Investigating Nonlinear and Stochastic Variability of Accreting Compact Objects via Recurrence Analysis

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University of Washington

Collaborators: Padi Boyd, Alan Smale, Brian Powell (NASA Goddard); Michael Vogeley, Gordon Richards (Drexel); Eric Bellm (UW); ZTF Collaboration

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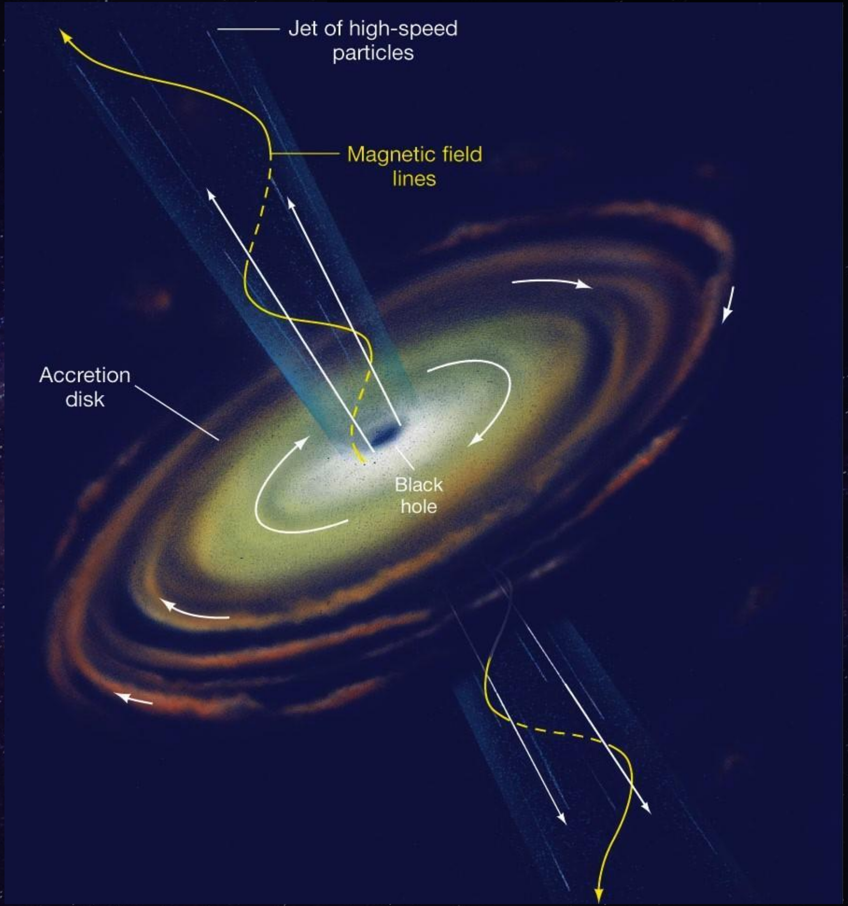
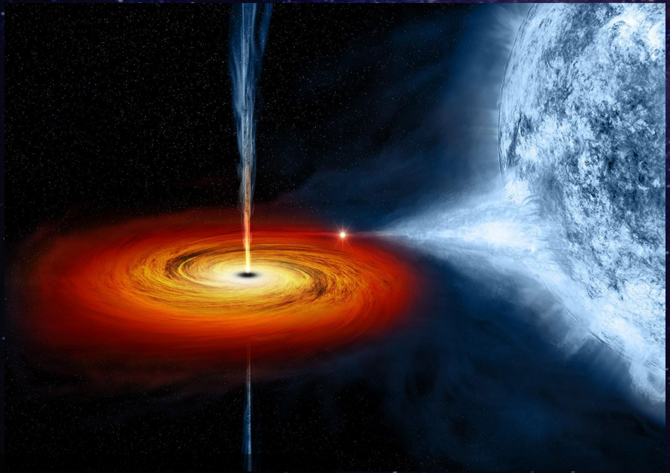
NSF Grant: AST-1812779 (UW)
Advisor/PI: Dr. Eric Bellm (UW)

Outline

- Motivation:
  - Long-term monitoring of X-ray Binaries and Active Galaxies
  - Traditional time series analysis
- Methods:
  - Phase Space and Topology
    - Example: 4U 1705-44
  - Recurrence Plots
  - Quantitative recurrence analysis
- Applications:
  - Distinguishing between stochastic and deterministic behavior
  - Identifying chaos
  - Outstanding challenges

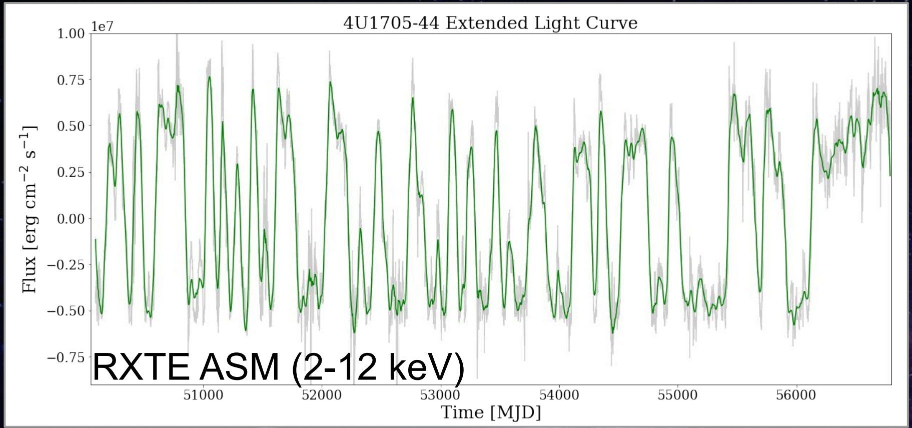
Active Galactic Nuclei (AGN)

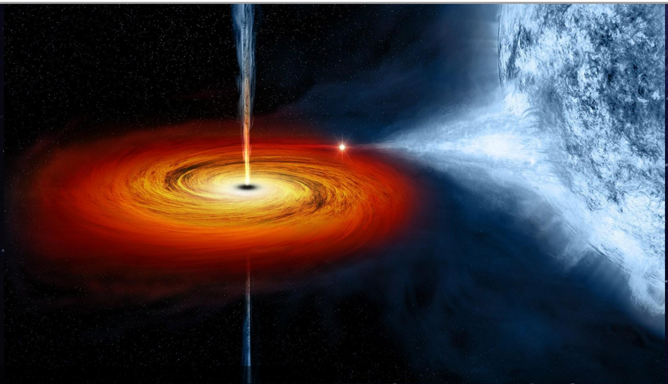




X-ray Binary (XRB)

Long-term variability provides a window into the dynamics of accretion





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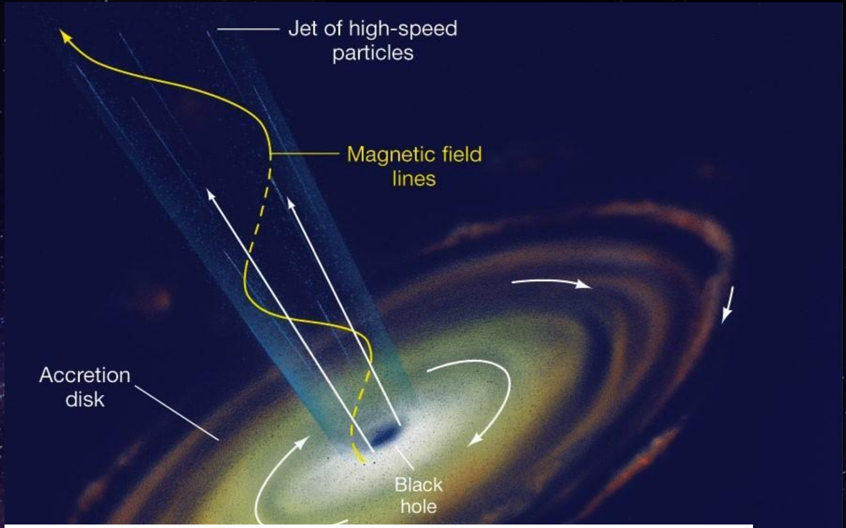

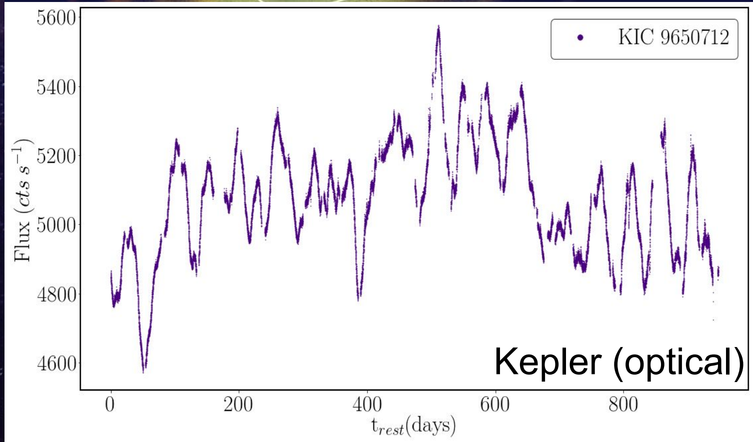

Time Series Analysis Guilds

- Domain based Methods
  - Frequency-Domain Methods
  - Time-Domain Methods

Examples:

- *Spectral Analysis*
- *Auto-correlation*

Time Series Analysis Guilds

Domain based Methods

Frequency-Domain Methods

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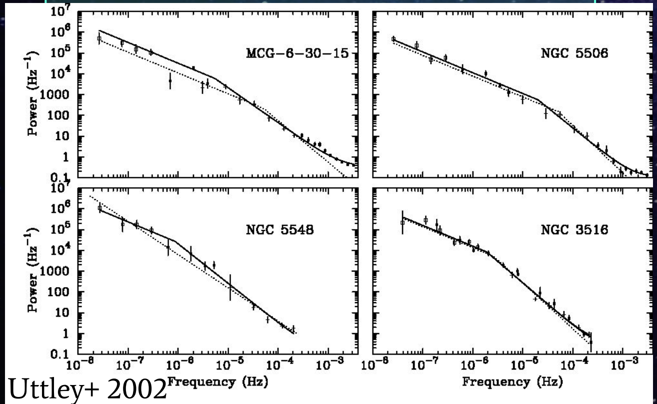

Uttley+ 2002

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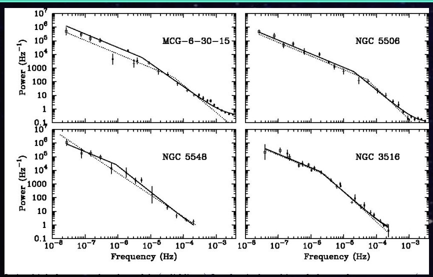

Statistical methods

- Parametric
- Non-parametric

Examples:

- *Moving Averages*
- *Kernel Regression*

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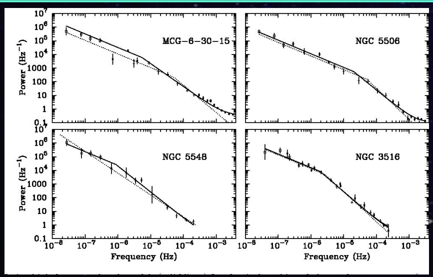

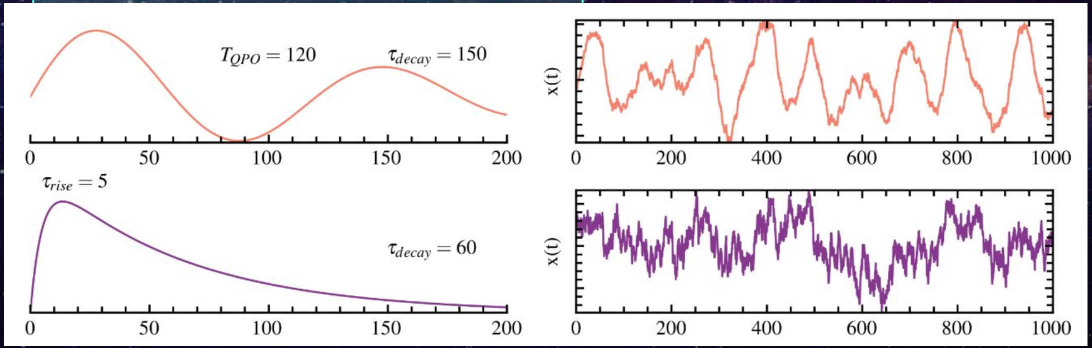

Moreno+ 2019

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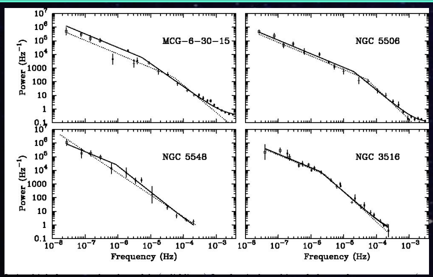

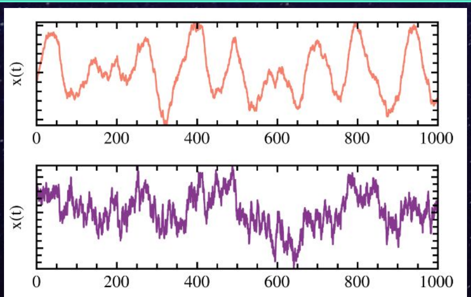

Goal:

- Connect power spectrum and statistical features to intrinsic physical properties (black hole mass, spin, etc)

Challenges:

- Assumptions of stationarity, linearity; inconsistencies across bandwidth; influence of noise

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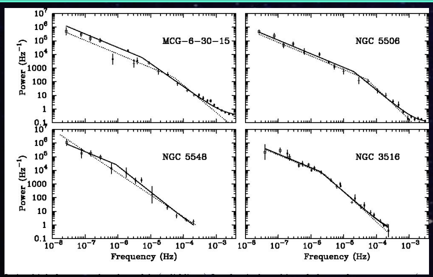
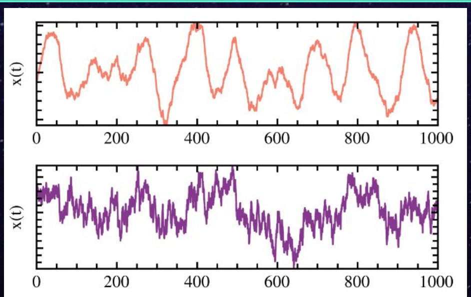
Phase-Space based methods

- Descriptive Diagrams
- Attractor Invariants

Examples:

- *Poincaré Plots (return maps)*
- *Recurrence Plots*





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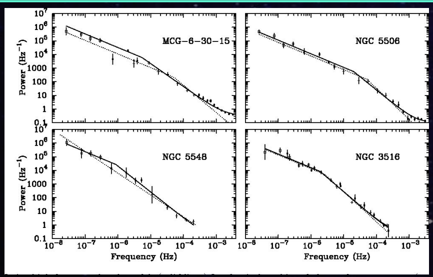

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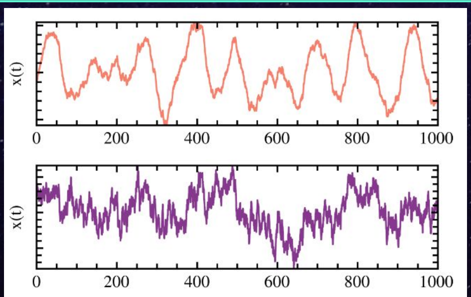

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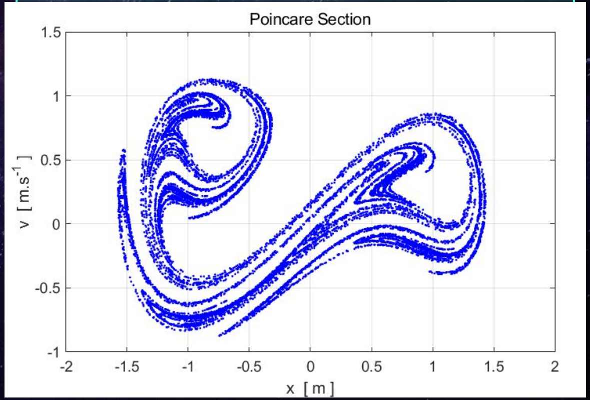

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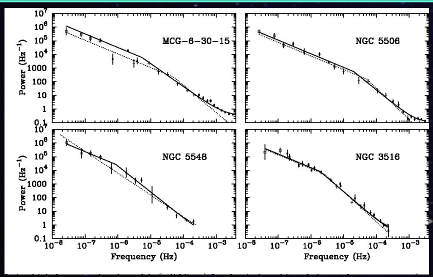
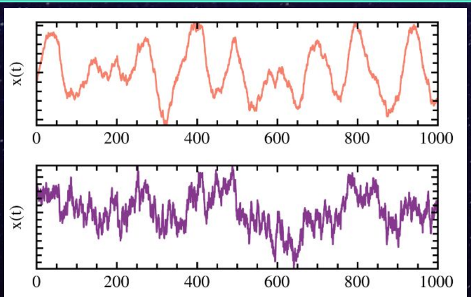
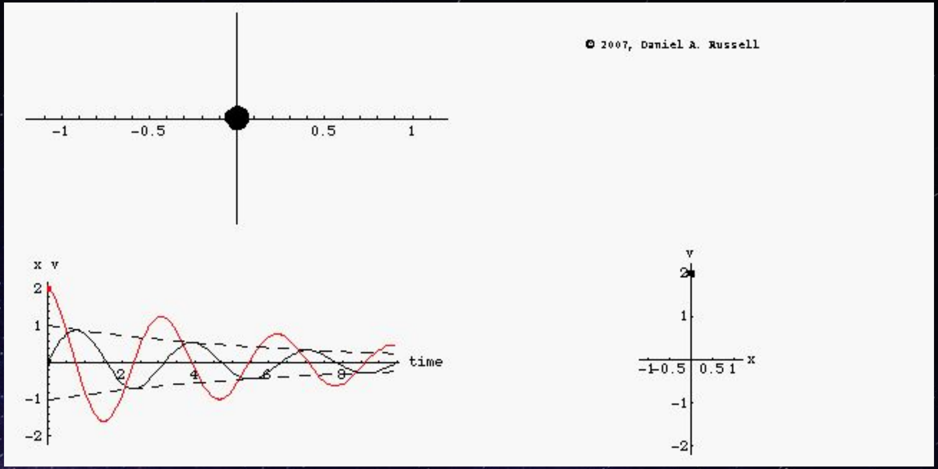




<u>Advantages:</u>
Analysis for nonlinear & linear systems; more direct probe of dynamics; can be applied to nonstationary, stochastic & deterministic time series

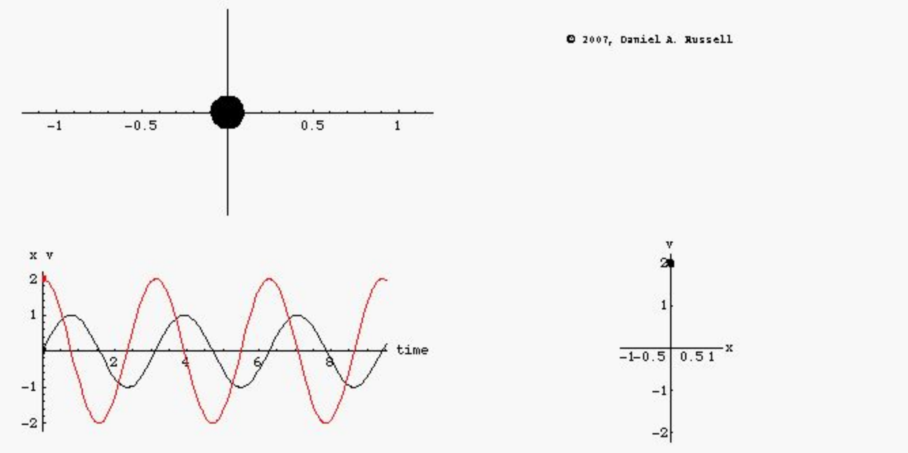
Phase Space

Classically: position versus velocity (or coordinate vs. first derivative)



Simple harmonic oscillator

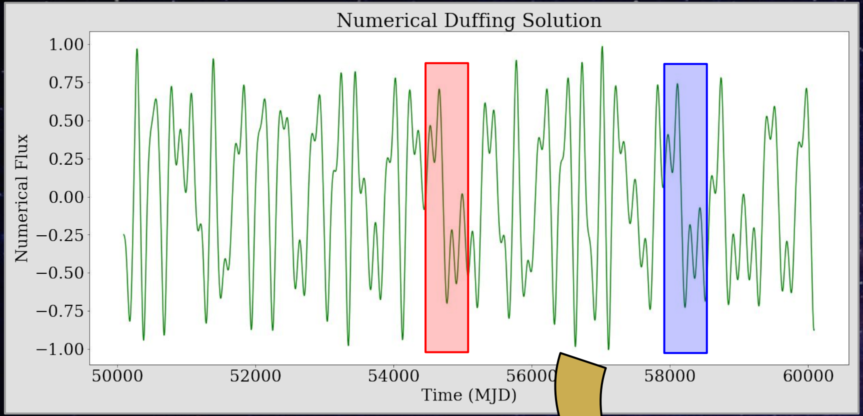
Damped harmonic oscillator

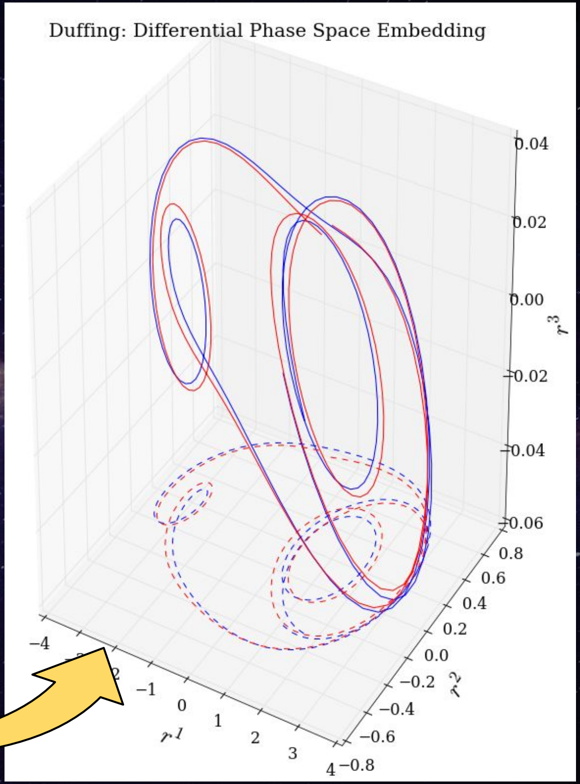


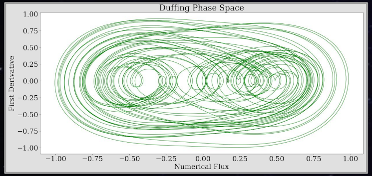
Phase Space

Damped & Driven Oscillator (Duffing equation): $\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = \gamma\cos(\omega t)$

Phase Space encodes dynamical information







Phase Space encodes dynamical information

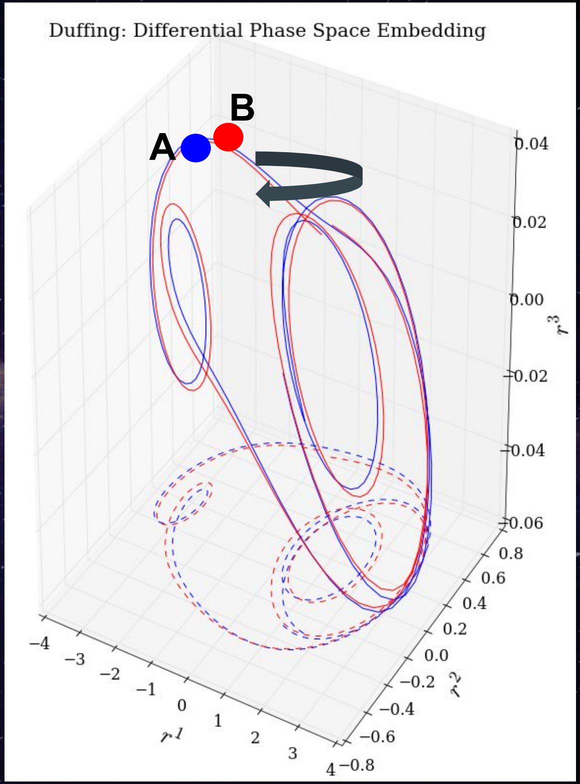
Relative Rotation Rates: How two trajectories (A and B) in phase space ‘wind’ around each other:

$$R_{ij}(A,B) = \frac{1}{2\pi\, p_A p_B} \oint \frac{\boldsymbol{n}\cdot(\boldsymbol{\Delta r}\times d\boldsymbol{\Delta r})}{\boldsymbol{\Delta r}\cdot\boldsymbol{\Delta r}}$$

$$\text{where } \boldsymbol{\Delta r} = [x_B(t)-x_A(t),\, y_B(t)-y_A(t)]$$

The set of RRRs (a set of integers) are **unique** to each class of differential equations.
(Solari & Gilmore 1988)

If the set of RRRs **are the same for two systems** -- they likely are produced by the same underlying attractor.
(Birman-Williams Theorem)



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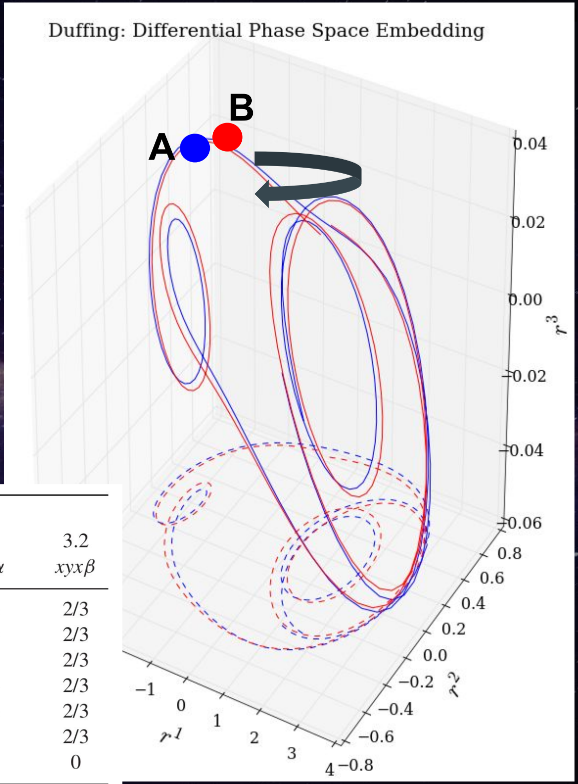

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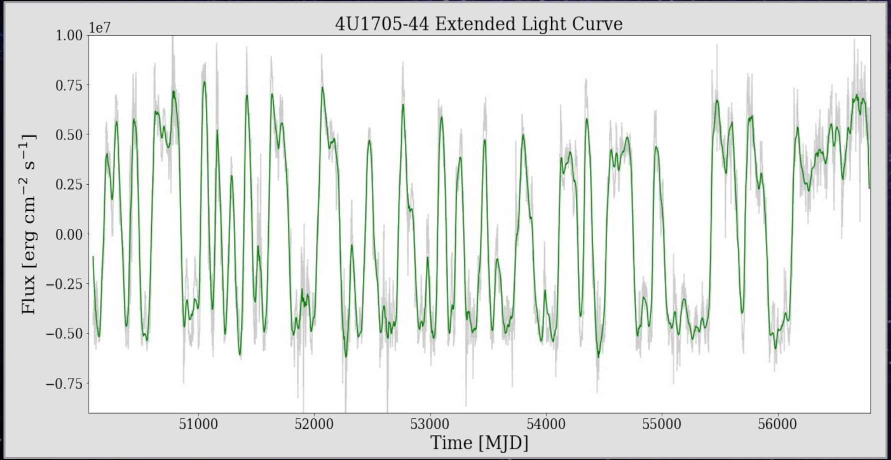
Duffing relative rotation rates

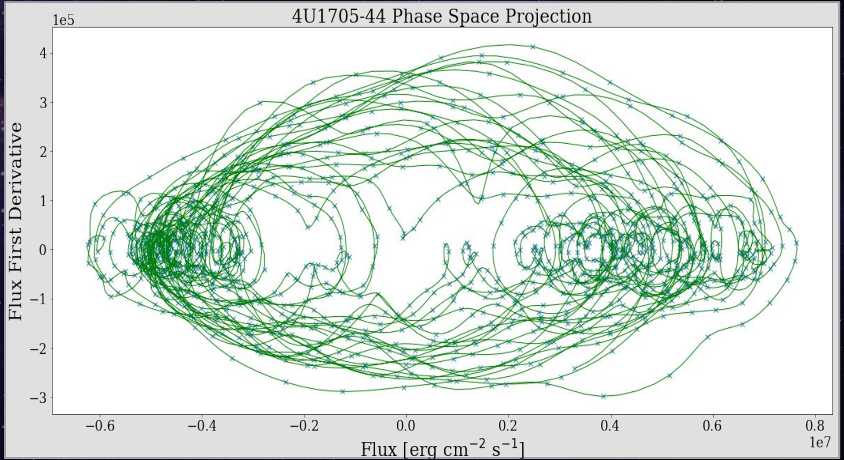
	1.1 $x\gamma$	1.2 $x\gamma$	1.5 $x\gamma$	2.1 $yy\alpha$	2.2 $yy\beta$	3.1 $xyx\alpha$	3.2 $xyx\beta$
1.1	0	1	1	1	1	2/3	2/3
1.2		0	1	1	1	2/3	2/3
1.5			0	1	1	2/3	2/3
2.1				0	1/2	2/3	2/3
2.2					0	2/3	2/3
3.1						0	2/3
3.2							0

Phase Space encodes dynamical information

4U 1705-44: a low-mass neutron star X-ray binary; **preface: has evidence for nonlinearity**

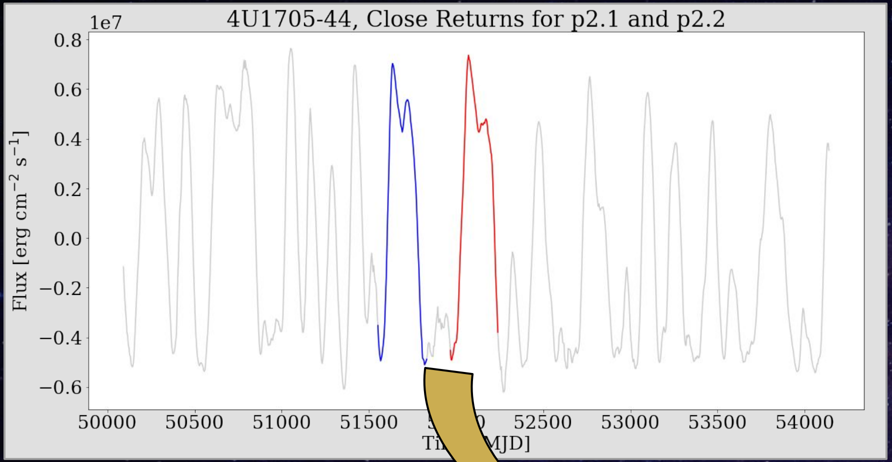
Left: light curve from RXTE All-sky monitor (2-12 keV)
Right: 2D phase from the numerical derivative of the flux

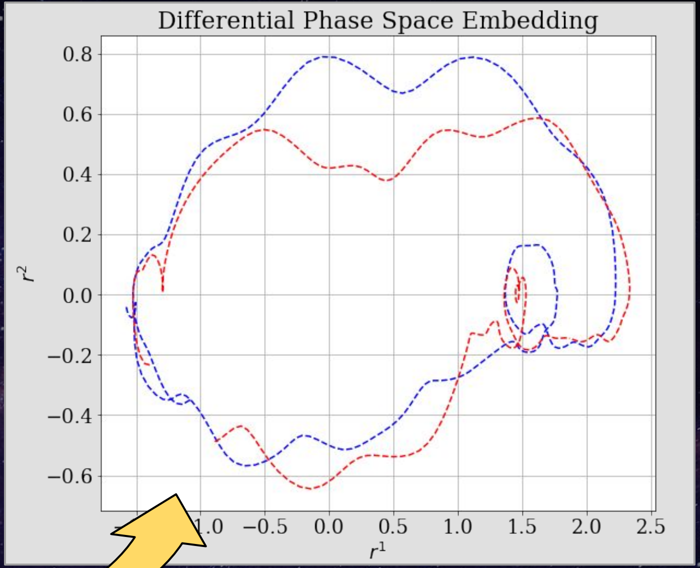


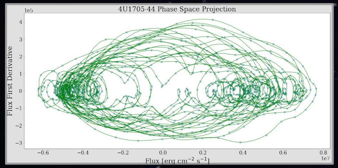


Phillipson+2018

Phase Space encodes dynamical information







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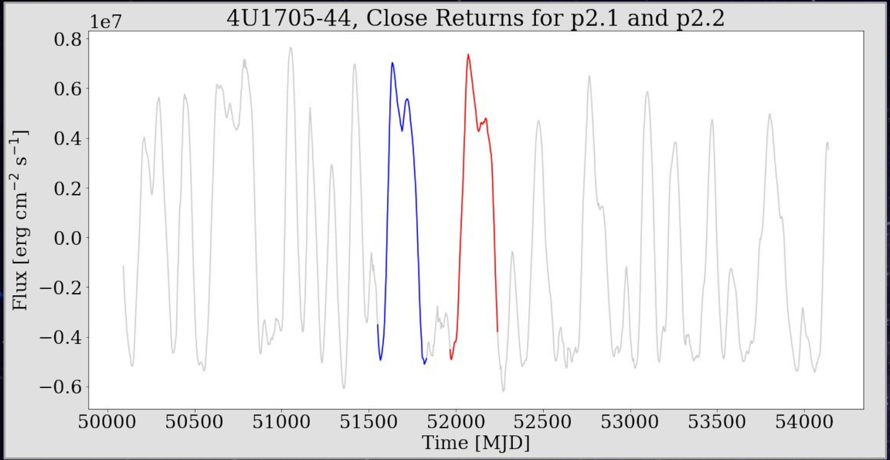

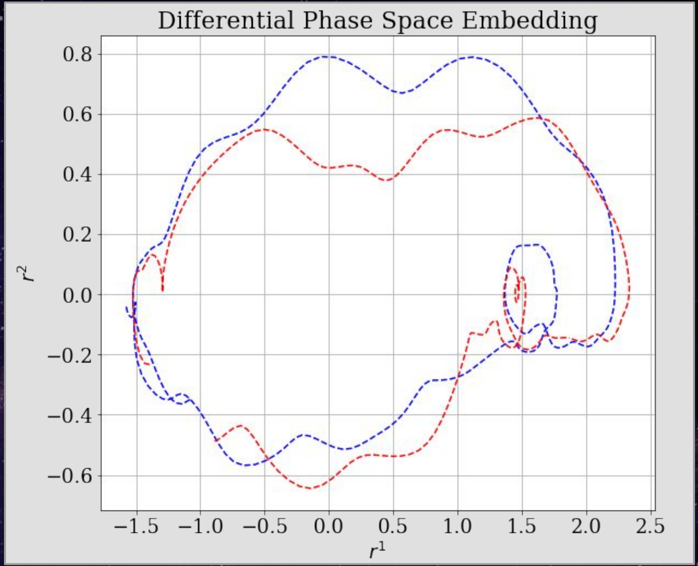

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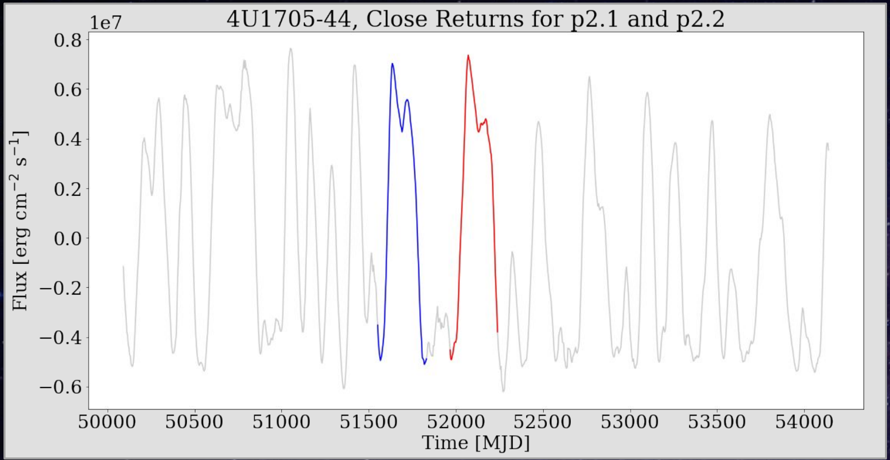

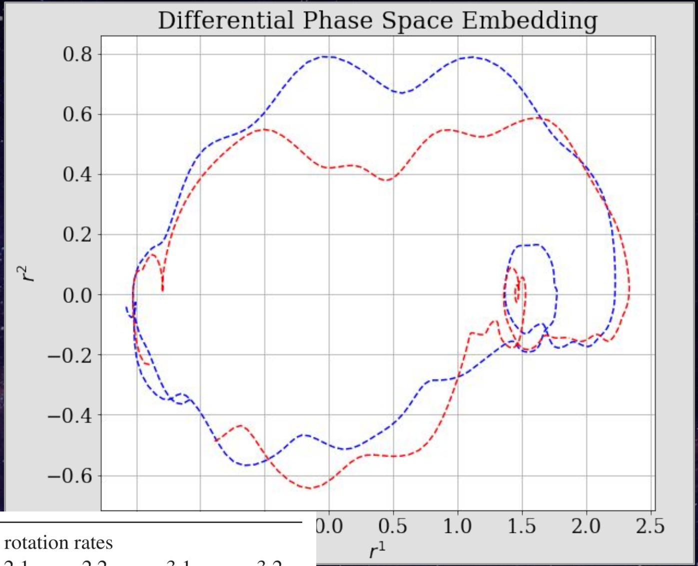

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Phillipson+2018

Phase Space encodes dynamical information

Q: How to generate phase space of unknown or stochastic systems?

Q: Are there ways to automate the extraction of information encoded in phase space?

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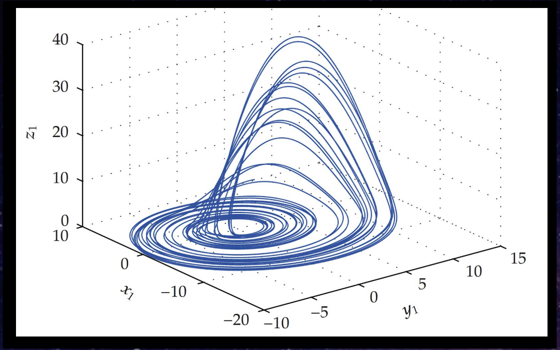
→ **The Time Delay Method**

Q: Are there ways to automate the extraction of information encoded in phase space?

→ **The Recurrence Plot**

Time Delay Embedding

Rossler Attractor
(in 3D differential
state space)



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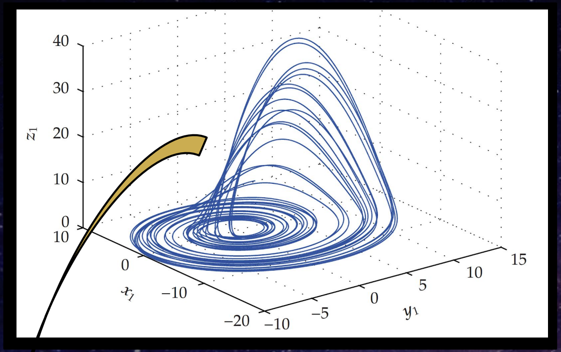

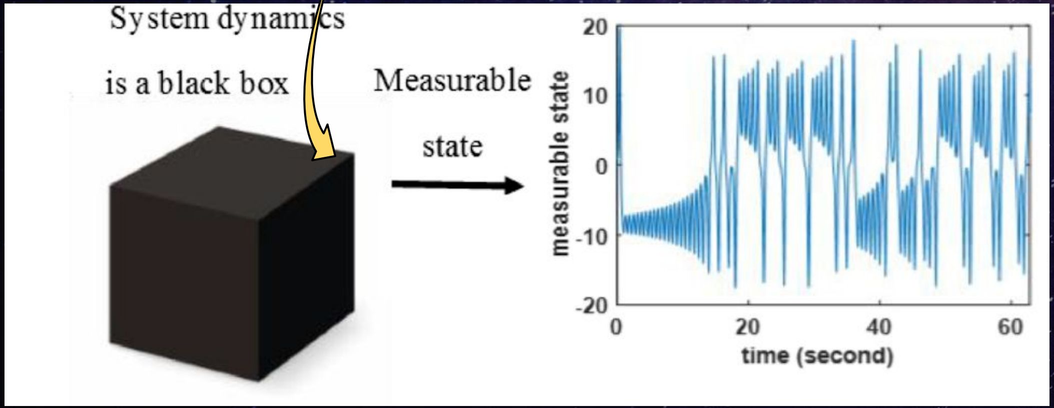

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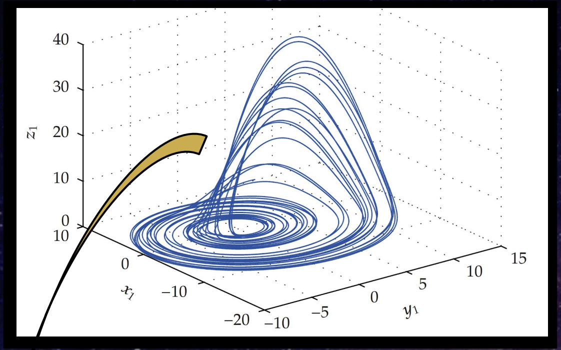
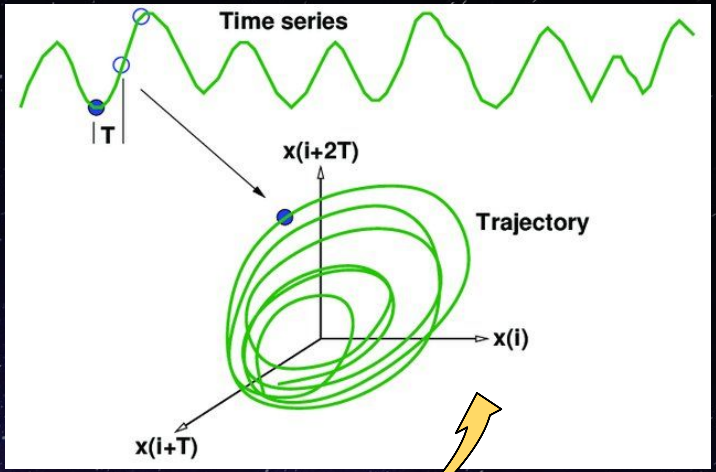
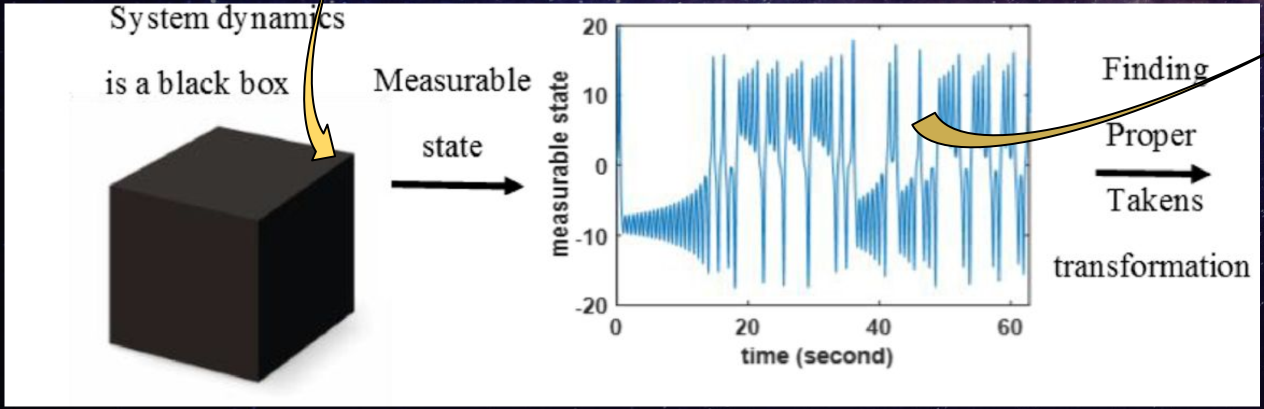




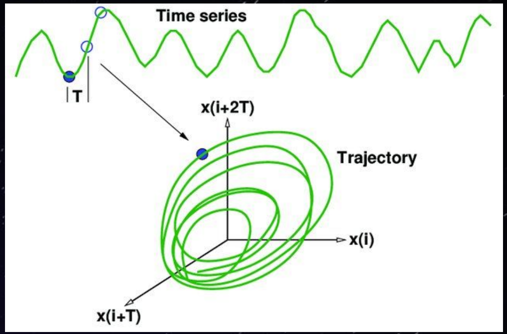


Takens 1981:

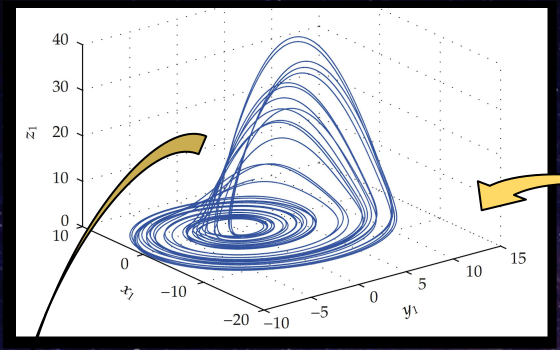
$$x(t) \rightarrow y(t) = (y_1(t), y_2(t), \ldots, y_n(t))$$
$$y_j(t) = x(t - \kappa_j), \quad j = 1, 2, \ldots, n,$$

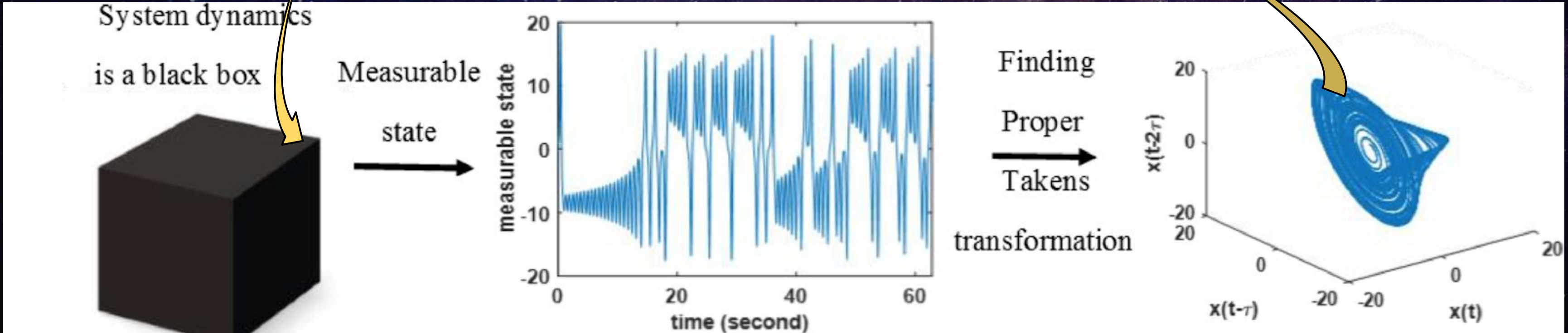
For right choice of time delay and dimension (n) recovers original attractor

Time Delay Embedding



Rossler Attractor (in 3D differential state space)





Kaveh+ 2018

The Recurrence Plot:

Given a dynamical system represented by the trajectory "x" in a d-dimensional phase space, the recurrence matrix is defined as:

$$\mathbf{R}_{i,j}(\epsilon) = \Theta(\epsilon - ||\vec{x}_i - \vec{x}_j||) \text{ for } i, j = 1, ..., N,$$

ϵ is a threshold distance $\quad \Theta(\cdot)$ is the Heaviside function

The following condition holds for two states less than the threshold distance apart:

$$\vec{x}_i \approx \vec{x}_j \Leftrightarrow \mathbf{R}_{i,j} = 1.$$

The result is a binary 2D matrix -- the positions of each entry corresponds to two points in time.

Translation:

Non-zero entries tell us when two points in time are close to each other in phase space.
The recurrence plot is the visualization of this binary matrix.

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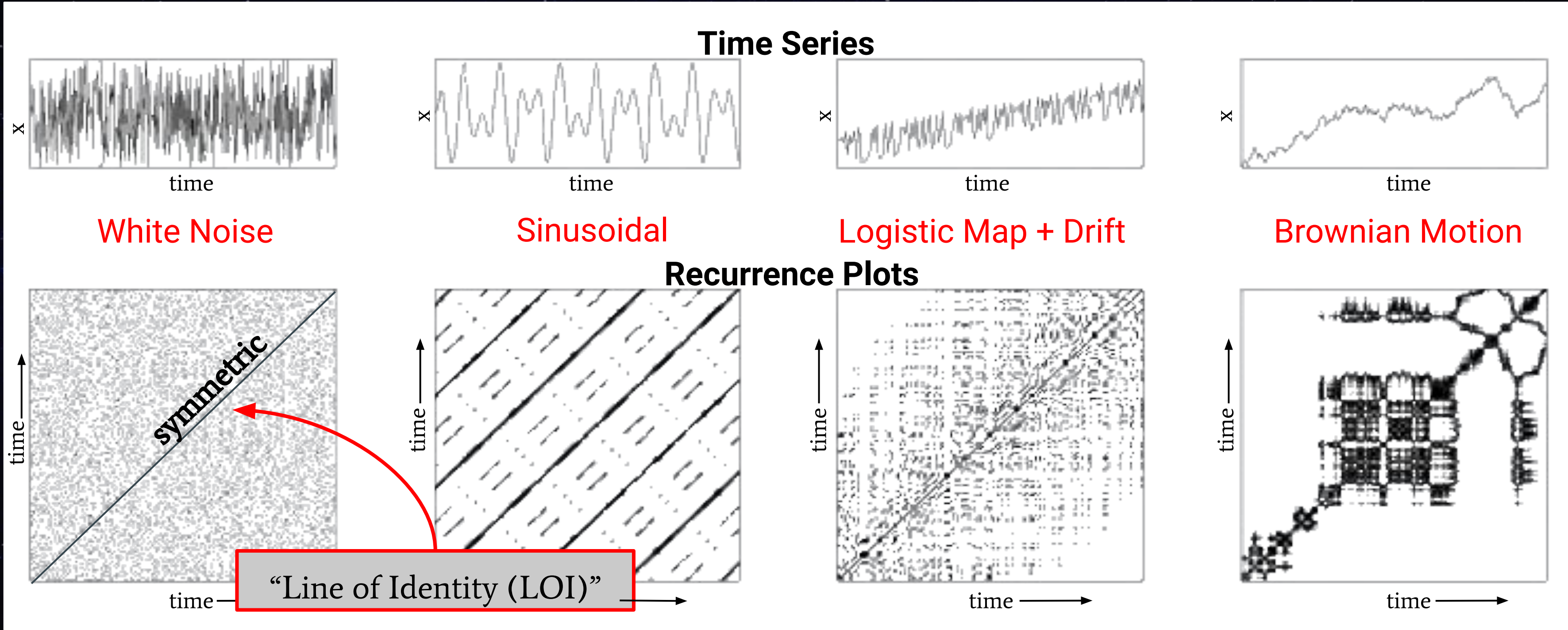

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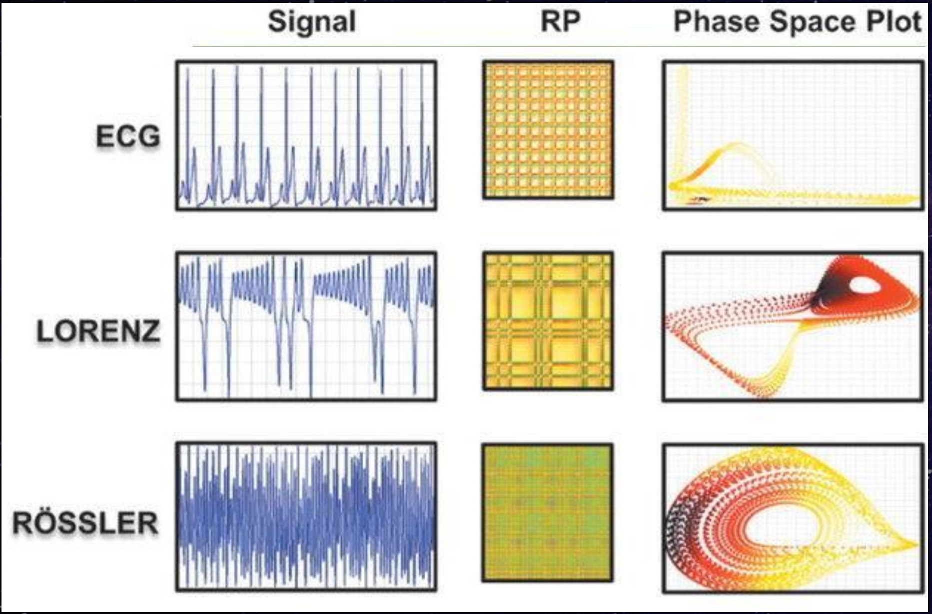
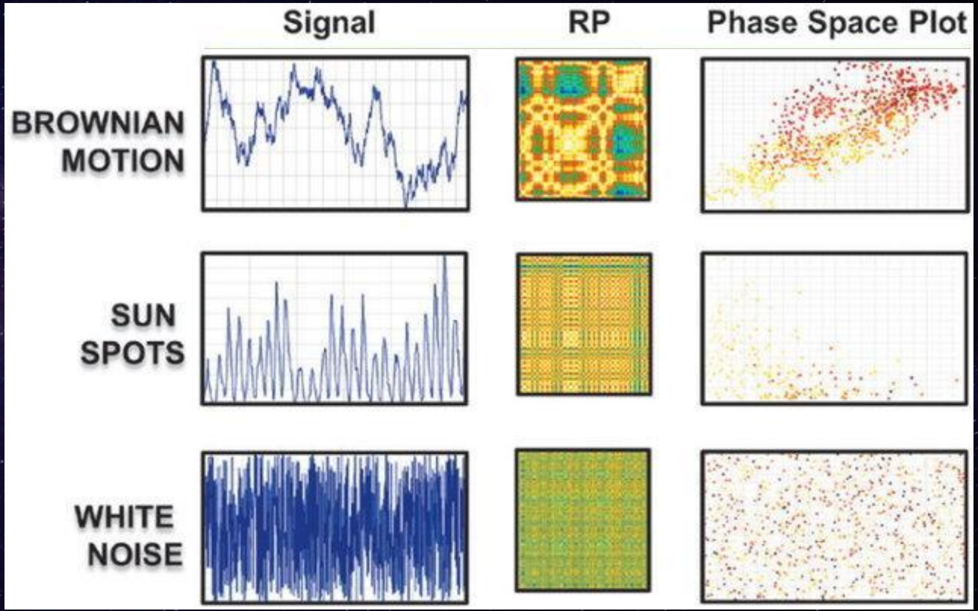
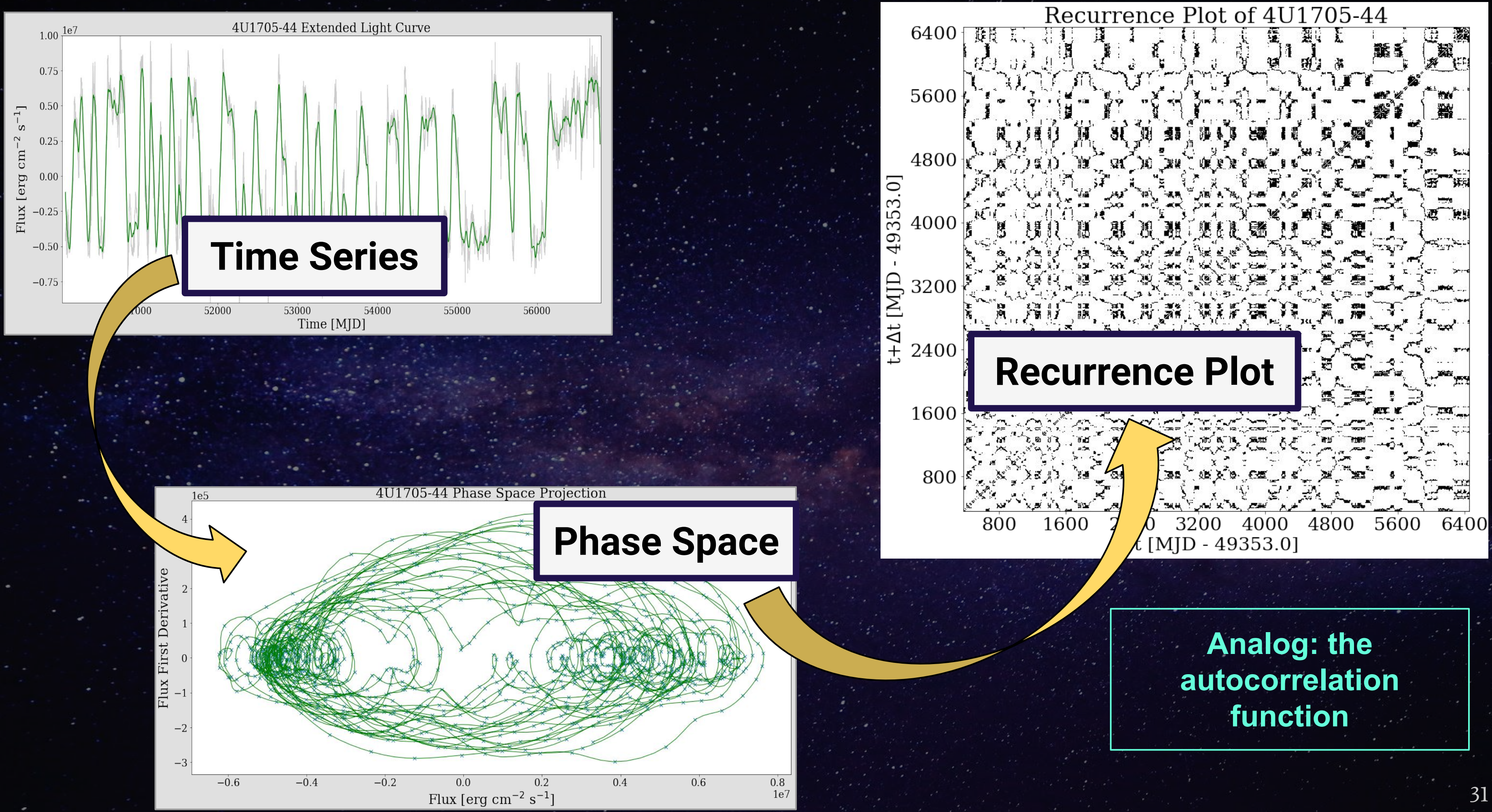




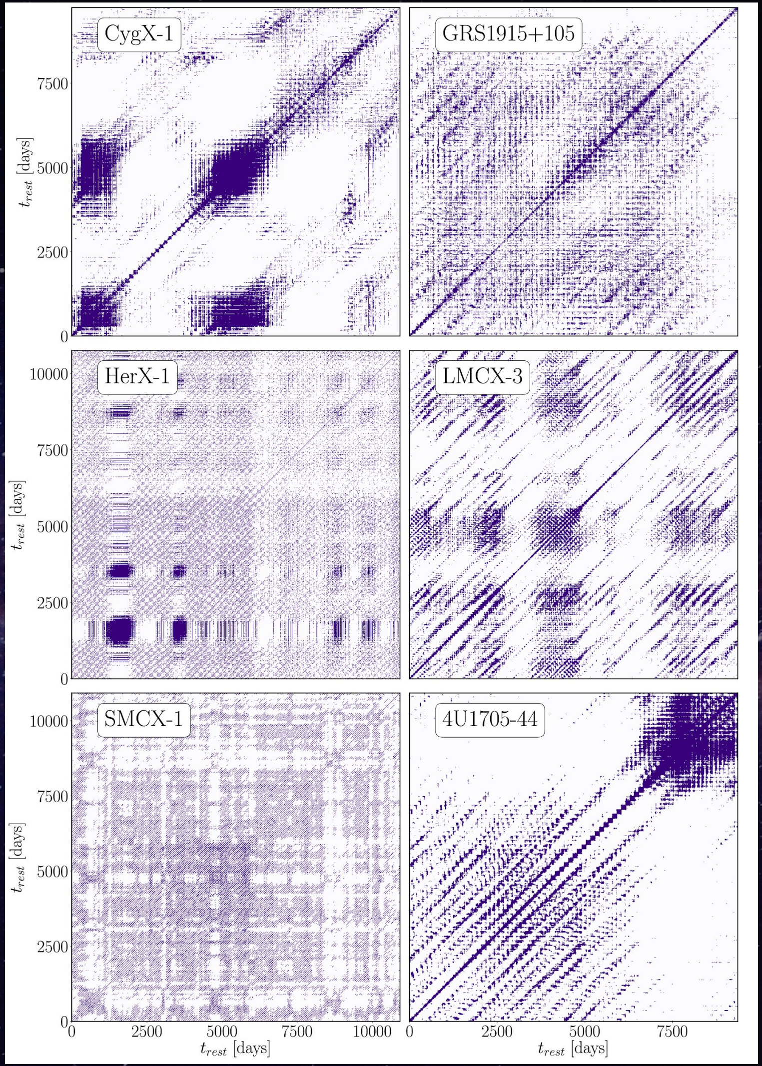
García & Romo 2013



Analog: the autocorrelation function

The Recurrence Plot:

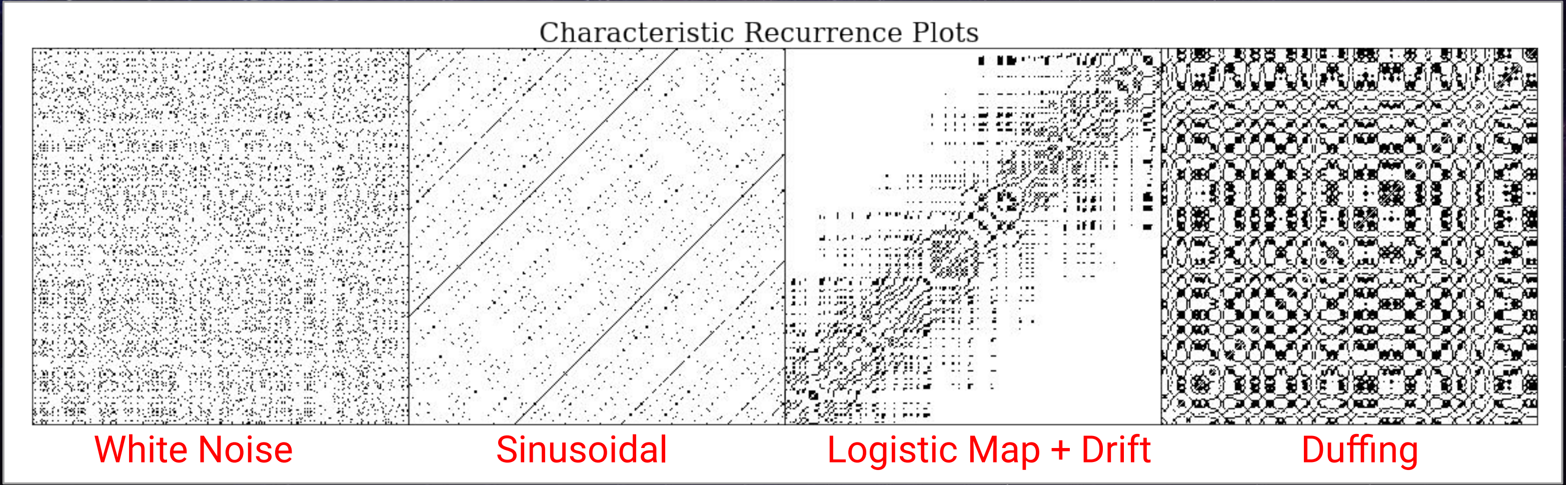
Example: X-ray Binaries!



The Recurrence Plot:

Quantify the structure in the RP:

- Recurrence Quantification Analysis (RQA)
- **Examples**: longest diagonal line, average length of diagonal or vertical lines, # lines part of a diagonal feature versus isolated points
- A total of 16 quantities
  - Diagonal features: periodicities, determinism
  - Vertical features: time invariance, state changes

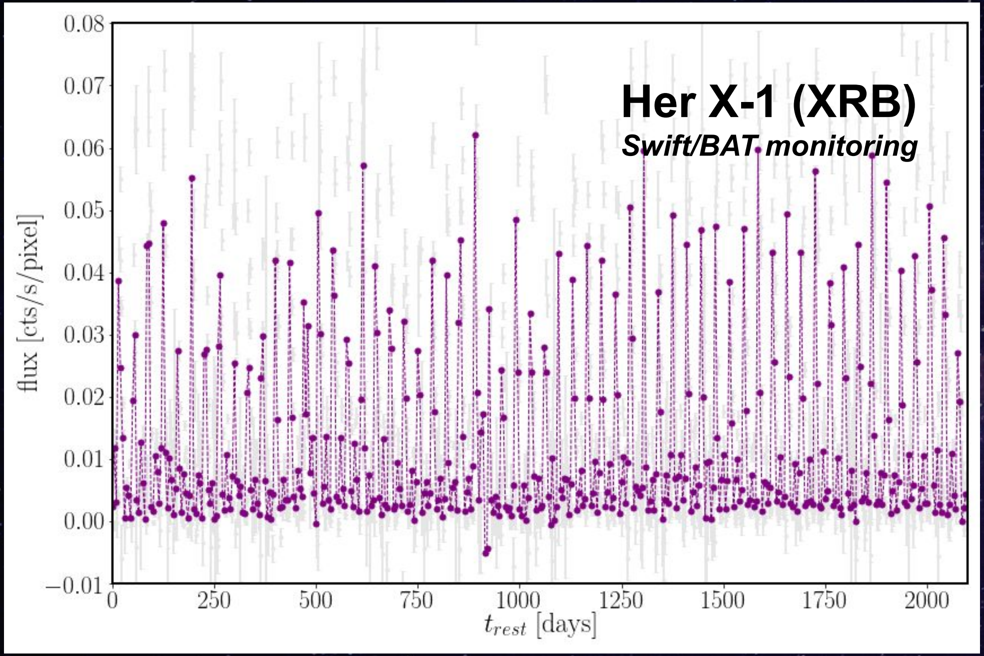


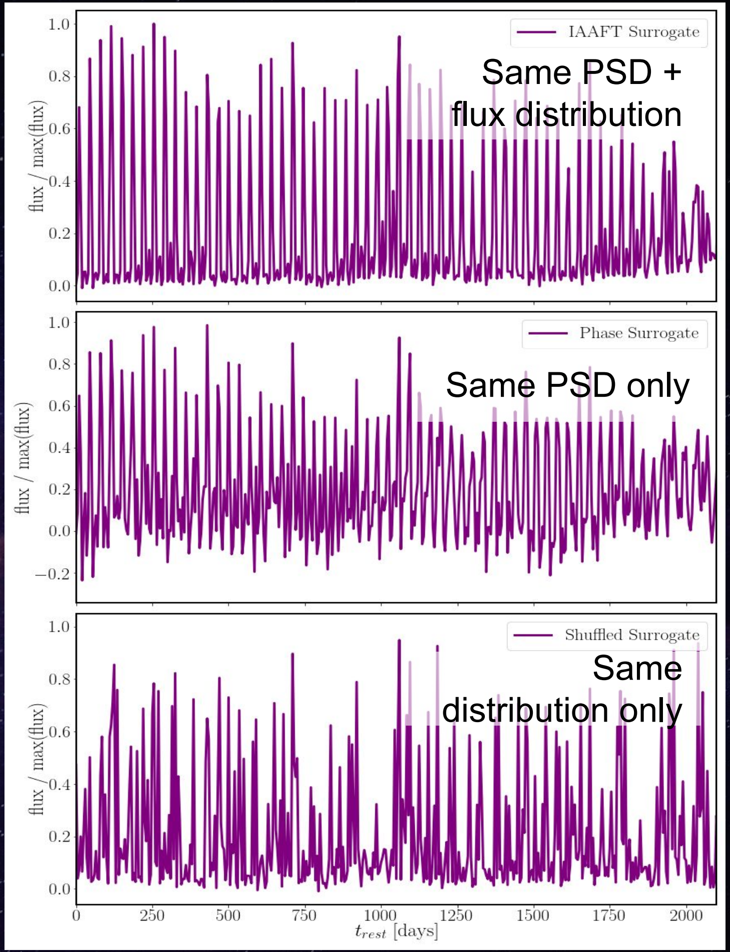
Significance of Recurrence Features

The Surrogate Data method (*Theiler et al. 2002*):

- Data-driven null hypothesis testing
- Generate surrogate light curves that have:
    - the same power spectrum (phase),
        - *i.e. take Fourier transform of time series, randomize the phases, and then inverse Fourier transform to obtain the surrogate*
    - the same flux distribution (shuffled),
    - or both (IAAFT)
- Apply statistical test to data and ensemble of surrogates :
    - if the data is significantly different, we rule out the hypothesis of the surrogates (e.g. correlated noise)
- Surrogates *do not* retain dynamical information and carry the same noise and systematics as the original light curve

The Surrogate Data Method





Swift/BAT AGN

Hard X-ray (14 - 150 keV) monitoring of 46 AGN from the 70-month catalog, previously observed by power spectra analysis:

- PSD slope of -0.8 for all sources but one;
  Shimizu & Mushotzky 2013

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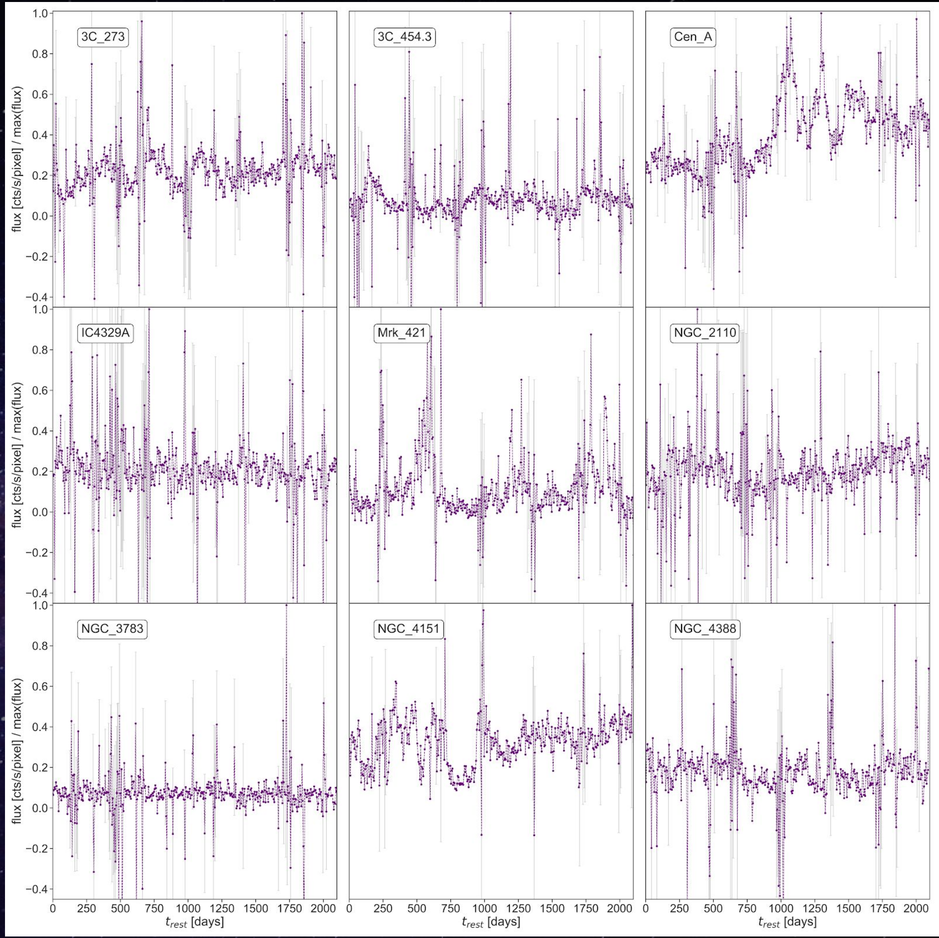
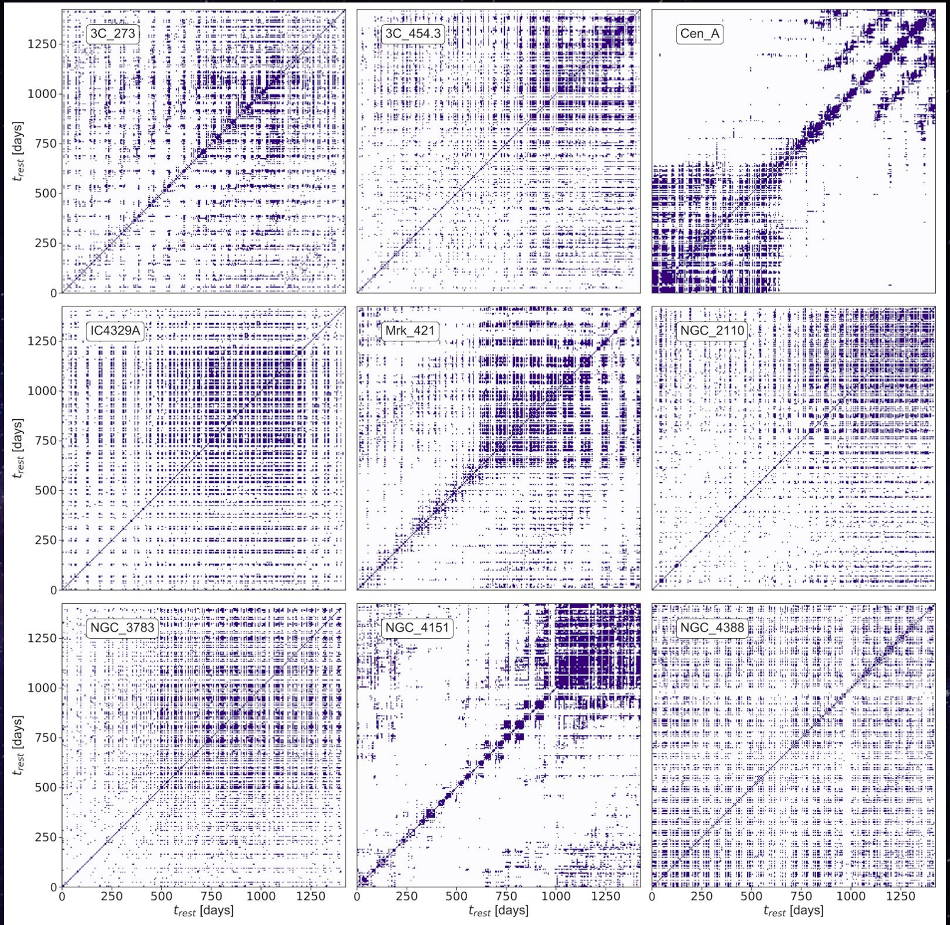


(Phillipson et al 2021a - in prep)

Swift/BAT AGN RPs

Variety of behaviors evident in RPs:

- diagonal structures: repeating behavior
- vertical/horizontal lines: trapped states
- large scale inhomogeneities: non-stationarity
- abrupt changes in texture: state changes



(Phillipson et al 2021a - in prep)

Swift/BAT AGN Recurrence Properties

Quantify the structure in the RP to find evidence for:

- Nonlinear behavior
  - Longest diagonal line length (Lmax)
- Determinism
  - Fraction of recurrences that are part of diagonal structures (DET)
- Stochastic behavior
  - Shannon entropy (randomness in the distribution of recurrences; Lentr)

Compare these measures to ensembles of surrogate data.

Are there correlations of significance of recurrence properties with physical characteristics:

- **Type 1 vs. Type 2**
- **Obscured vs. unobscured**
- **Radio loud vs. radio quiet**

(Phillipson et al 2021a - in prep)

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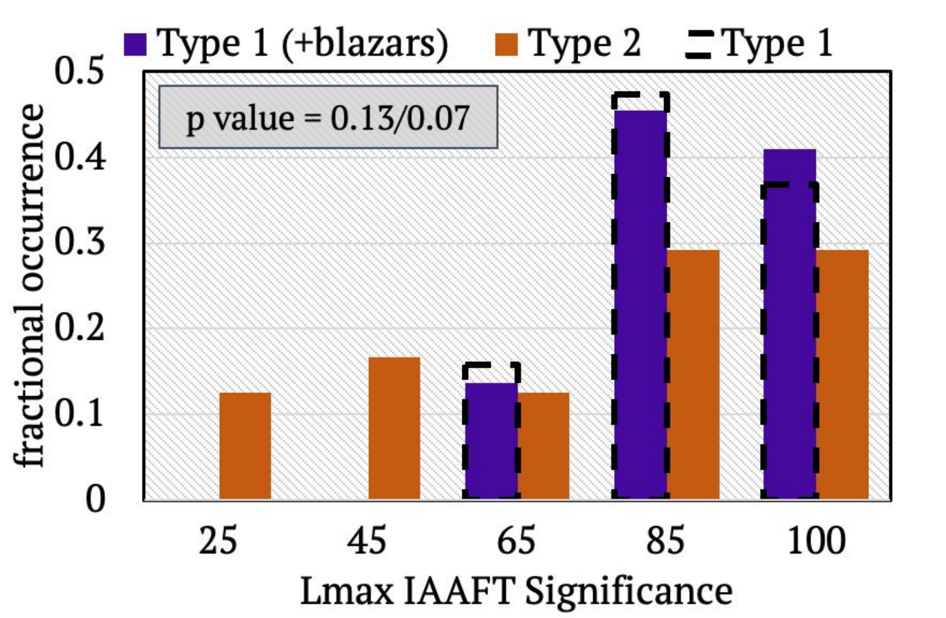

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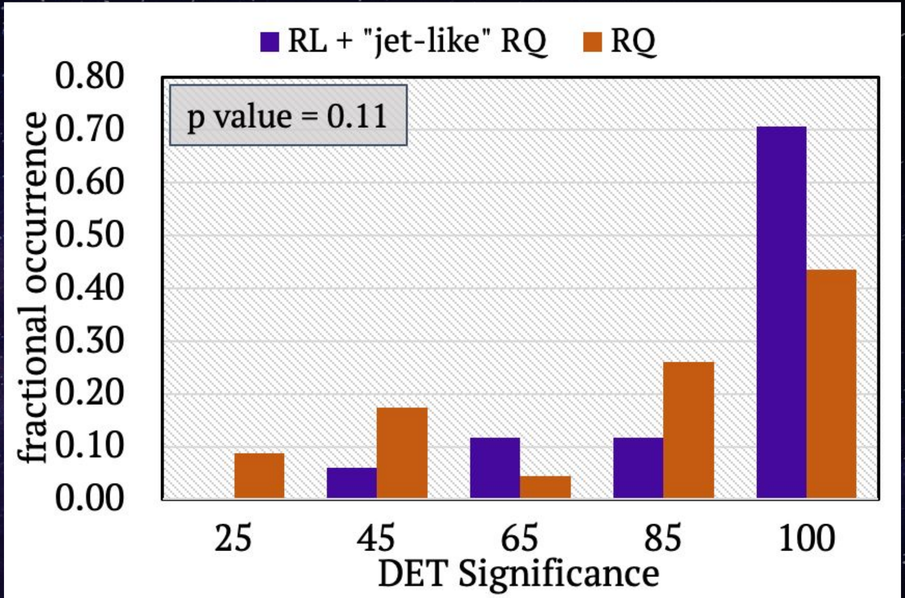

(Phillipson et al 2021a - in prep)

Ongoing Research (& challenges)

Swift/BAT AGN:

- Only nominal results comparing to physical characteristics of AGN
- Strong evidence for nonstationary behavior
- **Ongoing:** application to 157-month catalog

Correlated Timing and Spectral variations for XRBs:

- Recurrence Plots as a moving window: uncovers changes in the variability as function of time; overlaps with spectral state transitions

Irregularly Spaced Time Series:

- The time delay method for embedding in phase space depends on evenly sampled time series
- Other methods for embedding:
  - Legendre polynomials, numerical differentiation
- Developing python package for recurrence analysis, including an alternative recurrence plot that handles irregularly spaced time series (coded for ZTF light curves)

Generally: Classification of variable sources using recurrence quantities

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Slack/Zoom

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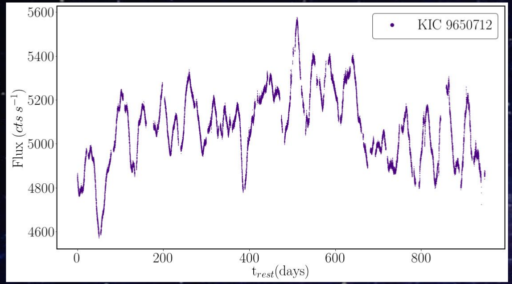
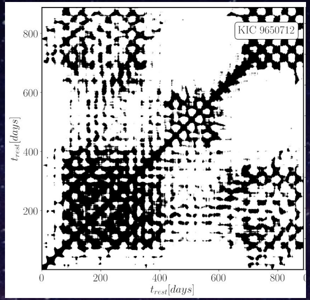
@raphillipson

@beckastrosaurus

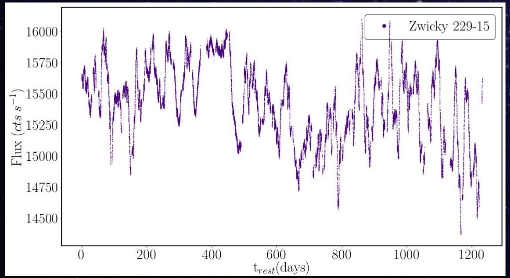
Fun with recurrence plots:
https://colinmorris.github.io/SongSim/#/rumourhasit

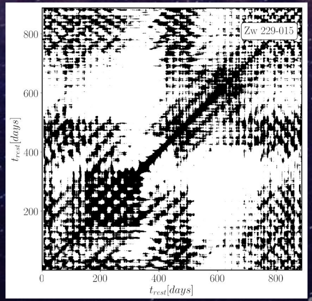
Example: Diverse Variability in Active Galaxies





From distribution of diagonal lines, obtain 'correlation entropy', compare to stochastic surrogates — long-term variability distinguishable from stochastic, linear mechanisms



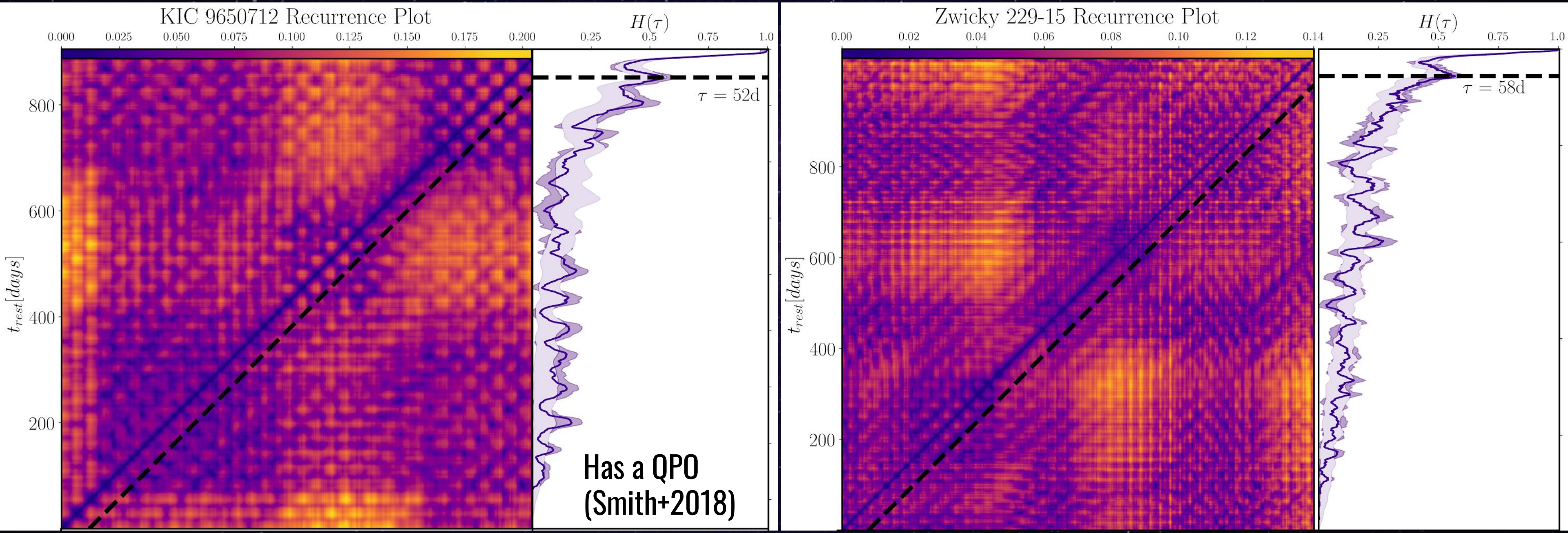


Obtain 'correlation entropy' — long-term variability **NOT** distinguishable from stochastic surrogates

(Phillipson et al 2020)

Diverse Variability in Active Galaxies

"Close Returns": pseudo-autocorrelation function -- quantifies diagonal lines as a function of time delay

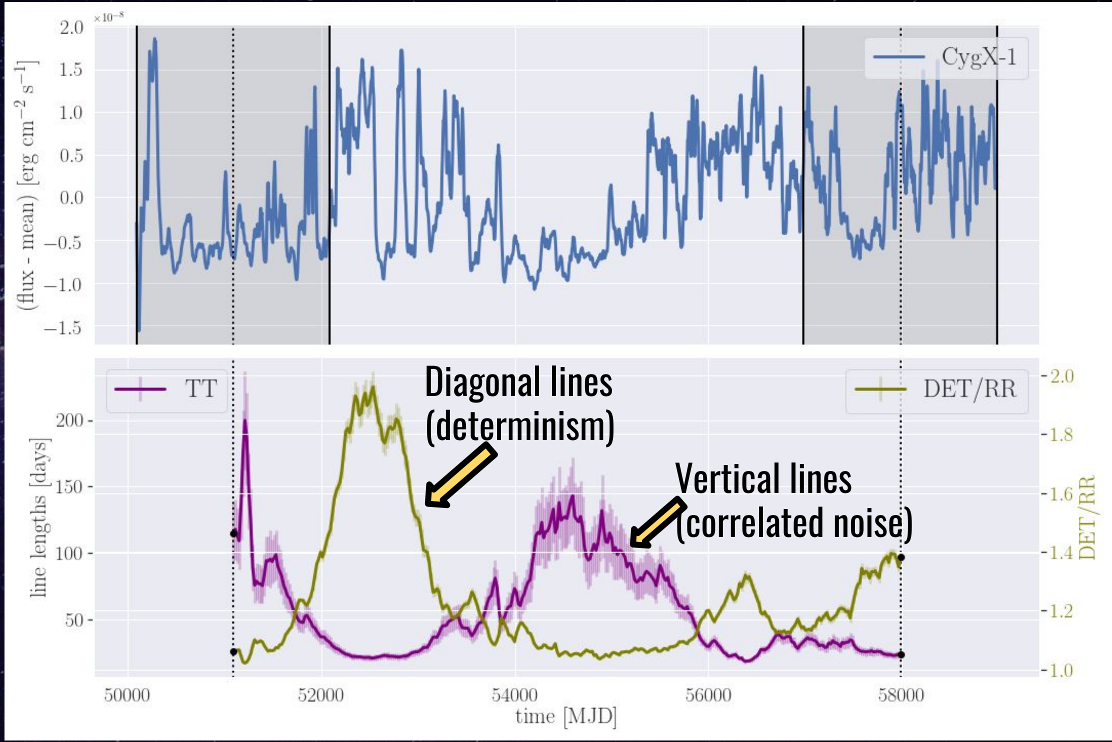


(Phillipson et al 2020)

(Phillipson et al 2021b - in prep)

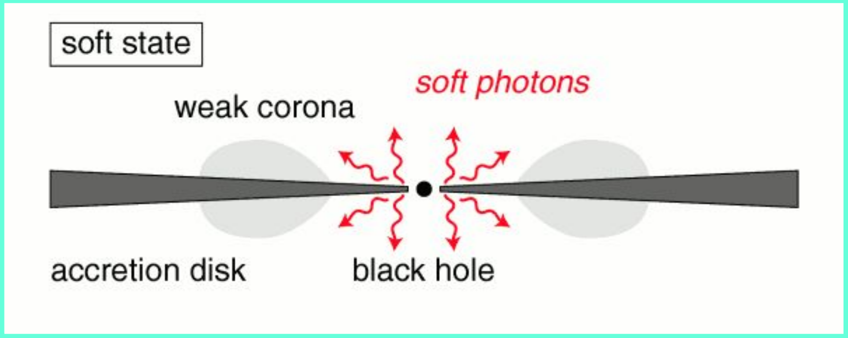
Example: Changes in Variability States of XRBs

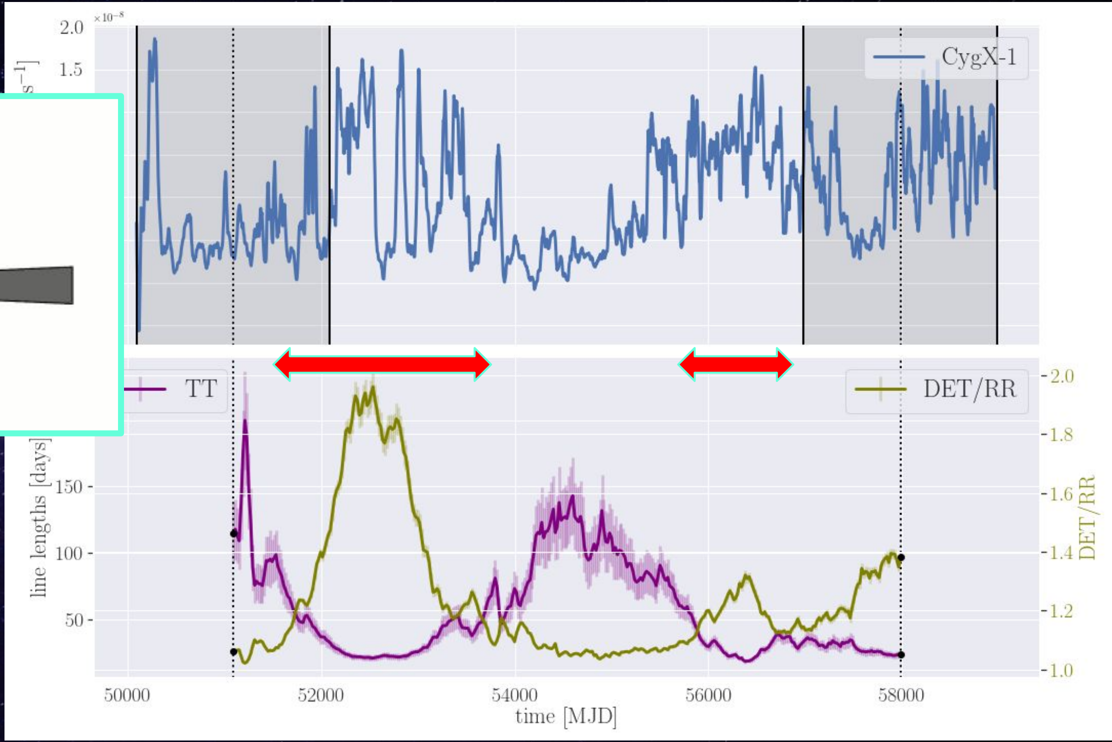
- Cyg X-1 experienced a series of failed state transitions and soft states (overall MJD 51,000 to MJD 53,900; *Grinberg et al. 2013*).
- A second, similar transition identified by DET/RR starts to occur at approximately MJD 56,000, where a second pro-longed, very soft X-ray period occurs in 2012 (*Grinberg et al. 2013*)



(Phillipson et al 2021b - in prep)

Example: Changes in Variability States of XRBs

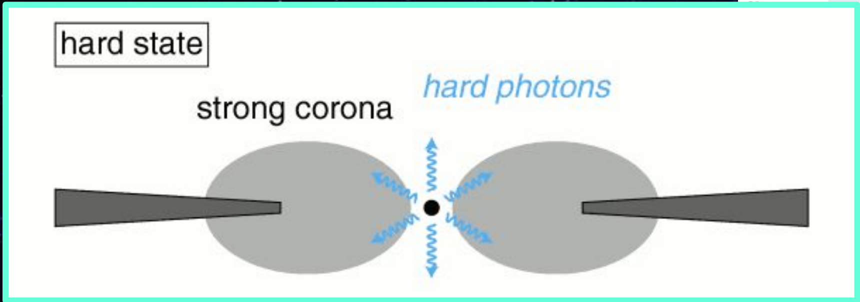


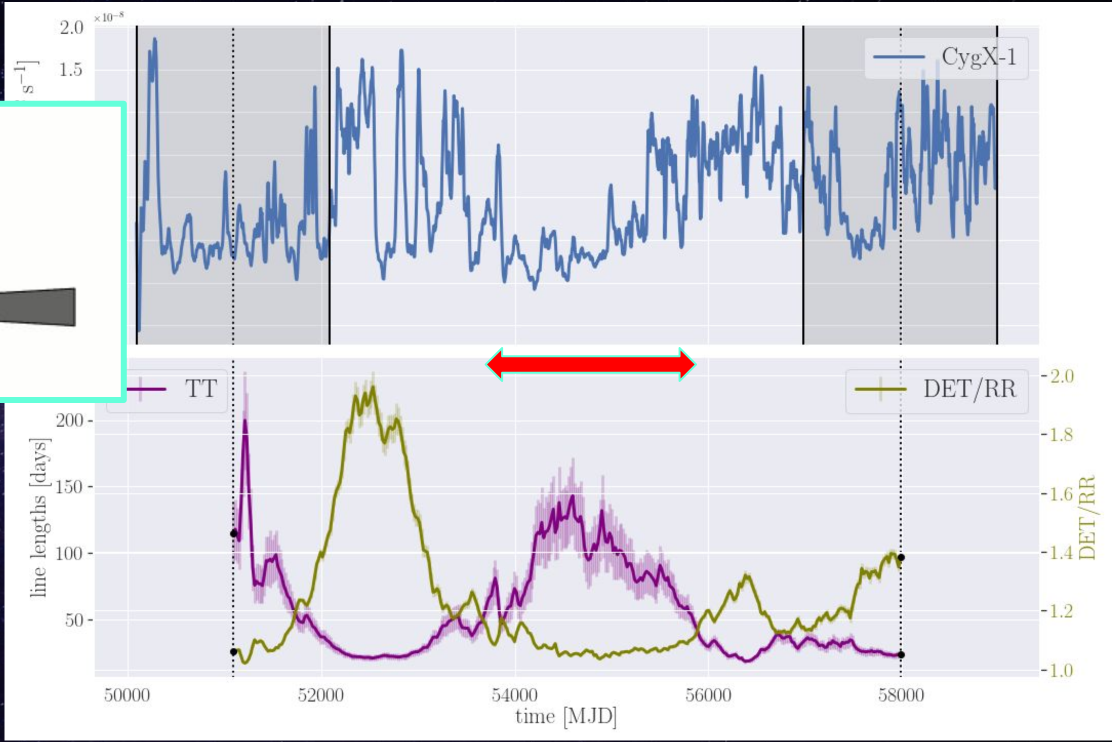


Possible Interpretation:
Disk-dominated “soft” state corresponds to high determinism and regularity

(Phillipson et al 2021b - in prep)

Example: Changes in Variability States of XRBs





Possible Interpretation:
Corona-dominated "hard" state corresponds to high trapping time (laminarity) and low determinism