Reducing Ground-Based Astrometric Errors with *Gaia* and Gaussian Processes

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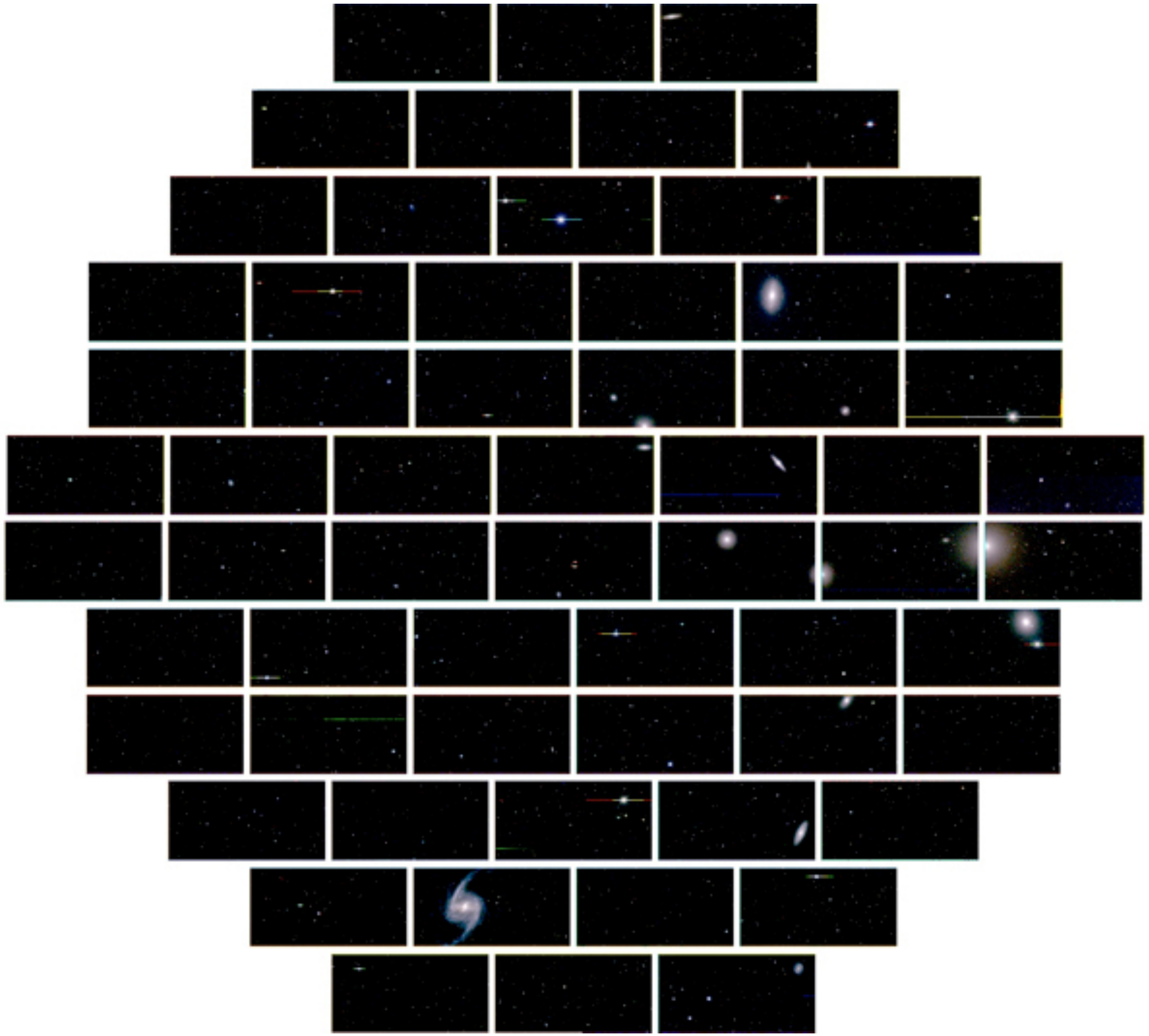
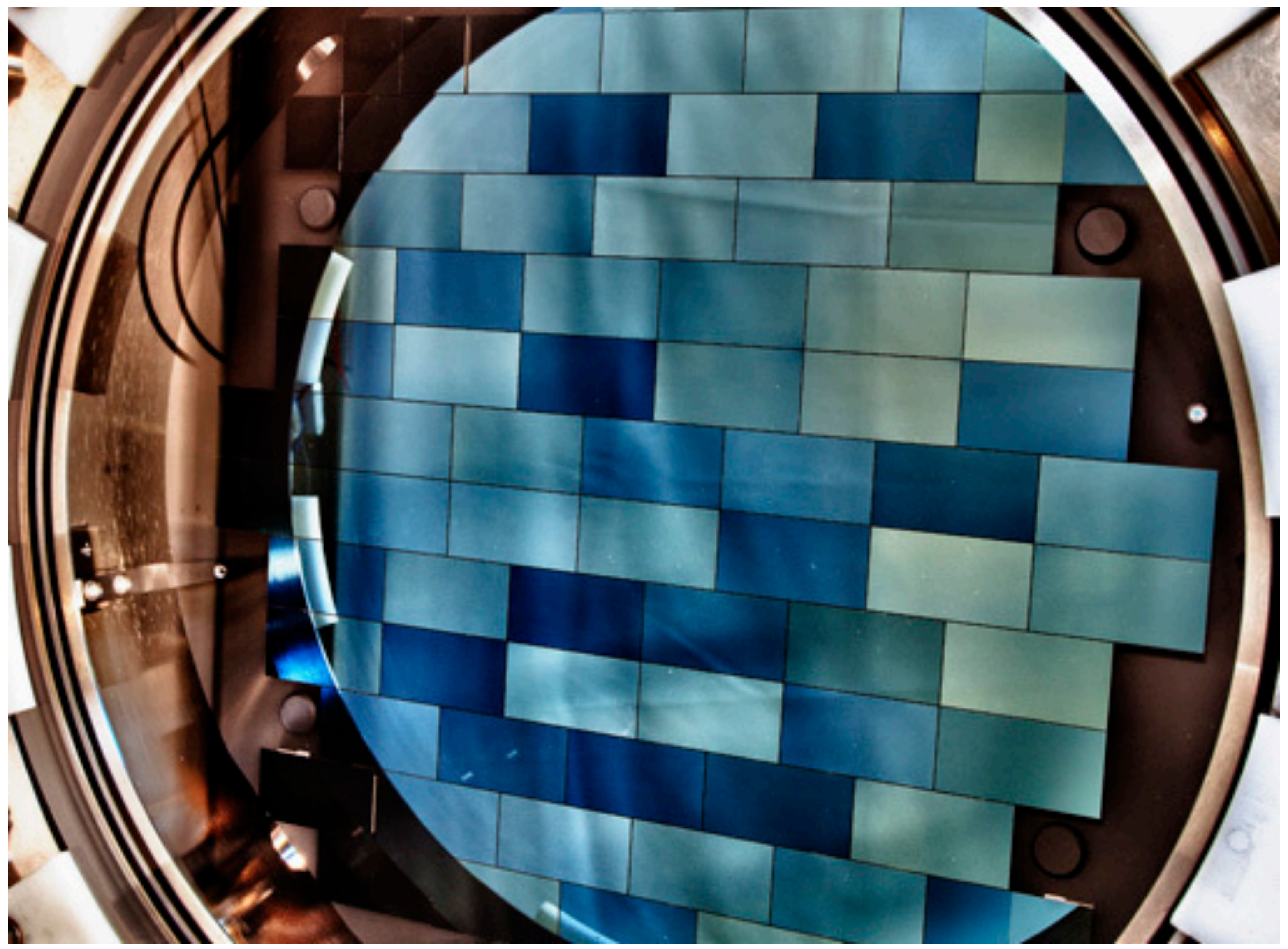
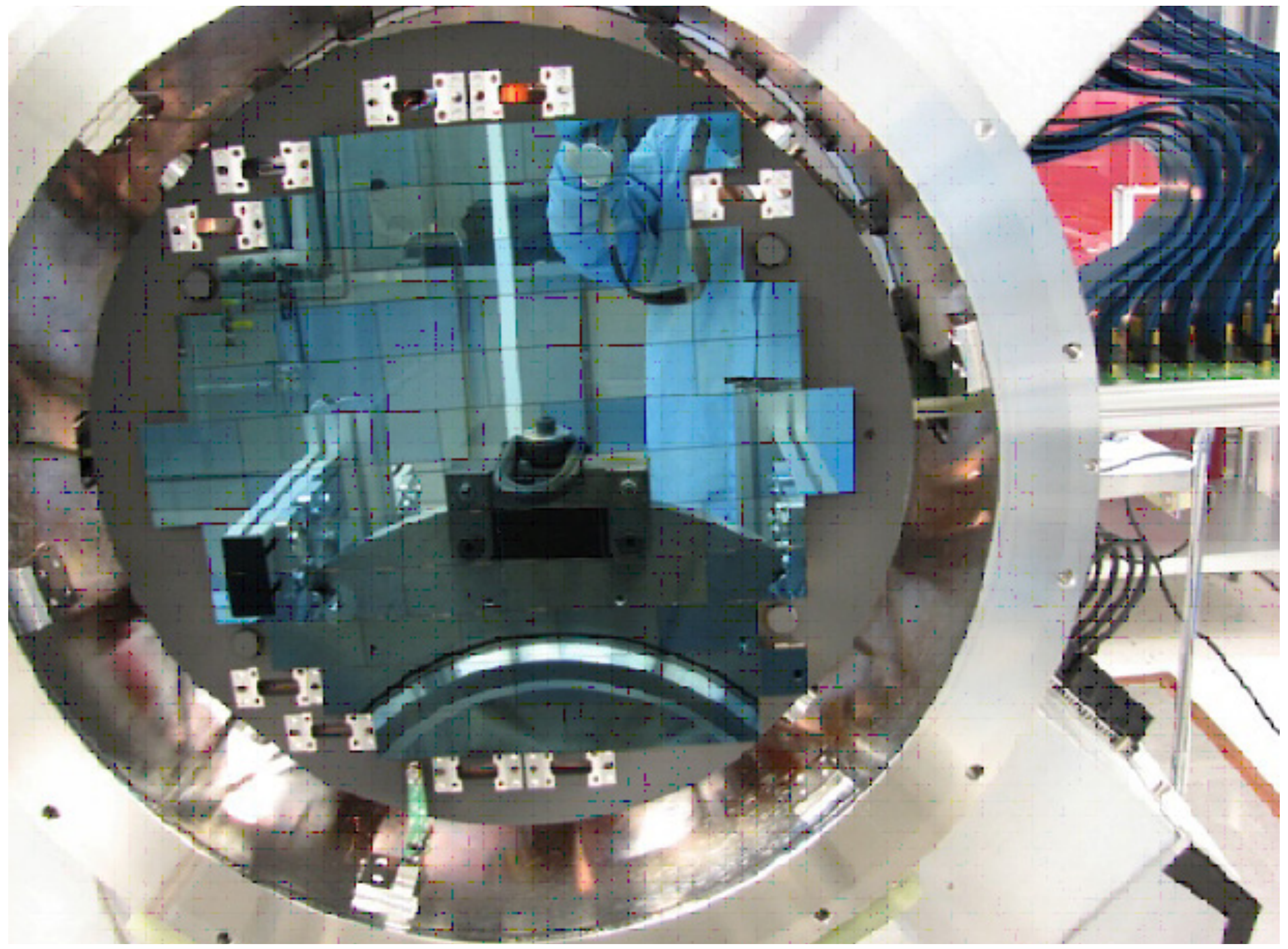
See W. F. Fortino *et al* 2021 *AJ* **162** 106
Also see arXiv:2103.09881 (Léget et al) for similar work on HSC

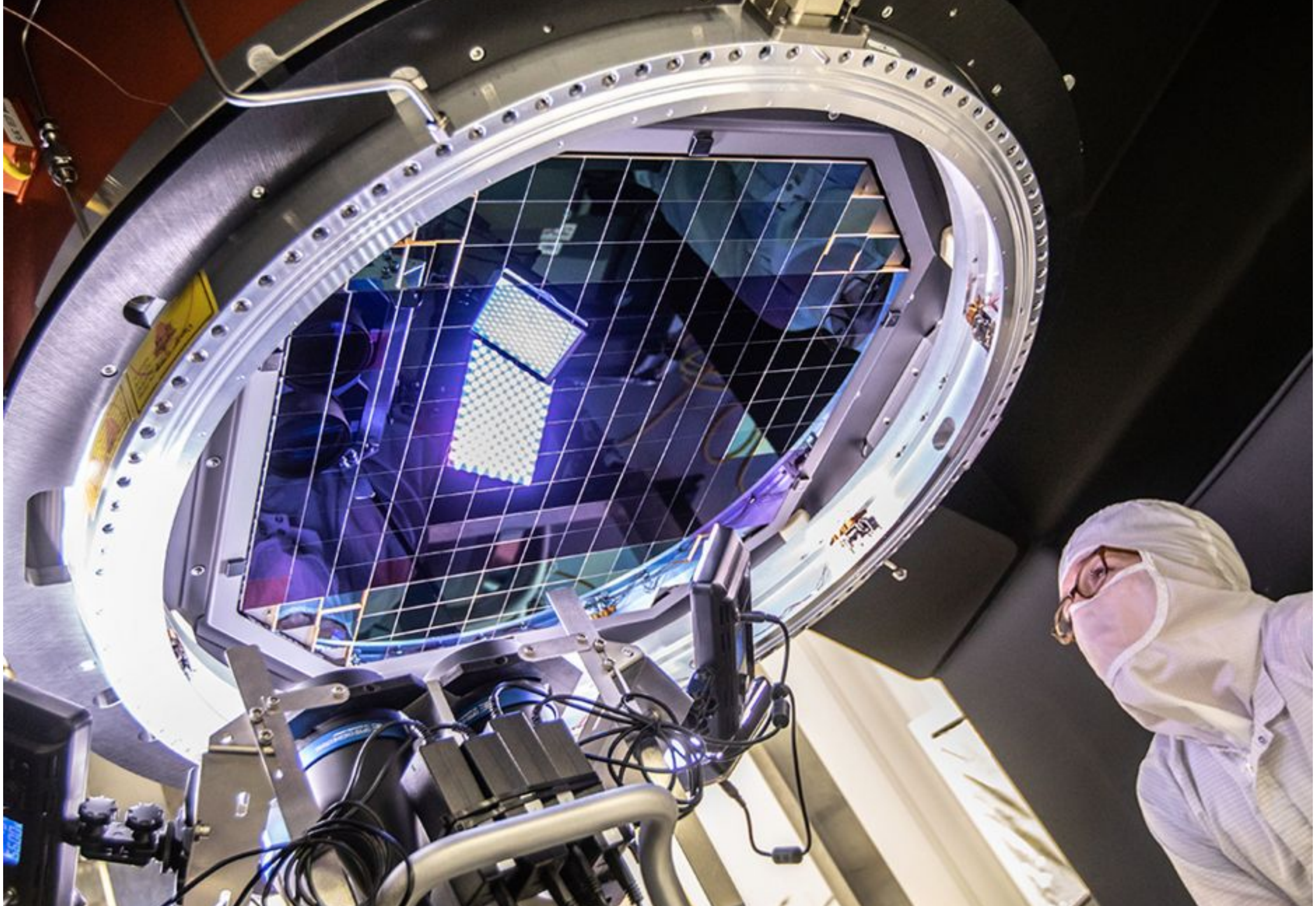






The Dark Energy Survey
and the Dark Energy Camera

- Victor M. Blanco Telescope (Blanco) at CTIO
- Dark Energy Camera (DECam)
  - Given to CTIO in exchange for observation time
  - 62 2048×4096 CCD tiles (520MP total)
  - iPhone 12 Pro has a 12MP camera.
- >5000 deg^2
- 758 Observing Nights over 6 years
- Goal: Looking for evidence of Dark energy and Dark matter, and more



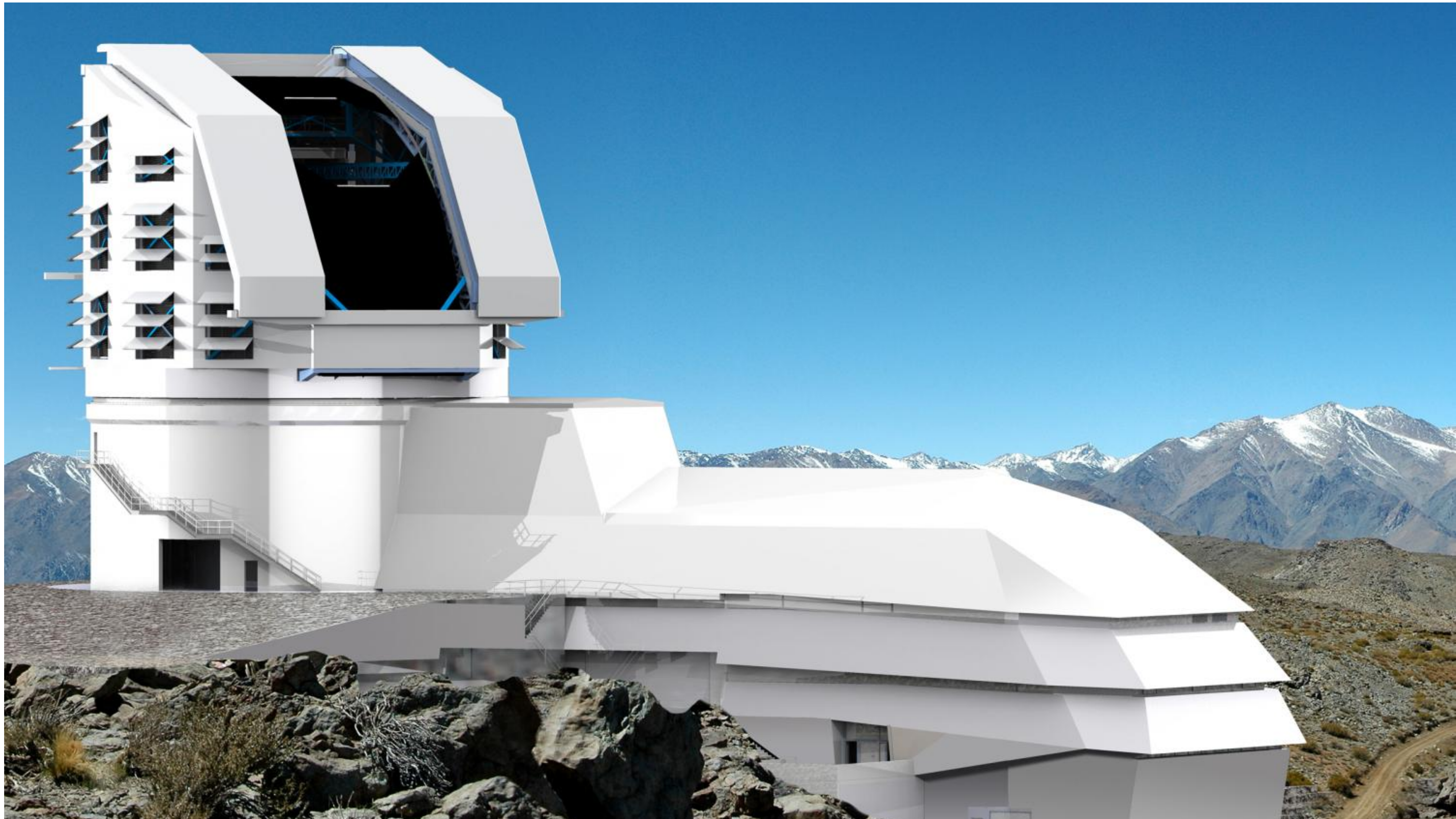
Focal plane
Utility Trunk—houses support electronics and utilities
Cryostat—contains focal plane & its electronics
1.65 m (5'-5")
L3 Lens
Filter
L2 Lens
L1 Lens


UA SCIENCE
RICHARD F. CARIS
MIRROR LAB
Steward Observatory
OHARA
GMT

The Vera C. Rubin Observatory

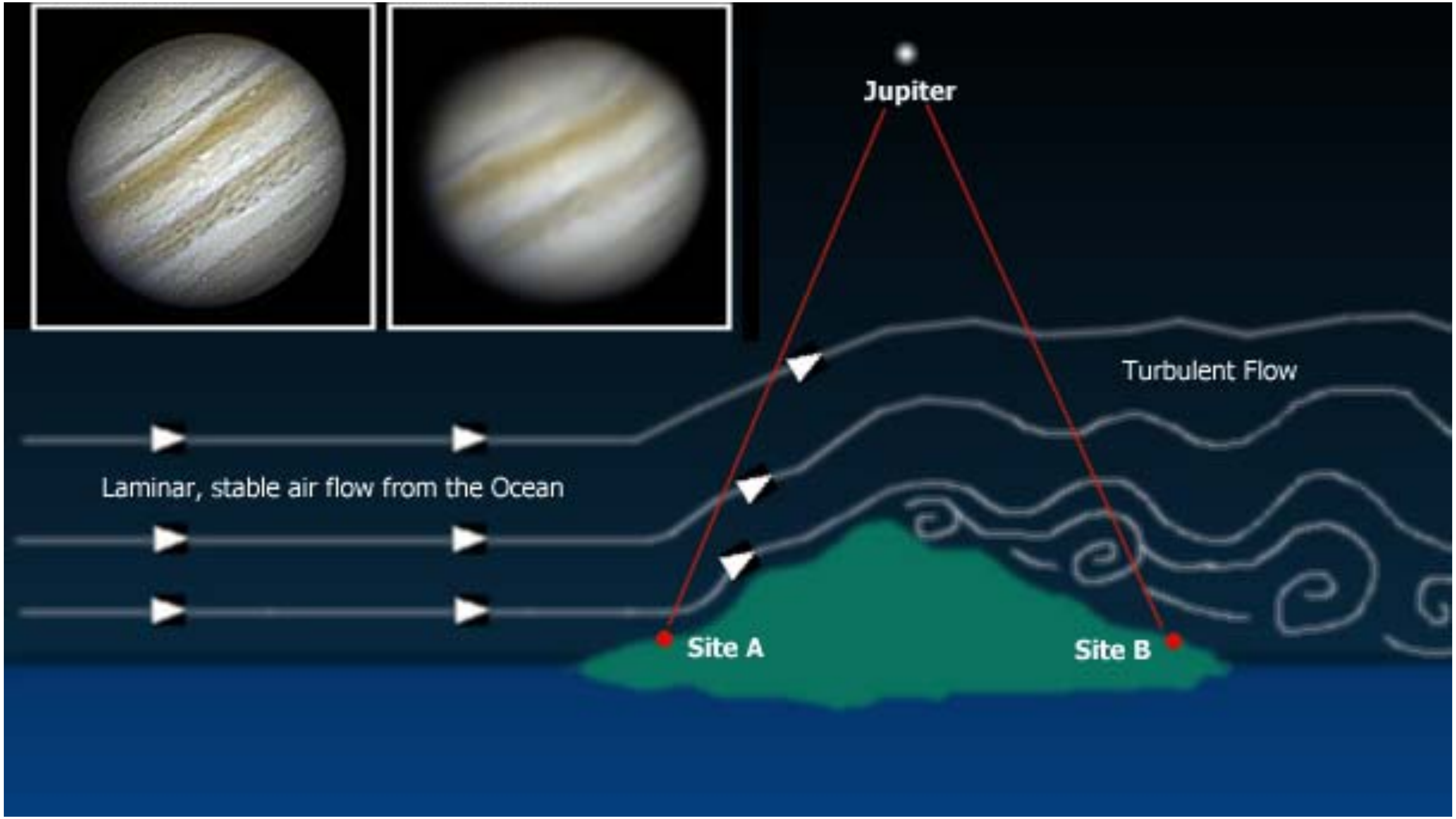
and the Legacy Survey for Space and Time

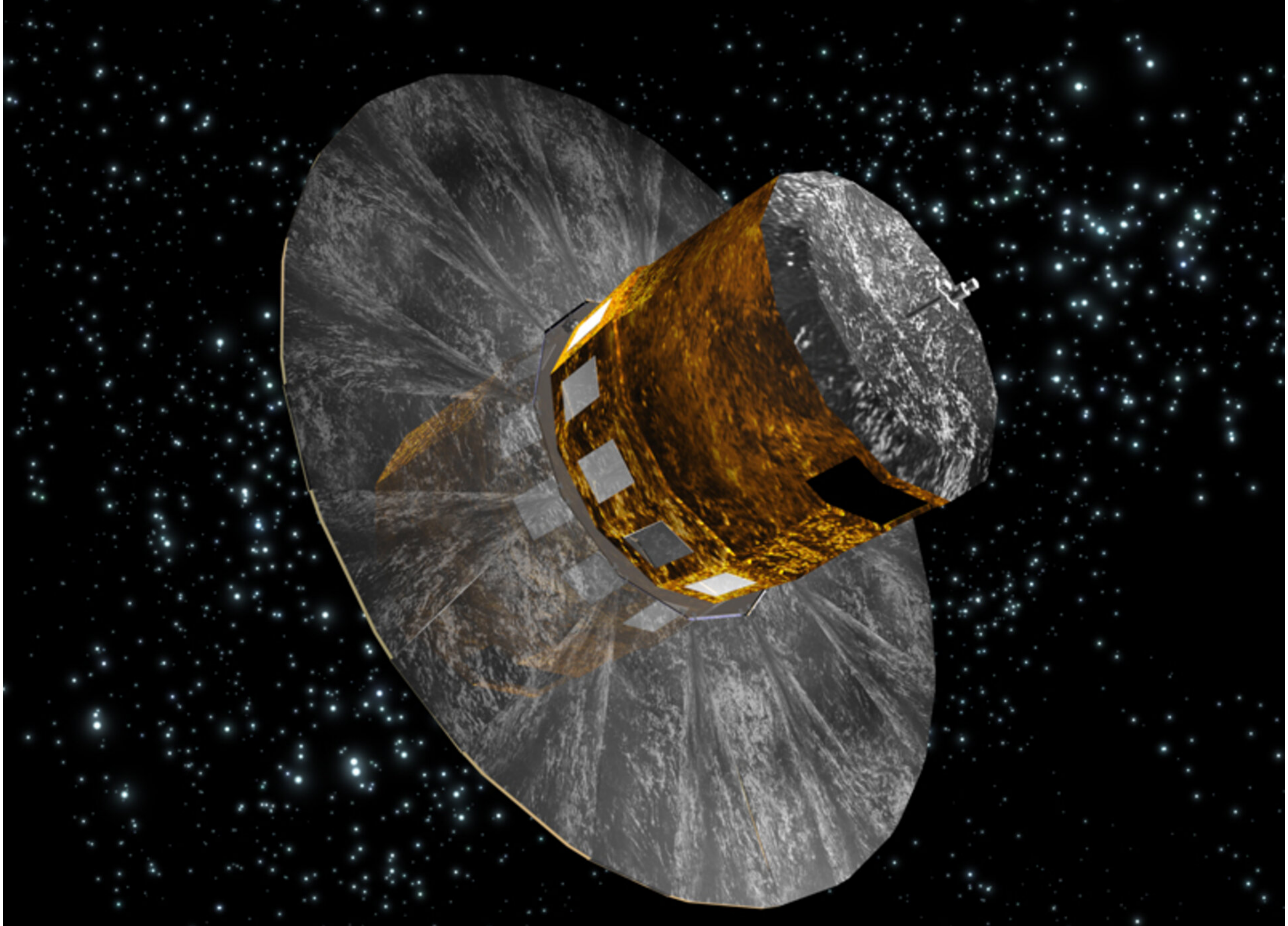
- Near CTIO
- The successor to DES
- >18000 deg^2
- 3200MP Camera
- 10 year survey
- Goal: Dark Energy, Dark Matter, +

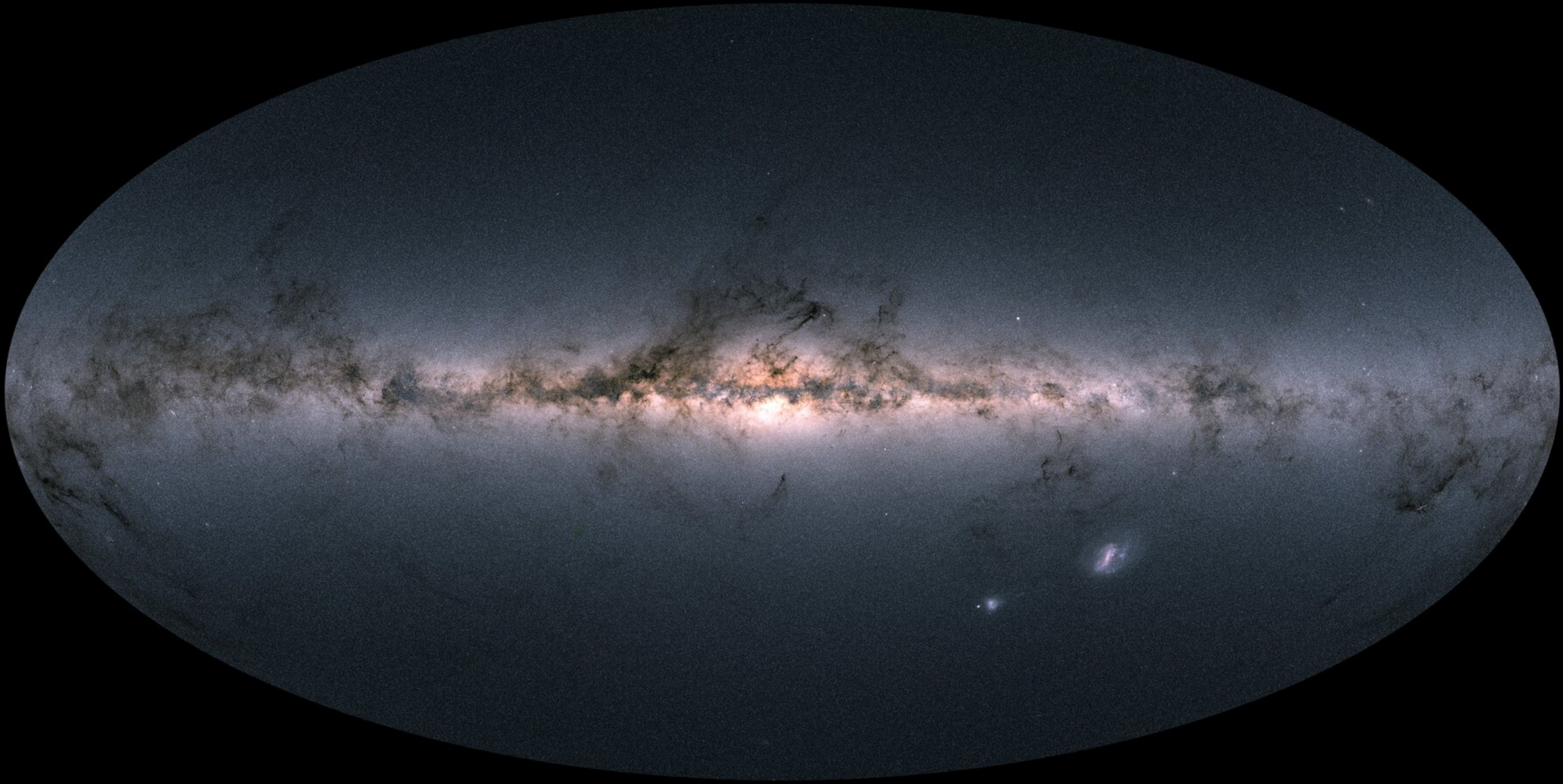


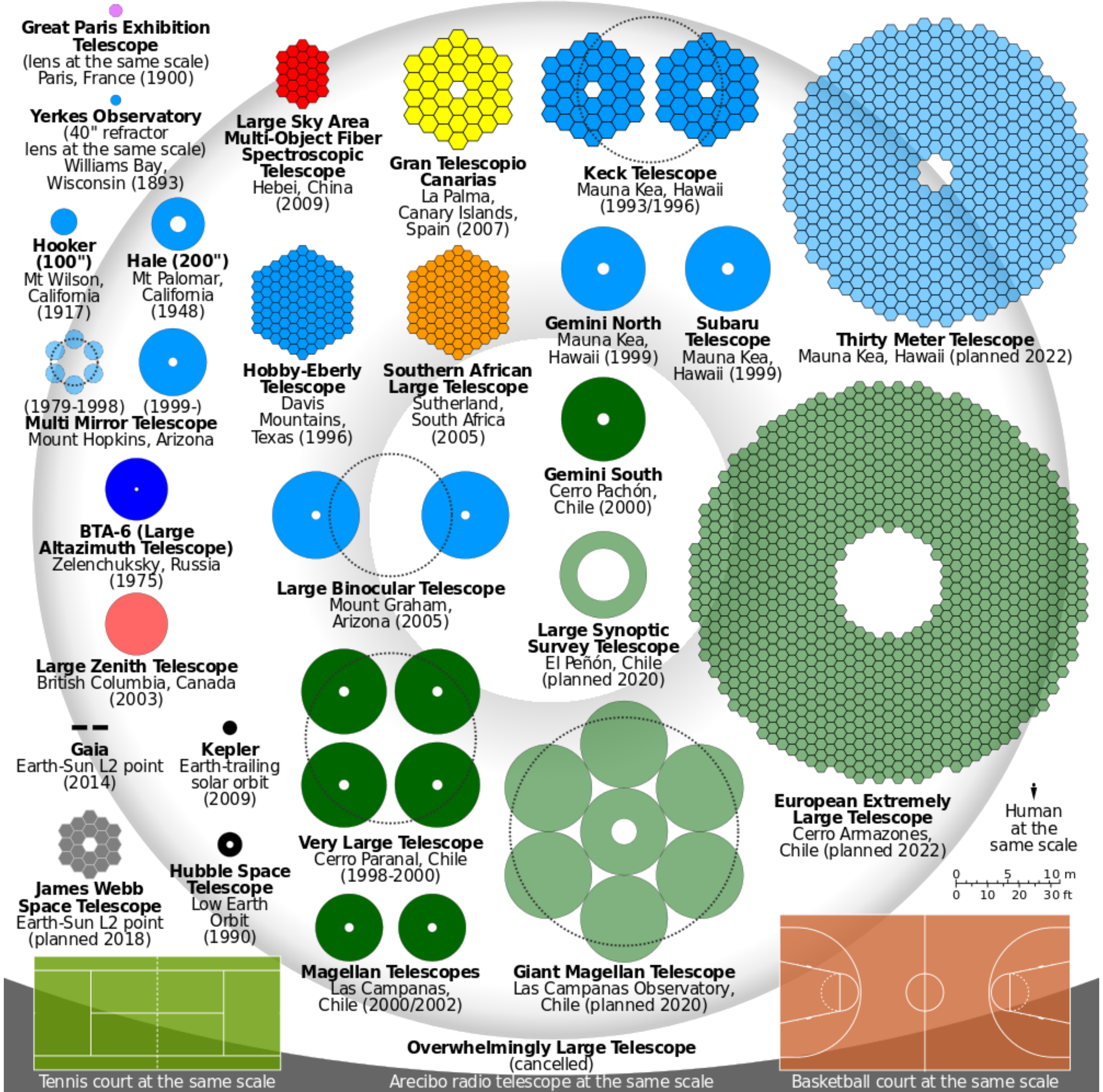
Atmospheric Turbulence

Why do stars twinkle?









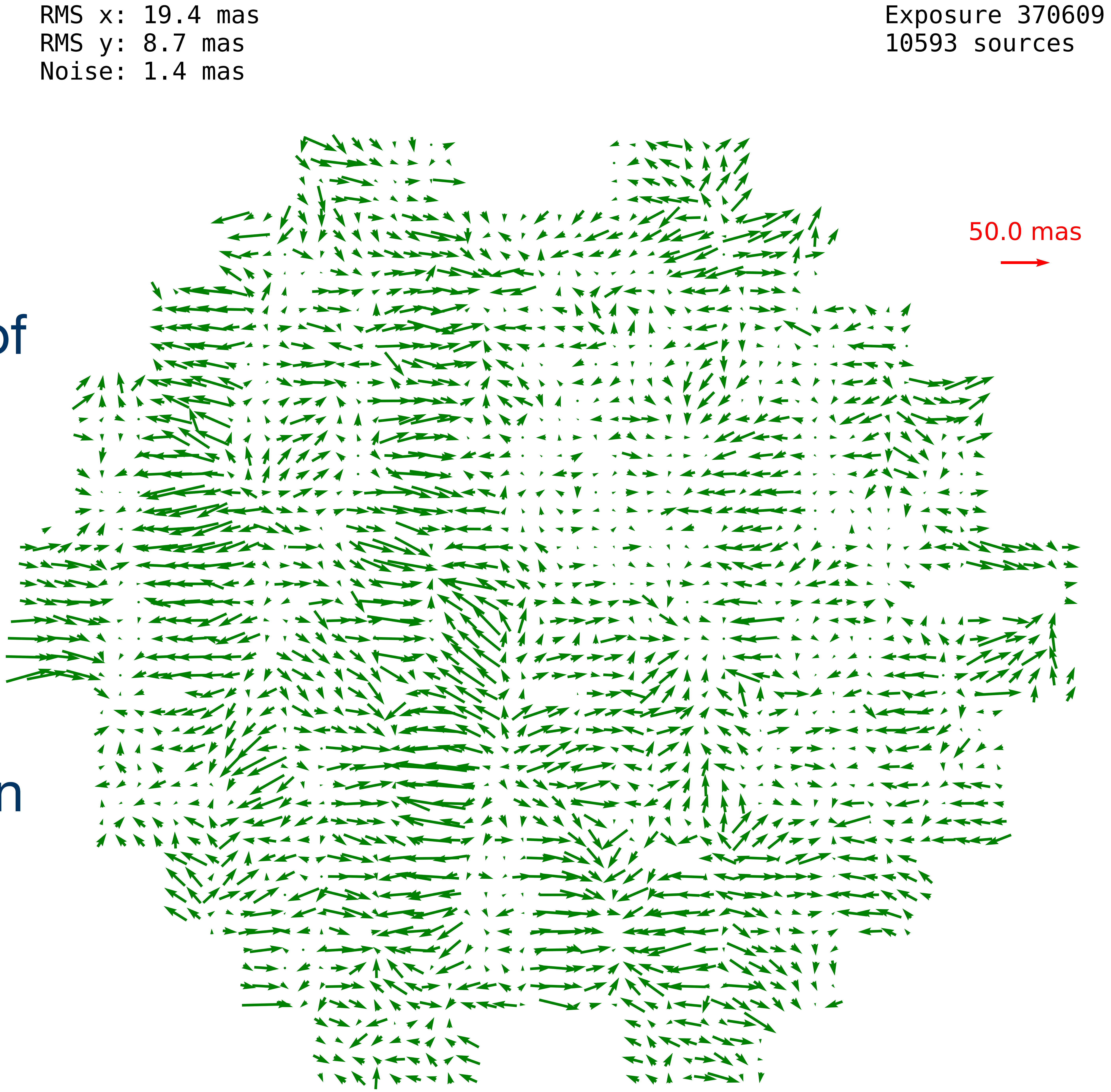
Great Paris Exhibition Telescope
(lens at the same scale)
Paris, France (1900)
Yerkes Observatory
(40" refractor lens at the same scale)
Williams Bay, Wisconsin (1893)
Hooker (100")
Mt Wilson, California (1917)
Hale (200")
Mt Palomar, California (1948)
(1979-1998) (1999-)
Multi Mirror Telescope
Mount Hopkins, Arizona
BTA-6 (Large Altazimuth Telescope)
Zelenchuksky, Russia (1975)
Large Zenith Telescope
British Columbia, Canada (2003)
Gaia
Earth-Sun L2 point (2014)
Kepler
Earth-trailing solar orbit (2009)
James Webb Space Telescope
Earth-Sun L2 point (planned 2018)
Hubble Space Telescope
Low Earth Orbit (1990)
Large Sky Area Multi-Object Fiber Spectroscopic Telescope
Hebei, China (2009)
Gran Telescopio Canarias
La Palma, Canary Islands, Spain (2007)
Keck Telescope
Mauna Kea, Hawaii (1993/1996)
Thirty Meter Telescope
Mauna Kea, Hawaii (planned 2022)
Hobby-Eberly Telescope
Davis Mountains, Texas (1996)
Southern African Large Telescope
Sutherland, South Africa (2005)
Gemini North
Mauna Kea, Hawaii (1999)
Subaru Telescope
Mauna Kea, Hawaii (1999)
Gemini South
Cerro Pachón, Chile (2000)
Large Binocular Telescope
Mount Graham, Arizona (2005)
Large Synoptic Survey Telescope
El Peñón, Chile (planned 2020)
Very Large Telescope
Cerro Paranal, Chile (1998-2000)
Magellan Telescopes
Las Campanas, Chile (2000/2002)
Giant Magellan Telescope
Las Campanas Observatory, Chile (planned 2020)
European Extremely Large Telescope
Cerro Armazones, Chile (planned 2022)
Human at the same scale
0 5 10 m
0 10 20 30 ft
Overwhelmingly Large Telescope
(cancelled)
Tennis court at the same scale
Arecibo radio telescope at the same scale
Basketball court at the same scale

The Method:

1. Registered Y6 DES to *Gaia* DR2
   - Constructed the residual field of positions

2. Trained a Gaussian Process regression model
   - Minimized correlated variance in the residual field
   - Used a custom kernel based on known physics

3. Calculated the reduction in correlated variance



Gaussian Process Regression (GPR)

Gaussian Process

- "A collection of random variables, any finite number of which has a joint gaussian distribution."

- Can be thought of as a "distribution over functions."

$$f(x) \sim \mathcal{GP}(m(x), k(x, x'; \Theta))$$

$$m(x) = \mathbb{E}[f(x)]$$

$$k(x, x'; \Theta) = \mathbb{E}[(f(x) - m(x))(f(x') - m(x'))]$$

The GPR Model

Model Inputs, X

- DES Astrometric Positions
- $x_i^{DES} = (\alpha_i, \delta_i)$

Model Targets, y

- DES – Gaia Residuals
- $y_i = x_i^{DES} - x_i^{Gaia}$

Kernel

- $k(x, x'; \Theta)$

Joint distribution of the observed target values, y, and the function values, f_*, at the test locations, X_*.

$$\begin{bmatrix} y \\ f_* \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} K(X,X) & K(X_*,X) \\ K(X_*,X) & K(X_*,X_*) \end{bmatrix}\right)$$

$K(X_*, X)$ is an $n_* \times n$ matrix of covariances evaluated at all pairs of training and test points.

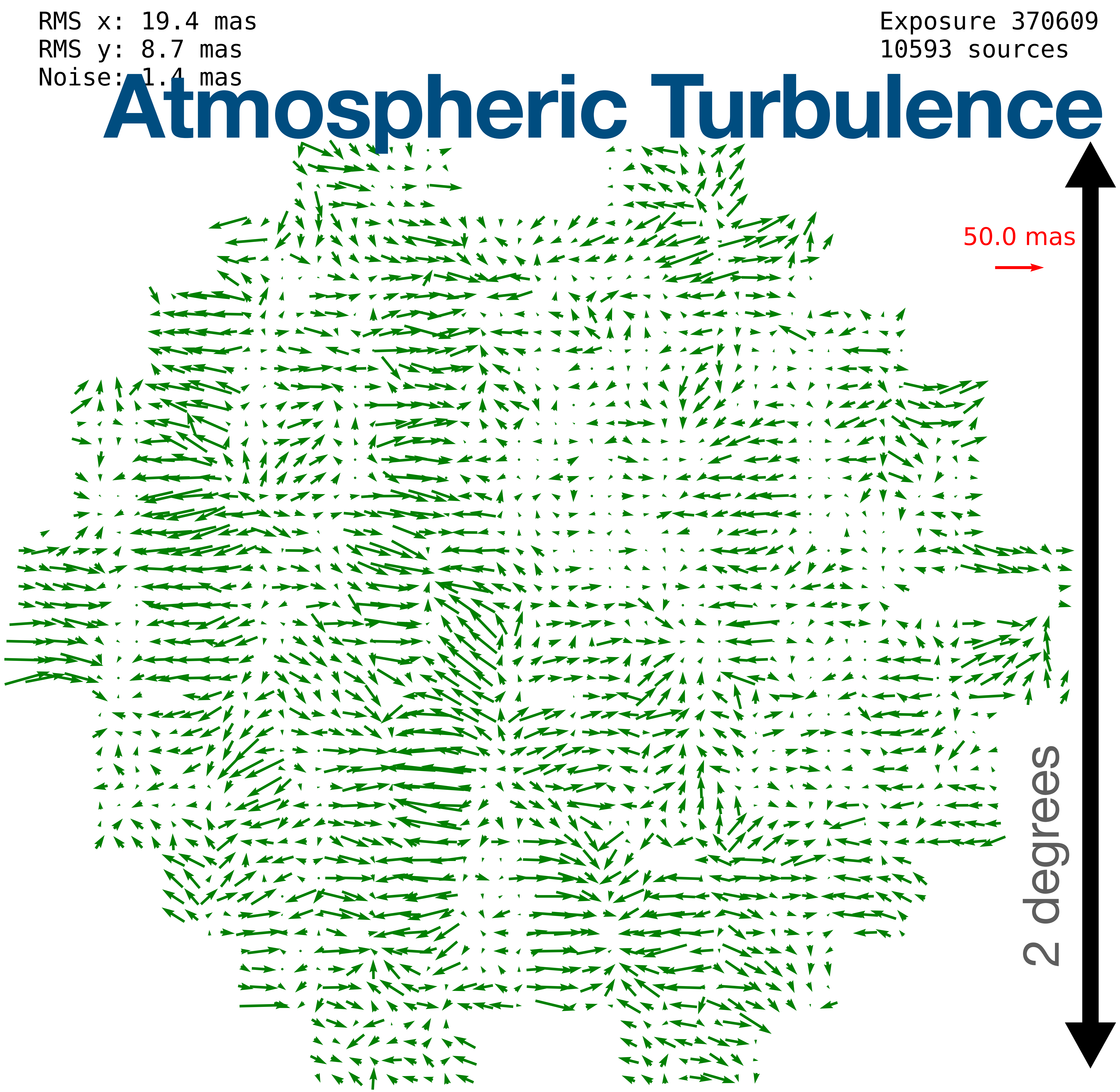
Posterior Predictive Mean

$$\bar{f}_* = K(X, X_*)^T K(X, X)^{-1} y$$

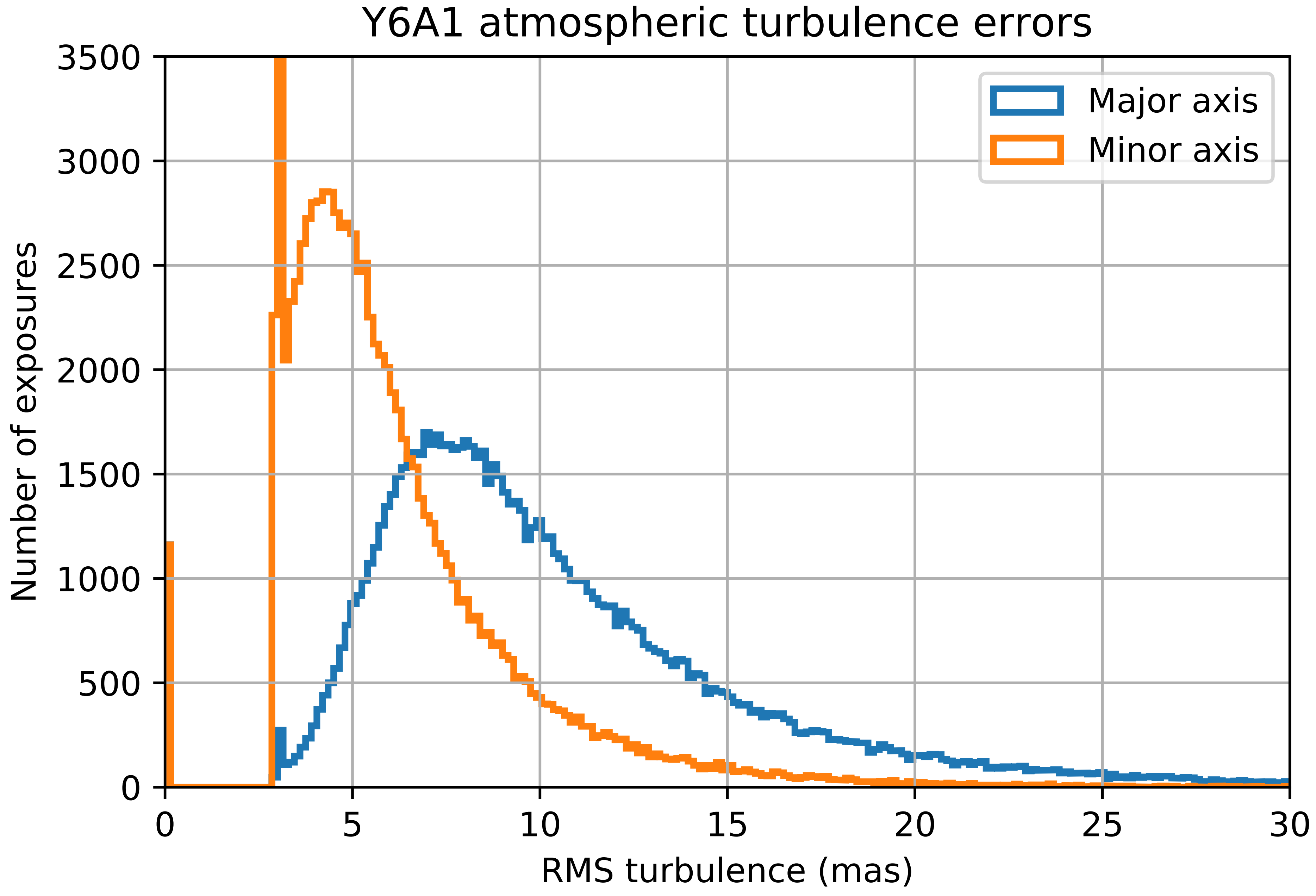
Noisy Posterior Predictive Mean

$$\bar{f}_* = K(X, X_*)^T (K(X, X) + W_{DES} + W_{Gaia})^{-1} y$$

Atmospheric Turbulence



- Coherence length of ~10 arcmin
- Amplitude and patterns change unpredictably
- Clearly anisotropic in both amplitude and coherence length



Gaia DR2 Astrometry

- < 1 mas RMS error for G < 20 mag
- ~ 1 Star per arcmin^2

Curl-Free = gradient of some field

Exposure 228645, *z*-band
30 seconds

ϕ
wind
$y = \nabla\phi$

A model for astrometric distortions

Caused by fluctuations in the index of refraction integrated along line of sight, $\phi(\mathbf{x})$

- $\mathbf{y}(\mathbf{x}) = \nabla \phi(\mathbf{x})$
- ϕ is well approximated as a Gaussian random field, with power spectrum $P_\phi(\mathbf{k})$

→ *Gaussian-process interpolation will be optimal if we choose the right power spectrum or correlation function for the field.*

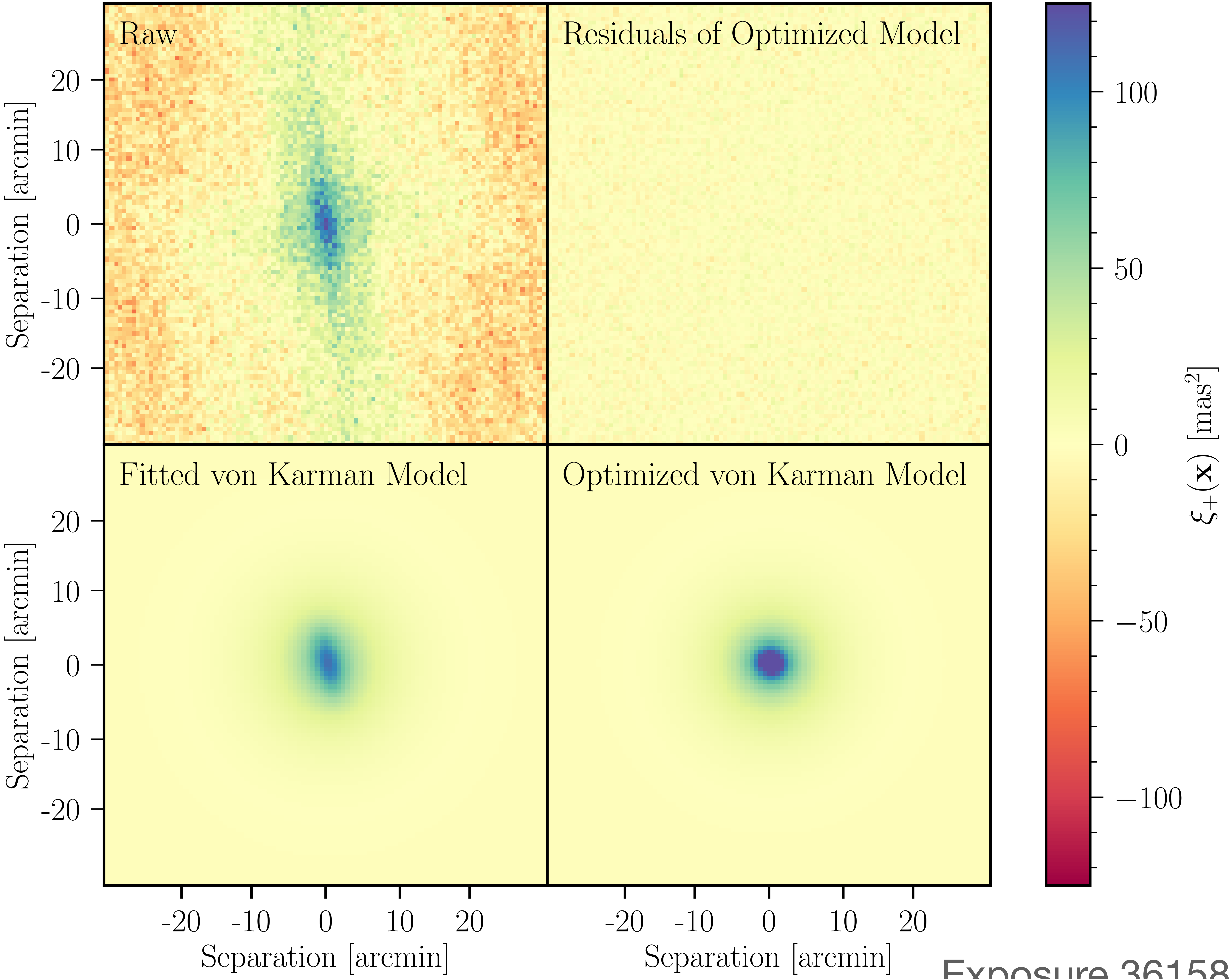
IF the turbulence is confined to a single layer in the atmosphere, then we expect a power spectrum of index fluctuations following this model:

Parameters

- Outer Scale, $\theta = 2\pi/k_0$
- Diameter, D
- Wind Vector, $\mathbf{w}$
- Total variance, σ^2

$$P_\phi(k) \propto \underbrace{\left(k^2 + k_0^2\right)^{-\frac{11}{6}}}_{\text{von Karman turbulence}} \underbrace{\left(\frac{J_1(kD/2)}{kD/2}\right)^2}_{\text{Integration over telescope aperture}} \underbrace{\text{sinc}^2\left(\frac{\mathbf{k}\cdot\mathbf{w}}{2}\right)}_{\text{Time integration of wind motion}}$$

$$\xi(r) \equiv \langle y(x)y(x+r) \rangle$$



Exposure 361582, *i*-band

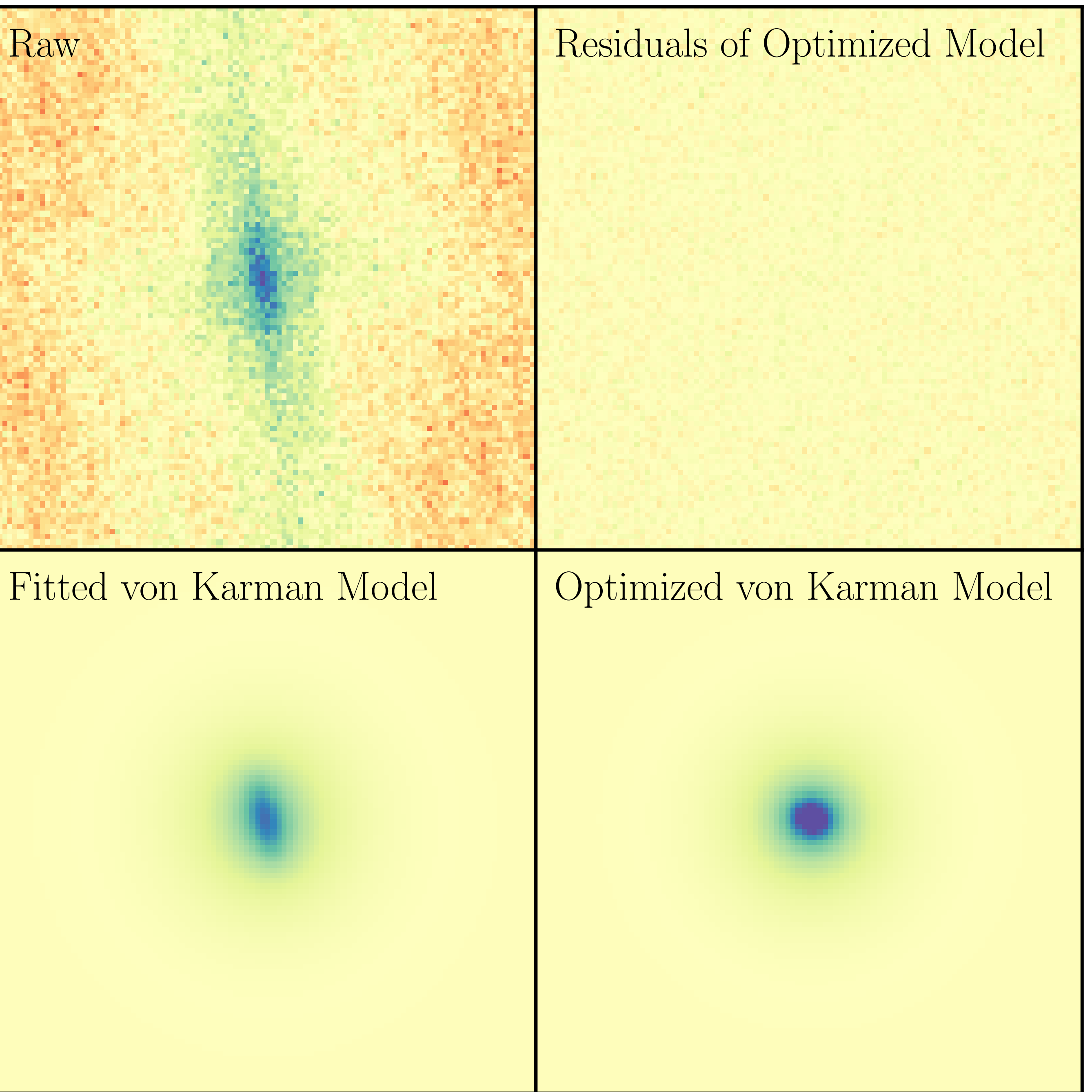
Non-standard GPR aspects

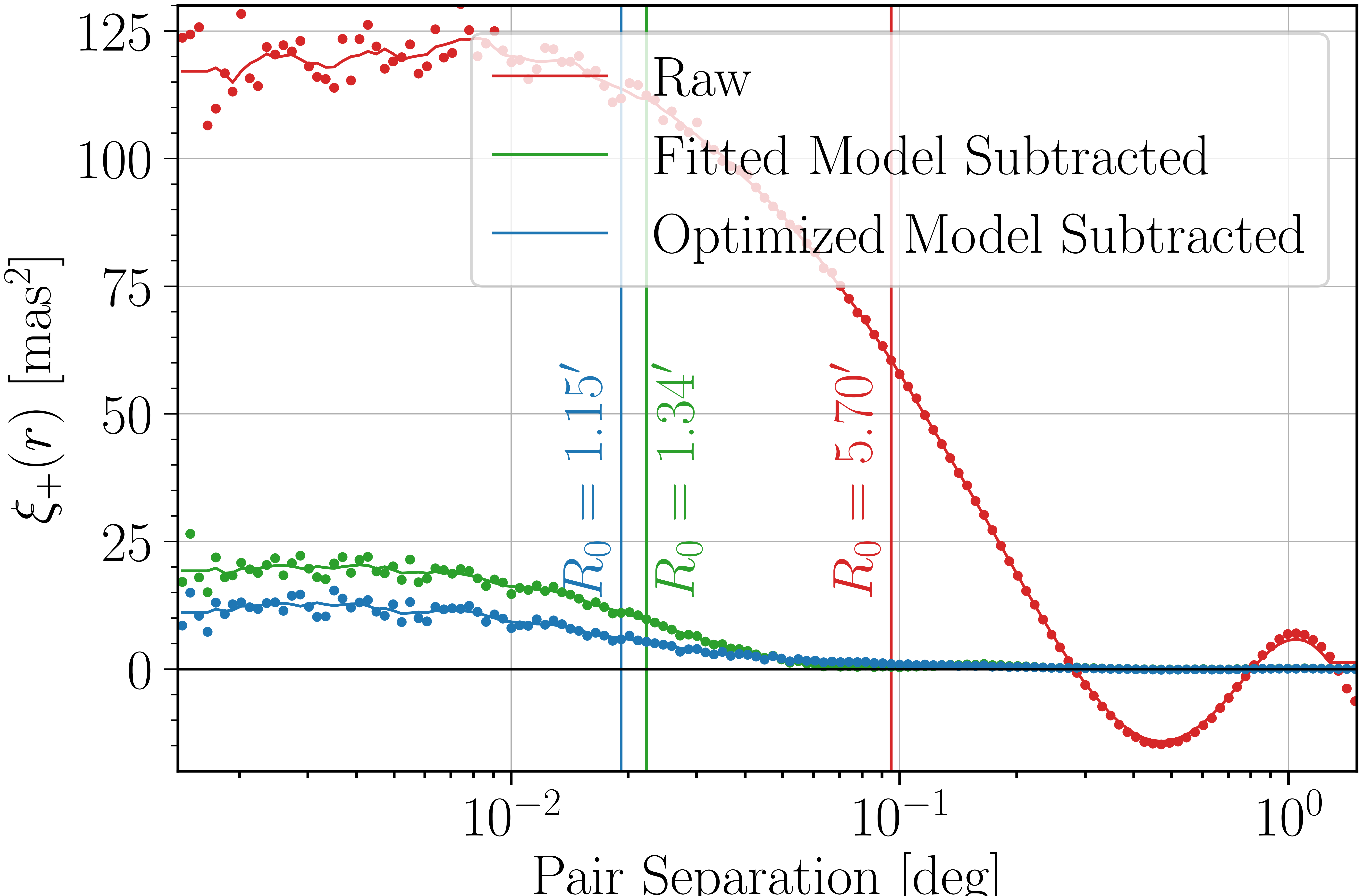
- Anisotropic kernel - not hard to include in the linear algebra
- Dimensions of **y** are not independent - expect a curl-free field.
  - x and y data can inform each other.
  - We derived a variant of the GPR formulae which enforces/exploits this, by forming a single (2Nx2N) covariance matrix for N points.
  - Also allows for anisotropic measurement errors (as in Gaia)

Procedure

- Acquire DES and Gaia information for bright stars of every exposure, calculate **y** for these & clip outliers.
- Calculate $\xi(\mathbf{x})$ of **y**
- Fit von Kármán model to ξ_{raw}
  - Execute GP, clip outliers in residuals
- Run an optimizer to minimize output $\xi_{\mathrm{resid}}(x \lesssim 0.5')$ of validation set over kernel parameters
  - This is *slow* since it requires 50-100 evaluations of GP

Results - avg of ~300 exposures spread over years

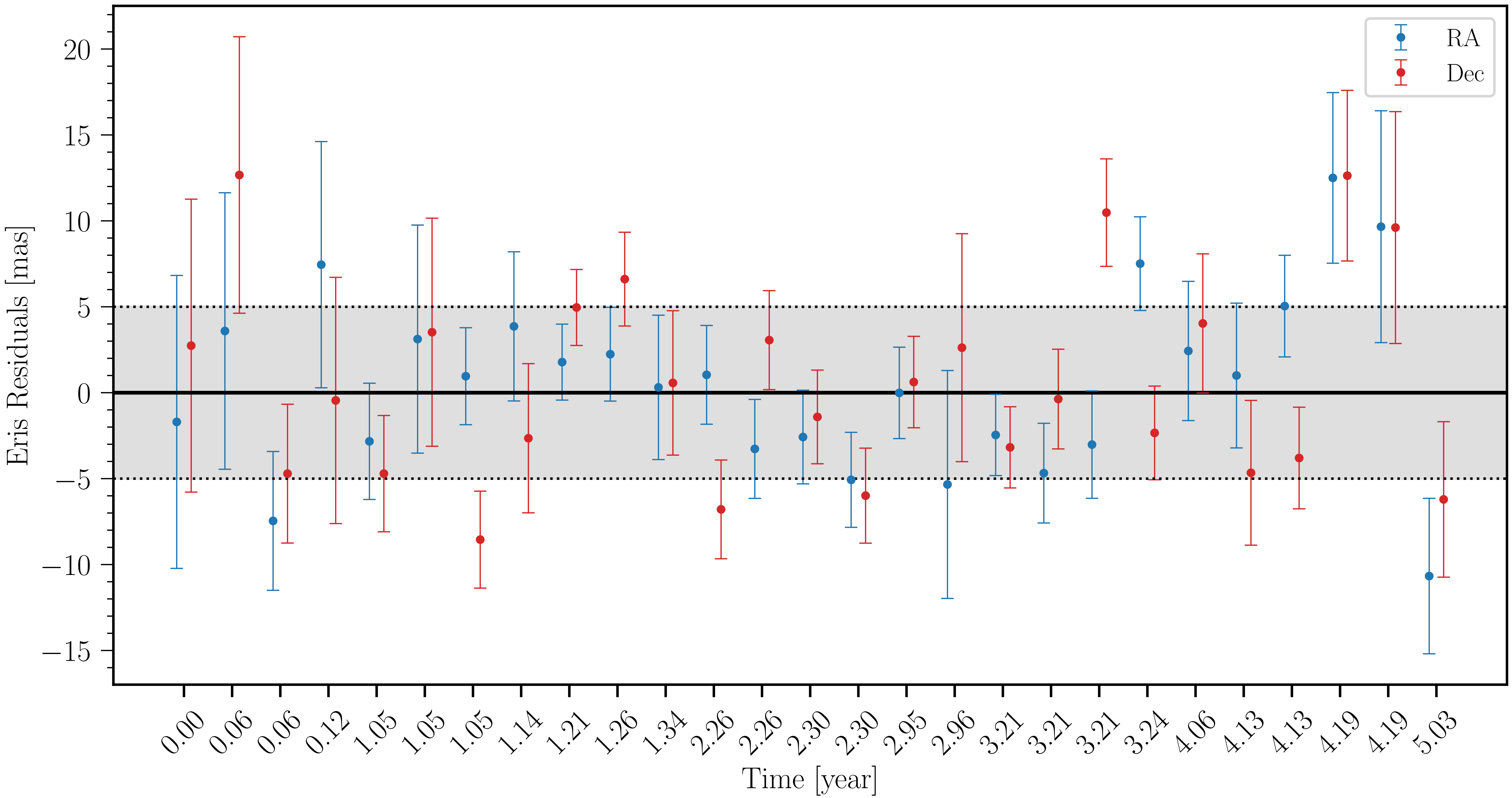




Mean ξ_0 Reduction
r band: 11.689 ± 1.675
i band: 12.446 ± 1.772
z band: 11.956 ± 1.604
Y band: 14.329 ± 2.264
No Change
5x Reduction
10x Reduction
20x Reduction
ξ_0 GPR Model Subtracted [mas^2]
ξ_0 Raw [mas^2]
0
10
20
30
40
50
60
70
80
0
100
200
300
400
500
600
700

No Change
2x Reduction
5x Reduction
r band
i band
z band
Y band
RMS Turbulence GPR Model Subtracted [mas]
RMS Turbulence Raw [mas]
Star Density [arcmin^{-2}]
3
4
5
6
7
8
9
10
20
0.6
0.7
0.8
0.9
1.0
2
3
4
5
6
7
8
0.2
0.4
0.6
0.8
1.0
1.2

A stringent test: calculate residuals to a Trans-Neptunian Object orbit fit as it moves a few degrees across the field over 5 years (Note it’s not so bright that shot noise is negligible)



Key Takeaways

- **Turbulence induced variance:**
  - 7 mas RMS $\rightarrow$ 2 mas RMS
- **Correlation length:**
  - 5.7 arcmin $\rightarrow$ 1.2 arcmin
- **Orbit Fitting — Eris:**
  - 10 mas RMS $\rightarrow$ 5 mas RMS
- **Room for Improvement:**
  - Y6 DES $\rightarrow$ LSST
  - Simultaneous solution for turbulence/proper-motion
  - von Kármán Kernel is not necessarily optimal


Thank you for listening!