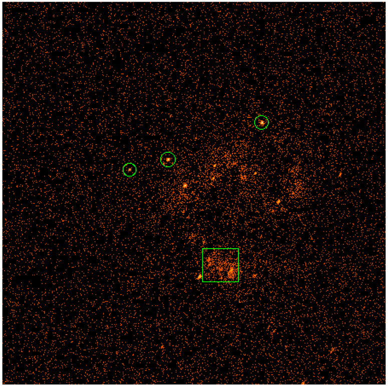
- Shows point sources and extended sources.
- Our task: separate the structure of sources from the background.



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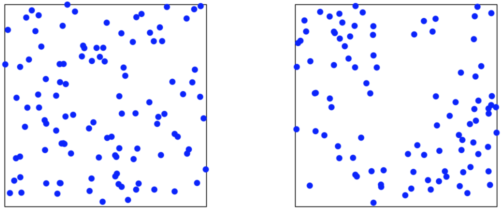
Inhomogeneous Poisson process

- Assumption: the detected photons follow an inhomogeneous Poisson process with density $\lambda(y)$.
- For any set A, $N(A) \sim \text{Pois}\left(\int_A \lambda(y)dy\right)$.
- $N(A)$: the number of photons contained in set A.



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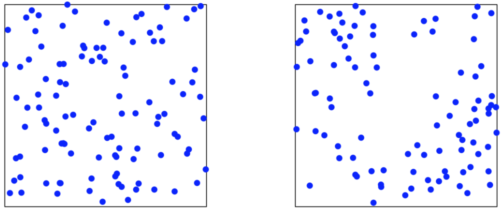
- We denote these photons as $\{p_1, p_2, \cdots, p_n\}$ as an realization of the Poisson process.



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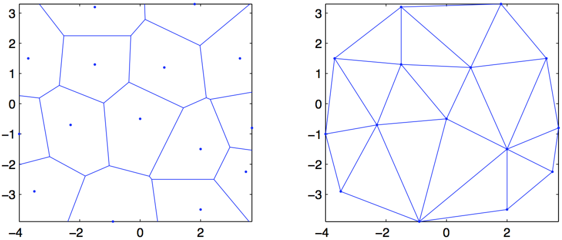
Voronoi tessellation

- Imagine that there are n points on the plane.
- Divides the plane into n cells $\{C_1, C_2, \cdots, C_n\}$ such that cell C_i contains all locations closer to point p_i than to any other point.



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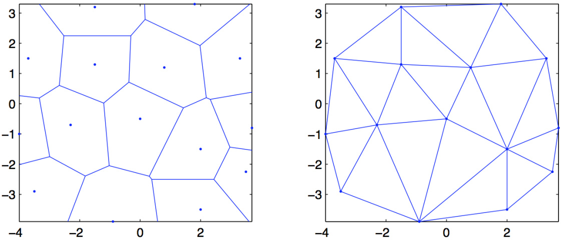
- Delaunay triangulation: the dual graph of Voronoi tessellation.



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Voronoi estimator

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- Construct the following graph:

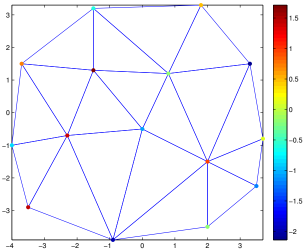


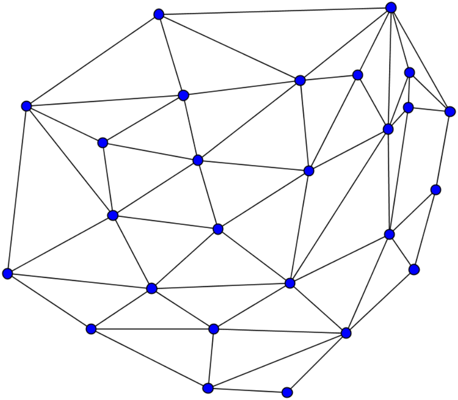
Figure : The graph constructed (each node has a value).

Graph-based seeded region growing (G-SRG)

- The SRG was first proposed by Adams et al. (1994).
- It is an algorithm used for image segmentation: separates an image into several regions such that each region is composed by connected pixels with similar values.
- We extend the usage of it from images to graphs.

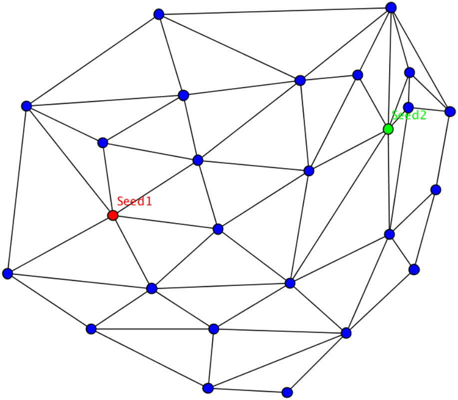
The algorithm: step 1

- Imagine that there is a graph, and each node of it has been assigned a value.



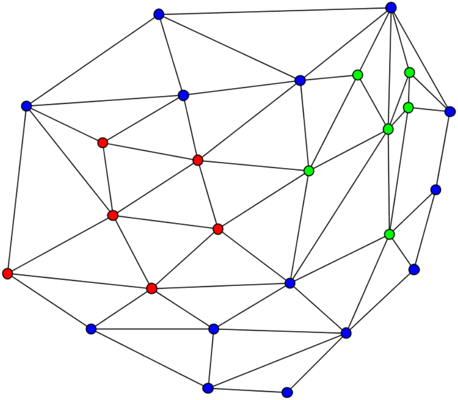
The algorithm: step 2

- Place a set of seeds in the graph, where each seed can be a single node or a set of connected nodes.



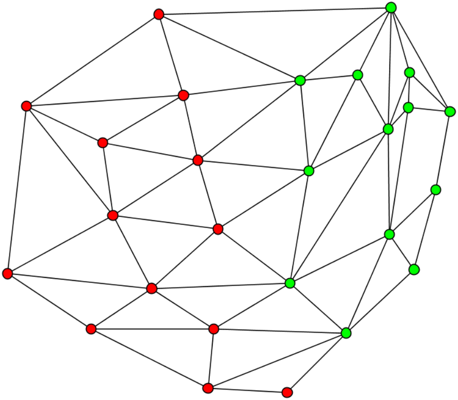
The algorithm: step 3

- Grows these seeds into regions by successively adding neighboring nodes.



The algorithm: step 4

- Finishes when all nodes in the graph are assigned to one (and only one) region.



The growing strategy

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- In detail, it chooses the pair of a growing region and its neighboring node such that the following criterion is minimized:

$$\delta(x, R) = \left| g(x) - \frac{\sum_i A(r_i) g(r_i)}{\sum_i A(r_i)} \right|.$$

- $g(\cdot)$: a function mapping a node index to its value.
  r_i: the i-th element of region R.
  $A(r_i)$: the area of the Voronoi cell containing r_i.

How to specify the seeds?

- The seeds of sources:
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  - We specify nearby nodes as the seeds of sources.
- The background seeds: they can be just specified manually.

Example one: two point sources

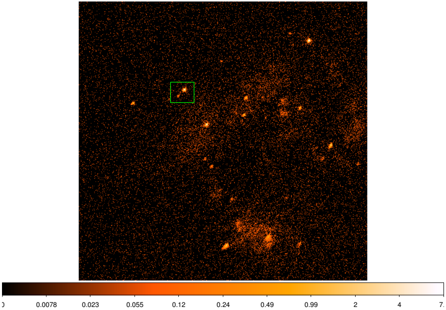

Figure : Region of interest (within the rectangle).

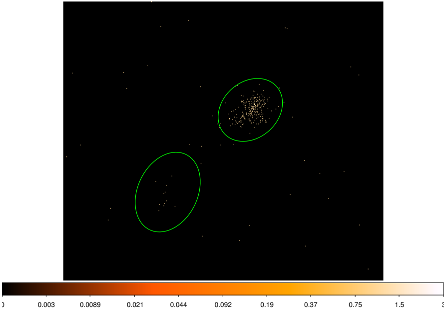

Figure : Region of interest after zooming in.

Example one: two point sources (cont.)

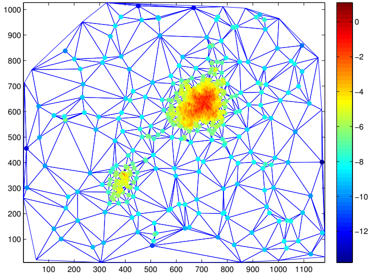

Figure : Graph constructed by Delaunay triangulation (after log transformation).

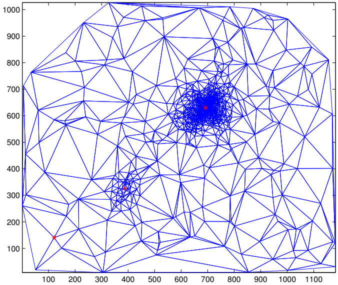

Figure : Seeds specified by wavdetect (three red dots).

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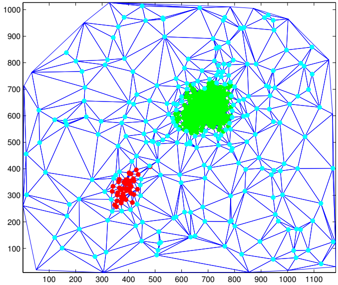


Figure : Result of G-SRG (clustering of photons)

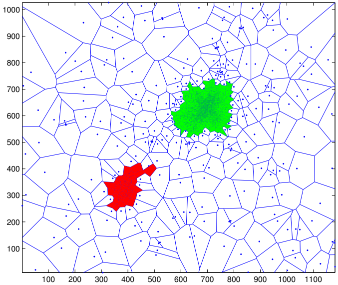
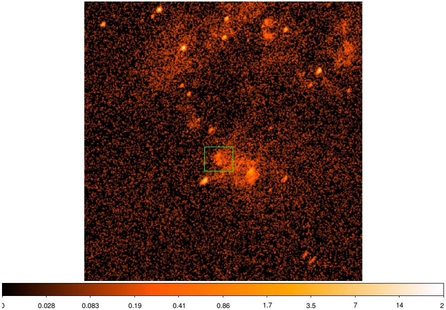


Figure : Result of G-SRG (clustering of Voronoi cells)

Example two: two embedded point sources in a field of structured extended emission



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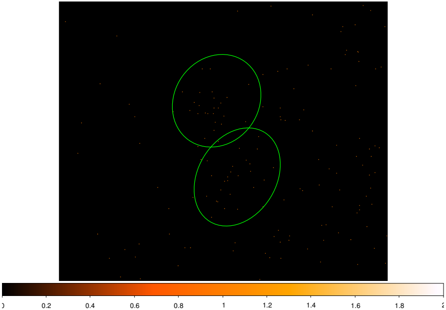
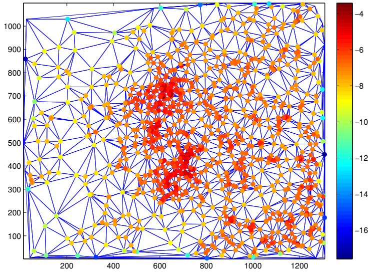


Figure : Region of interest after zooming in.

Example two: two embedded point sources in a field of structured extended emission (cont.)



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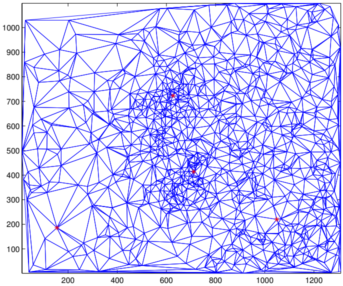

Figure : Seeds specified by wavdetect (four red dots).

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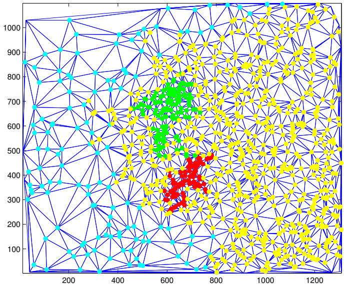

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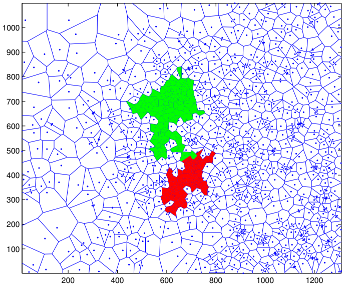

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Pros and Cons

- Pros:
  - Robustness: the result is not affected by the parameters, e.g., the bin size and the location of the background seeds.
  - Fast computation: the computational speed depends on the number of photons. The time complexity of Voronoi tessellation is $O(n \log n)$. The time complexity of G-SRG is at most $O(n^2)$. (On macbook, 10 seconds for $n = 1500$.)

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- Cons:
  - G-SRG is an ad-hoc method, which lacks a theoretical support.
  - It requires the specification of the seeds of sources, which affects the outcome of G-SRG significantly.

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