A Revolution in DEM Analysis (with Application to Nanoflare Heating)

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The DEM Reconstruction Problem

GOAL: Want to ascertain some measure of n_e and T_e of the optically thin plasma in the solar atmosphere.

For a line of sight through a heterogeneous distribution of plasma, the Differential Emisison Measure is a fundamental measure of the plasma that can be determined with intensity observations of atomic transitions.

$$\mathrm{Obs}|_{\mathrm{channel}} = \int_{T_a}^{T_b} \mathrm{Response}(T)|_{\mathrm{channel}} \times \mathrm{DEM}(T)\ \mathrm{d}T$$

$$\mathrm{Response}(T)|_{\mathrm{channel}} = \int_{\lambda_1}^{\lambda_2} \mathrm{Eff.Area}(\lambda)|_{\mathrm{channel}}\ \times\ S(\lambda, T)\ \mathrm{d}\lambda$$

$\mathrm{DEM}(T)\ \mathrm{d}T$ = Emission measure at T → T+dT
= (Total number electrons at T → T+dT) x (average electron density over T → T+dT)

Make discrete formulation…

$$d_c = \sum_T R_{cT}\ \times \mathrm{DEM}_T$$

How to invert linear system of equations to get DEM_T?

The DEM Reconstruction Problem: CONCERNS

$$d_c = \sum_T R_{cT} \times \mathrm{DEM}_T$$

- Finding the best (or an adequate) fit to the data.

- Keeping the DEM (mostly) positive definite. (I.e.: all DEM values ≥ 0)

- Dealing with the underdetermined problem ($N_T > N_c$). Handling the multiplicity of solutions and selecting for one solution.

- The problem is "ill-posed", meaning that it can be difficult to find a contiguous variation in DEM solutions for a corresponding variation in the data.

- Reasonable computation speed (i.e.: fast enough to be useful).

Historical Approaches to DEM Reconstruction

$$d_c = \sum_T R_{cT} \times \mathrm{DEM}_T$$

Can *broadly* categorize solution methods:

- Direct inversion
- Parametric models (assume a shape)
- Iterative fitting

A principal characteristic shared by these methods:

They all map one observation set to one DEM solution.

→ This necessarily confounds the handling of the issues (fitting, p.d., selection from underdetermined multiplicity, ill-posedness).

→ Selecting one solution from the underdetermined multiplicity is equivalent to applying some a priori information that is independent of the evidence of the data.

Historically, regularization has emerged as the preferred way to simultaneously get better results on all issues. (E.g., fitness criterion is a tradeoff of fit to data and maximizing the information entropy.)

Historical Approaches to DEM Reconstruction: Consequences

$$d_c = \sum_T R_{cT} \times \mathrm{DEM}_T$$

When the solution method returns one solution and confounds all issues…:

… the underdetermined multiplicity is opaquely filtered. No chance to investigate alternate shapes that satisfy the data. Was a better fit to the data possible?

… difficult to understand/pursue the ramifications of the implicit (or explicit) a priori information in selecting the solution.

… the ill-posed property may be somewhat mitigated by selecting for "smoothest" solutions, but there is no guarantee. As the community is venturing into correlating emission measures across space and time, the ability to control for contiguous solutions will gain importance.

A (R)Evolution in DEM Methodology

A new method for solving the DEM inversion:

- Finds all "globally best fit" pos.def. solutions that fit the data. If no exact solutions exist, then finds closest possible fit.

- This is equivalent to finding the subspace of solutions that fit the data. One may (must) still apply a priori constraints to filter for a unique solution.

- Working with the set of all global solutions, it is possible to avoid ill-posedness and find contiguous solutions for contiguous changes to the observations.

The (R)Evolution is…:

- One can know/explore ALL of the best possible (pos.def.) fits to the data (even if one wants to do trade-offs against other criteria).

- The "ill-posed" aspect of the problem can be cleanly circumvented.

- The issue of fitting the data is cleanly separated from the issue of applying a priori information to select a solution.

- There is a framework for mapping the "geometry" of the data and DEM vector spaces, as determined by the response functions and discretized temperature grid.

- By these properties, the new method is not only suitable for reconstructing DEMs, it may also be used for *meta-analysis of other solution methods*.

The "Convex Hull + Singular Value Decomposition" Framework (CH+SVD)

Convex Hull ←→ "Geometry" of observation (ratio) vector space:

- For ANY observation set that corresponds to a physical (p.d.) DEM, there MUST be at least one solution that uses only N_c of the N_T temperature bins, or there is no exact solution.

- Strategy → It is trivial to solve for a DEM with N_c unknowns, using data with N_c knowns. Just try every combination of "N_T choose N_c" bins.

- There is a very nice analogy to center of mass problems that provides geometric insight for thinking about the solutions in the observation vector space.
  - Example #1: one can visualize how far an observation set is from being consistent with an isothermal solution.
  - Example #2: one can visualize whether an observation set has an exact solution, or where the closest solvable set lies.
  - Example #3: one can see that it is possible to find solutions that vary contiguously with changes in the data, and thereby circumvent the ill-posedness problem.

- If none of the tried combinations work, then there is no exact solution. However, there are known linear programming methods for finding the absolutely closest observation set that CAN be solved.

- This method gives ALL DEM solutions that use N_C (or fewer) temperature bins. To get the ones that use more bins, we turn to SVD.

The "Convex Hull + Singular Value Decomposition" Framework (CH+SVD)

SVD ←→ "Geometry" of DEM vector space:

- For a response matrix R_{cT} on a specific T-grid, SVD will provide an orthonormal set of N_T basis functions for DEMs.

- In that set, there will be N_C "real" basis functions $\theta^{(i)}{}_T$ whose coefficients $a^{(i)}$ are uniquely fixed by the data d_C.

$$d_c = \sum_T R_{cT} \times \left(\sum_{i=1}^{N_c} a^{(i)} \theta_T^{(i)} \right)$$

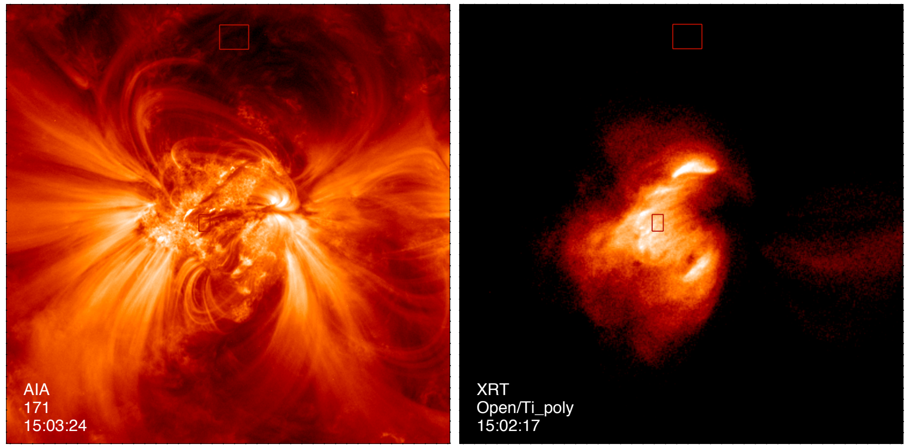
- There will also be (N_T-N_C) "null" basis functions $\phi^{(j)}{}_T$ that solve the homogeneous DEM equation, and therefore their coefficients $b^{(j)}$ are irrelevant for fitting to the data.

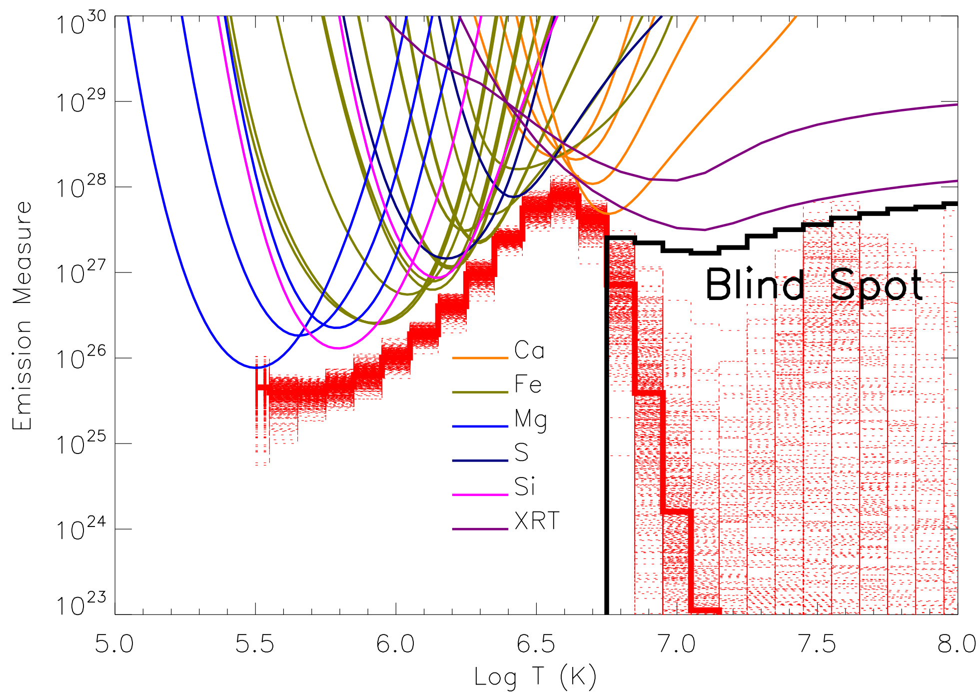
$$0 = \sum_T R_{cT} \times \left(b^{(j)} \phi_T^{(j)} \right) \quad , \forall\ j = \{1 ... (N_T - N_c)\}$$

$$\mathrm{DEM}_T = \left(\sum_{i=1}^{N_c} a^{(i)} \theta_T^{(i)} \right) + \left(\sum_{j=1}^{N_T - N_c} b^{(j)} \phi_T^{(j)} \right) \quad , \forall\ b^{(j)} \mathrm{s.t.}\ \mathrm{DEM}_T \geq 0$$

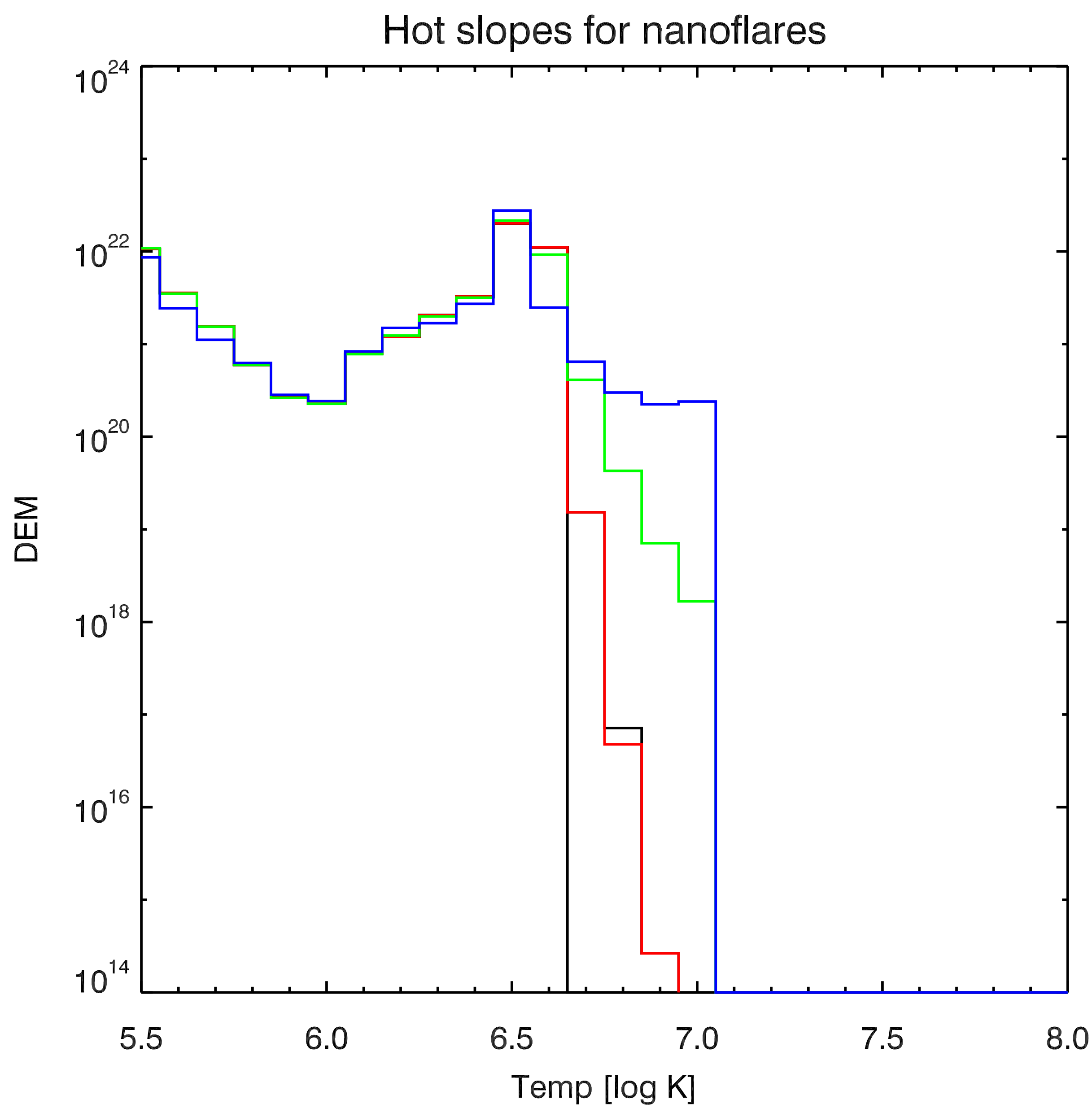
Application to Nanoflares

(Winebarger et al., 2012, ApJ Lett. 746, 17)

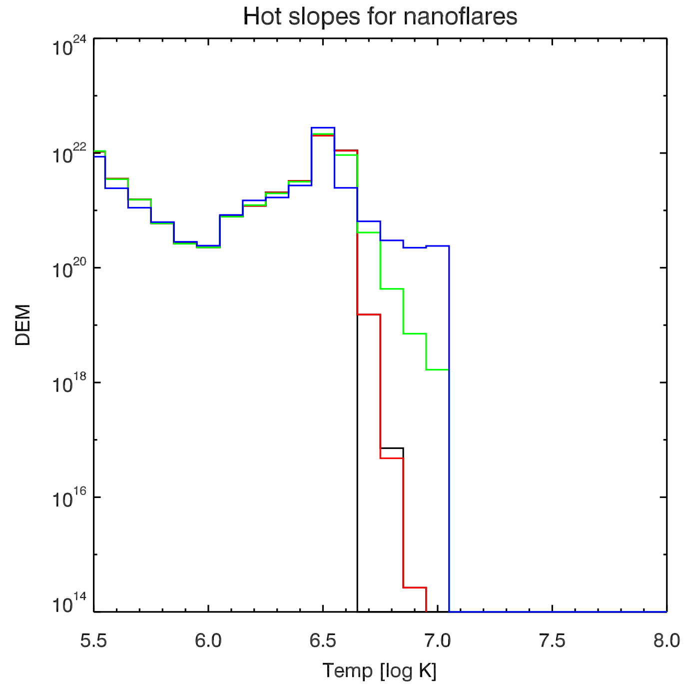




Application to Nanoflares



Application to Nanoflares



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