

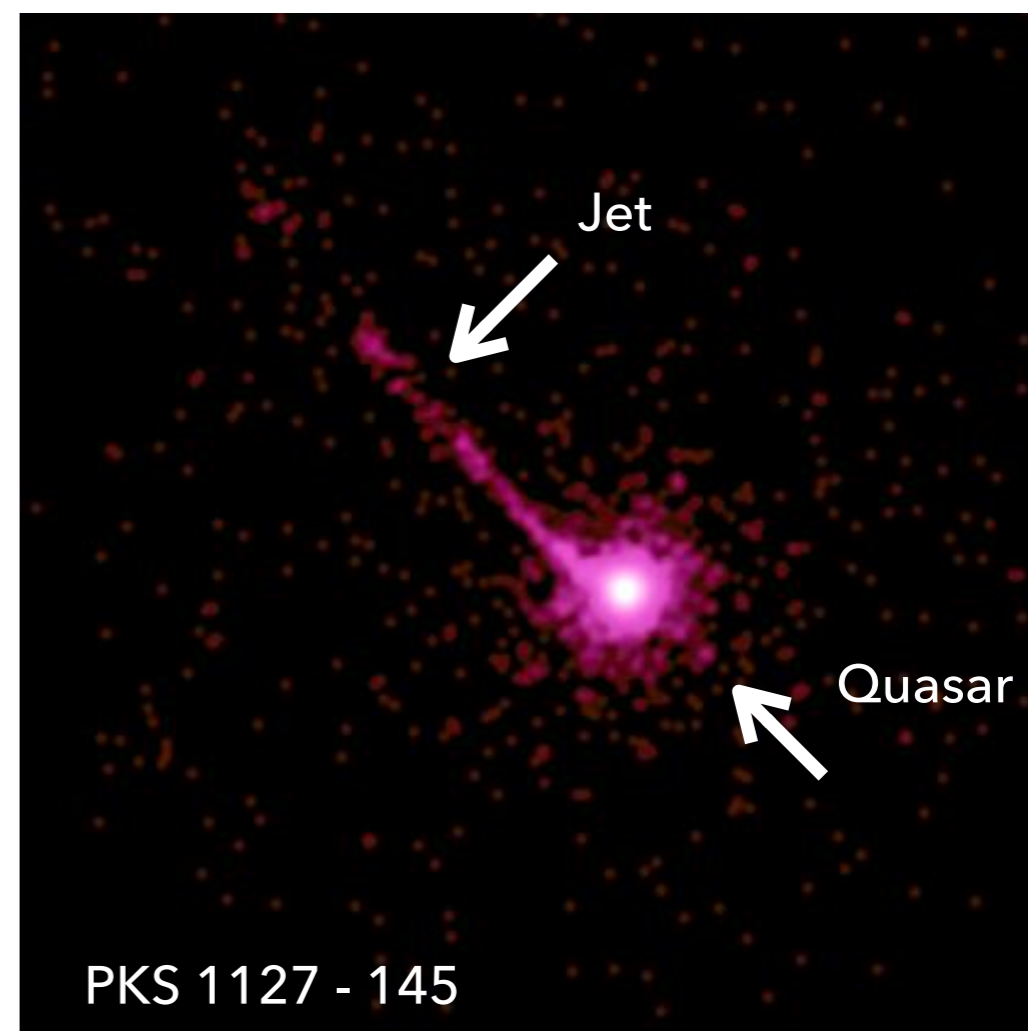
KATY MCKEOUGH

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**DEFINING REGIONS THAT CONTAIN COMPLEX
ASTRONOMICAL STRUCTURES**

SCIENTIFIC MOTIVATION

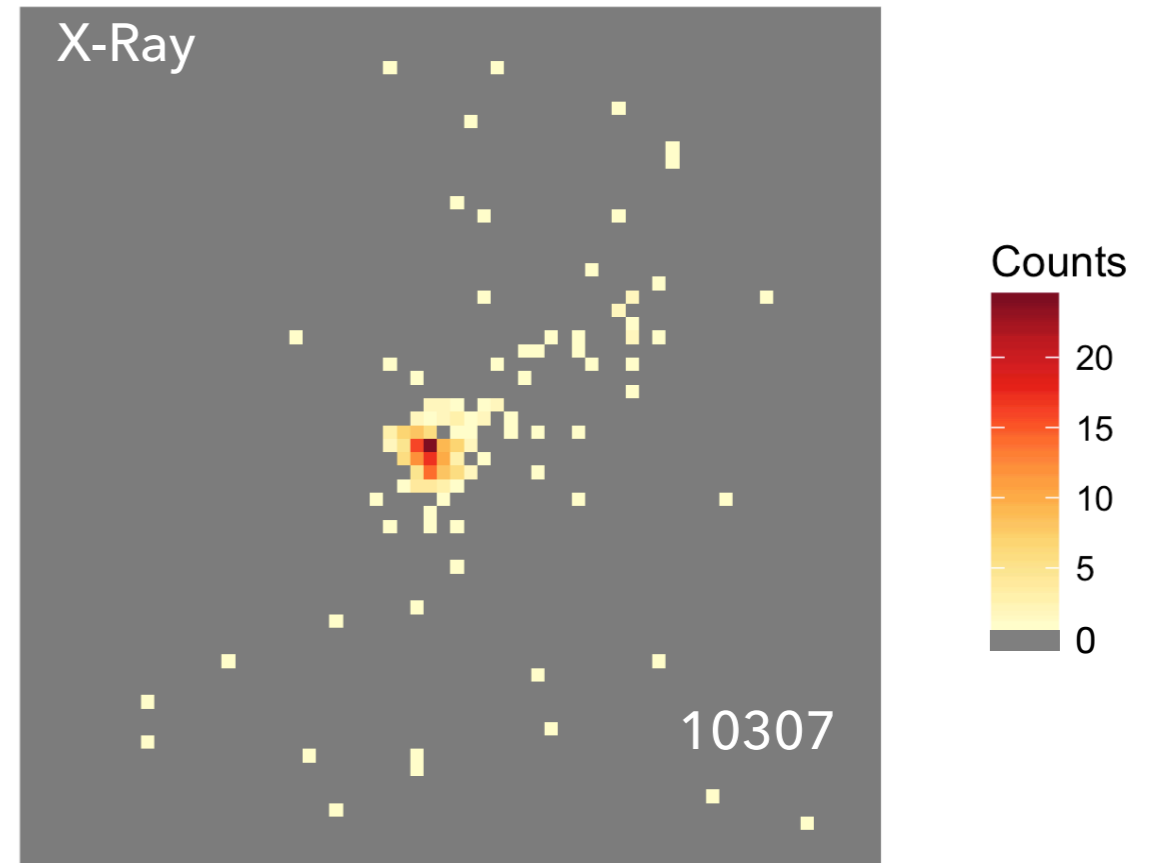
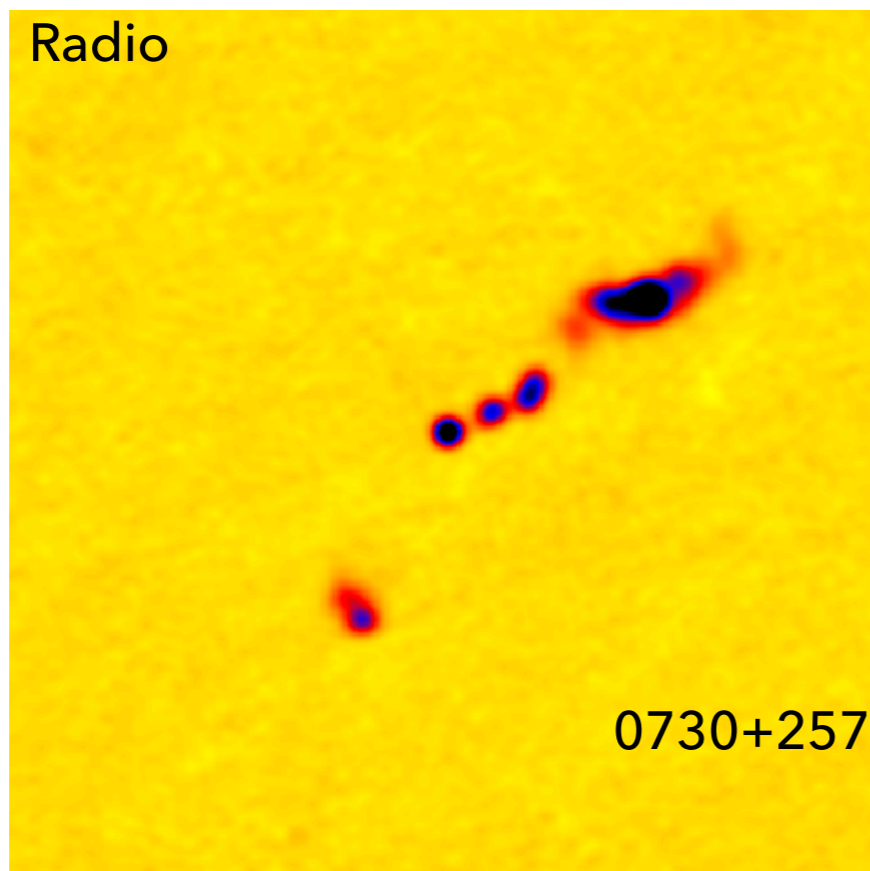
- ▶ We are interested in defining an outline around extragalactic jets coming from quasars at high redshift ($z > 2.1$) in X-ray images
- ▶ Defining this boundary is important for accurate luminosity and flux calculations.
- ▶ Detecting jets is difficult because they are diffuse sources (no edges, or center) and dim compared to the quasar.
- ▶ Images of high redshift jets are of low resolution and few X-ray photons



NASA/CXC/A.Siemiginowska(CfA)/
J.Bechtold(U.Arizona)

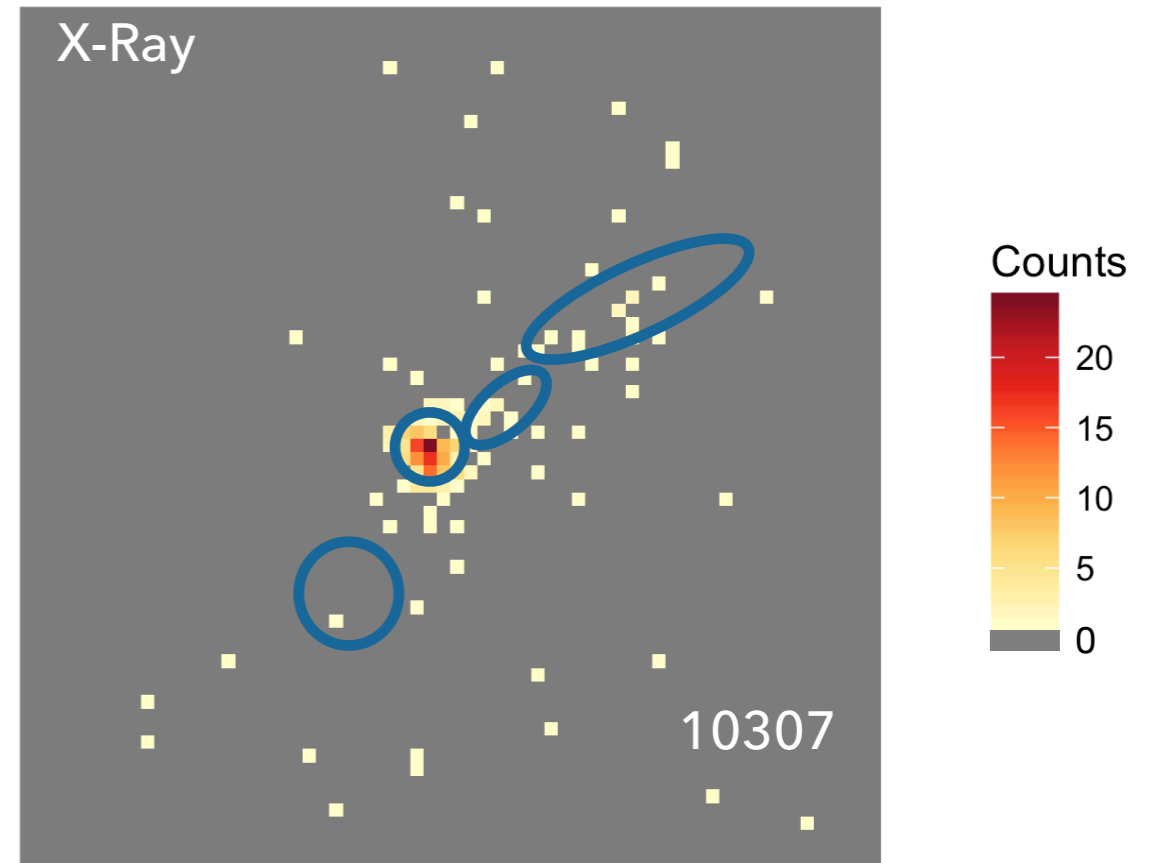
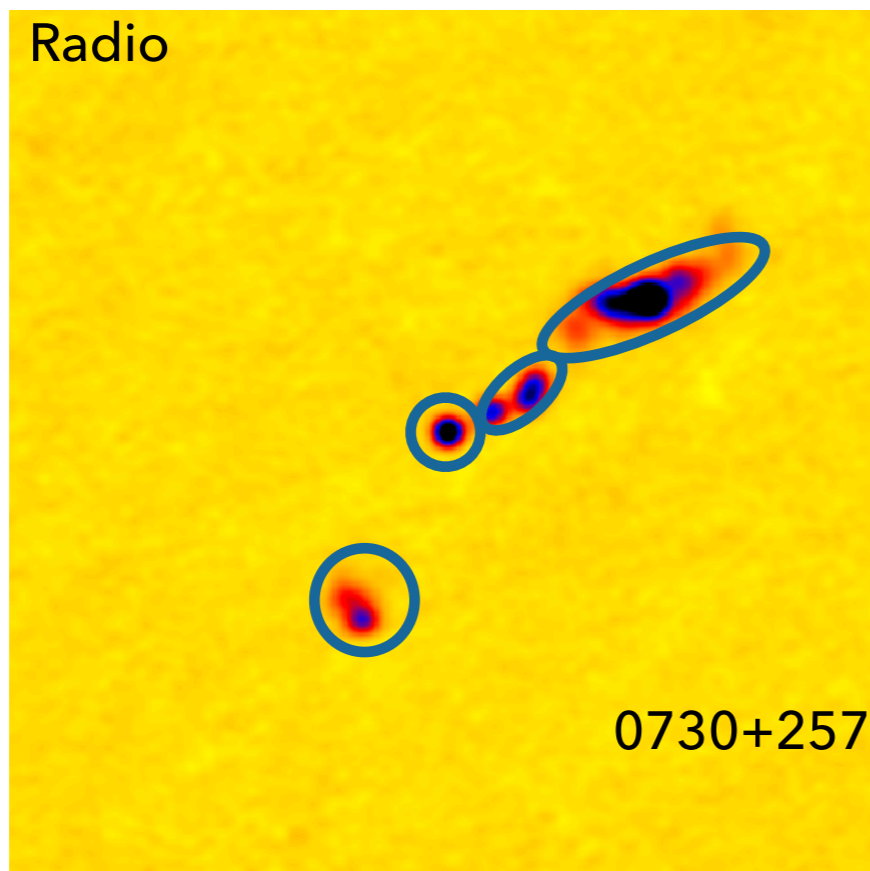
OBSERVATIONAL DATA

- ▶ Chandra X-ray Observatory - ACIS
- ▶ 64 x 64 or 128 x 128 pixel image centered on quasar
- ▶ High to intermediate redshift ($2.10 < z < 4.72$)



REGION OF INTEREST

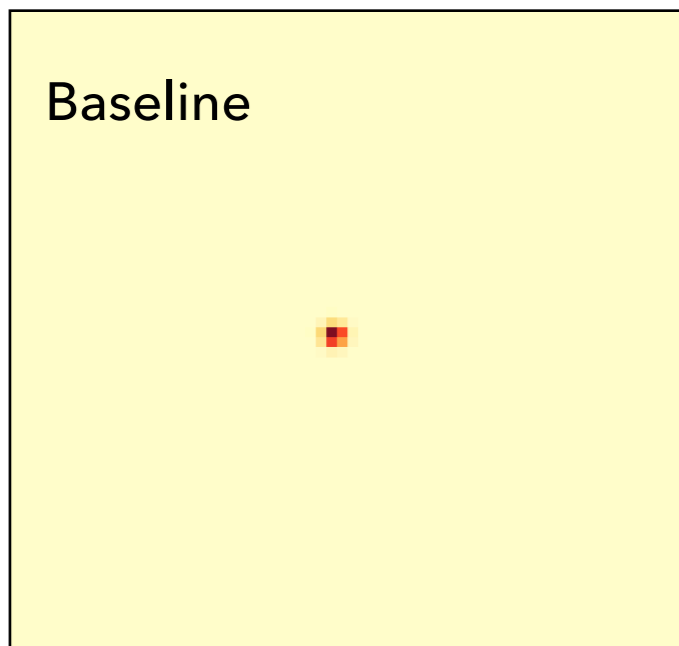
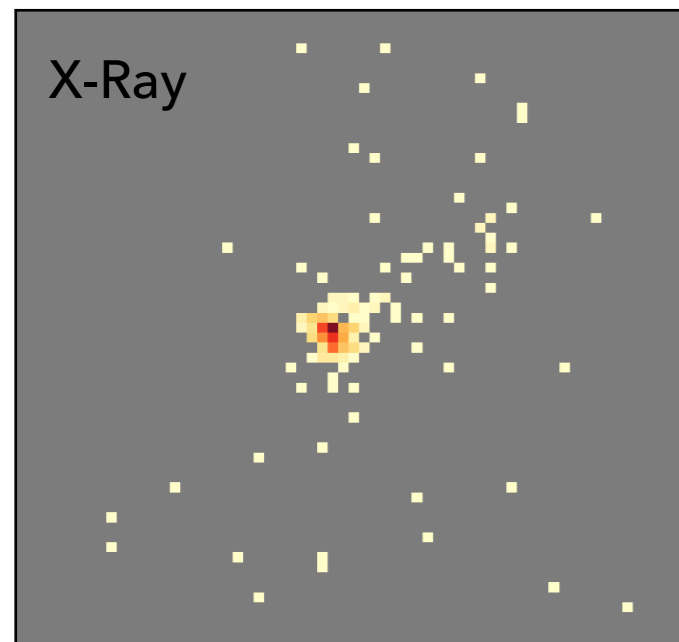
- ▶ **Region of Interest (ROI)** - region containing the jet or a partition of the jet (e.g. node or lobe)
- ▶ Previous work tests whether or not a jet exists in a predefined ROI (McKeough et al. 2016, Stein et al. 2015)



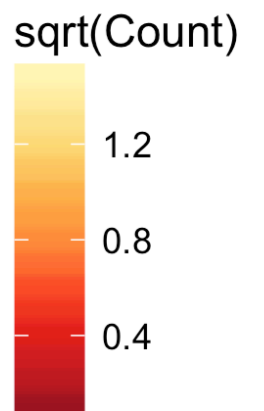
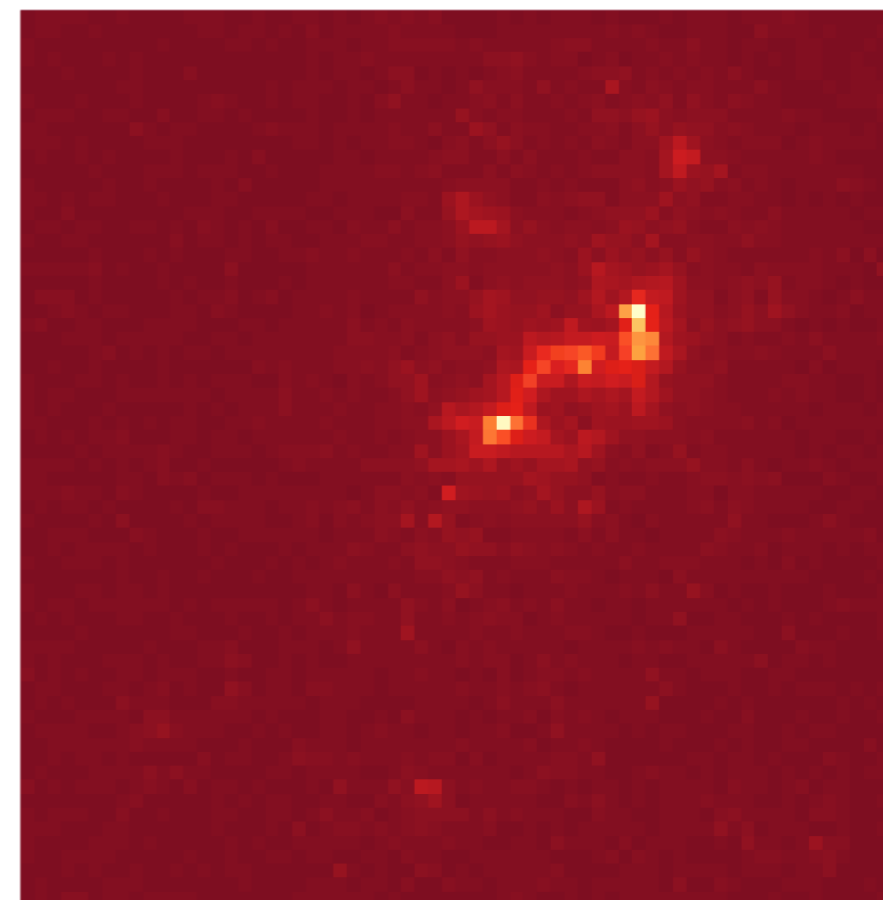
GOAL

Define a probabilistic ROI around a diffuse source as a post-processing step to LIRA.

LOW COUNT IMAGE RECONSTRUCTION AND ANALYSIS (LIRA)



Deconvolve
PSF
+
Extract
Baseline



LIKELIHOOD

$$\sqrt{\tilde{\lambda}_{ij}} | Z, \tau_{\pm}, \sigma_{\pm}^2 \sim \text{Normal}(\tau_{-}, \sigma_{-}^2) \mathbb{I}_{z_{ij}=-1} + \text{Normal}(\tau_{+}, \sigma_{+}^2) \mathbb{I}_{z_{ij}=+1}$$

- ▶ We are given observation Y from which we draw the LIRA output:

$$\tilde{\lambda} | Y$$

- ▶ We want to assign each pixel to either the background (-1) or the ROI (+1):

$$z_{ij} = \{-1, +1\}$$

- ▶ Each pixel assignment will have its own average intensity:

$$\tau_{-}, \tau_{+}$$

- ▶ We suspect the variance of the source will be greater than the background:

$$\sigma_{-}^2, \sigma_{+}^2$$

2D ISING PRIOR

$$p(z|\beta) = \frac{\exp(\beta \sum_{ij, i'j' \in |ij-i'j'|=1} z_{ij} z_{i'j'})}{\tilde{Z}(\beta)}$$

▶ Inverse temperature:

 β

▶ Higher β induces more correlation between pixels

▶ Partition function:

 $\tilde{Z}(\beta)$

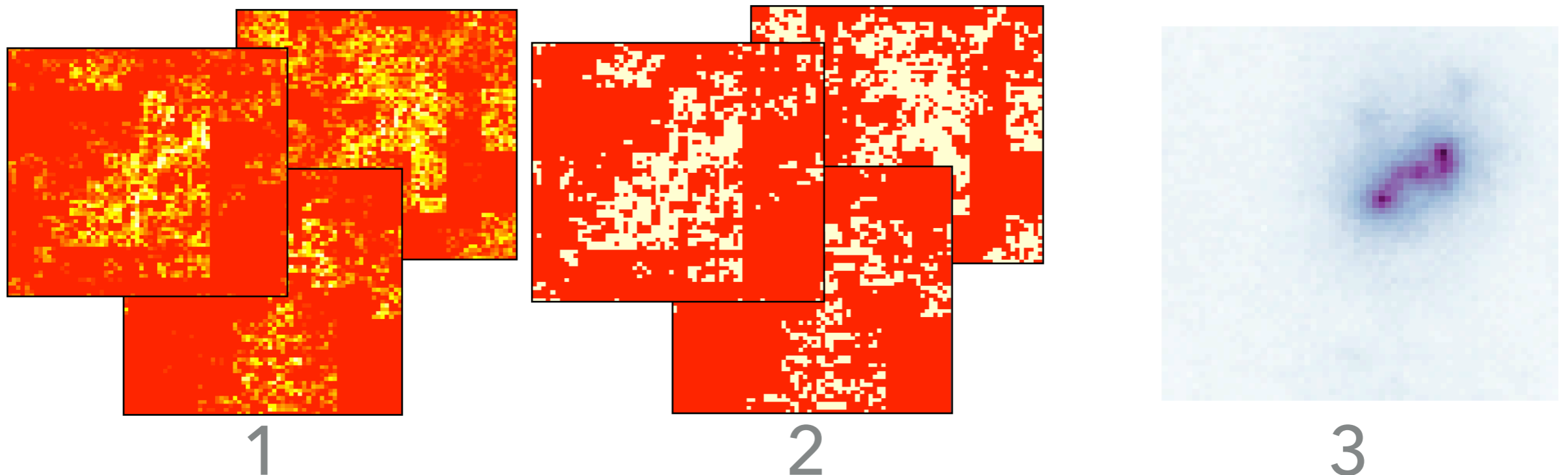
▶ Estimated via Beale (1996) assuming periodic structure

▶ Commonly used in modeling ferromagnetism.

▶ Induces spatial correlation; adjacent pixels will tend to have the same assignment.

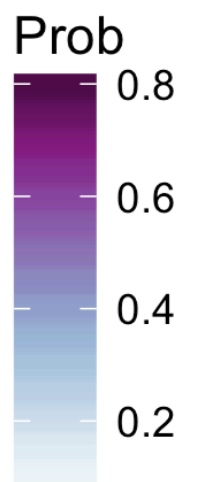
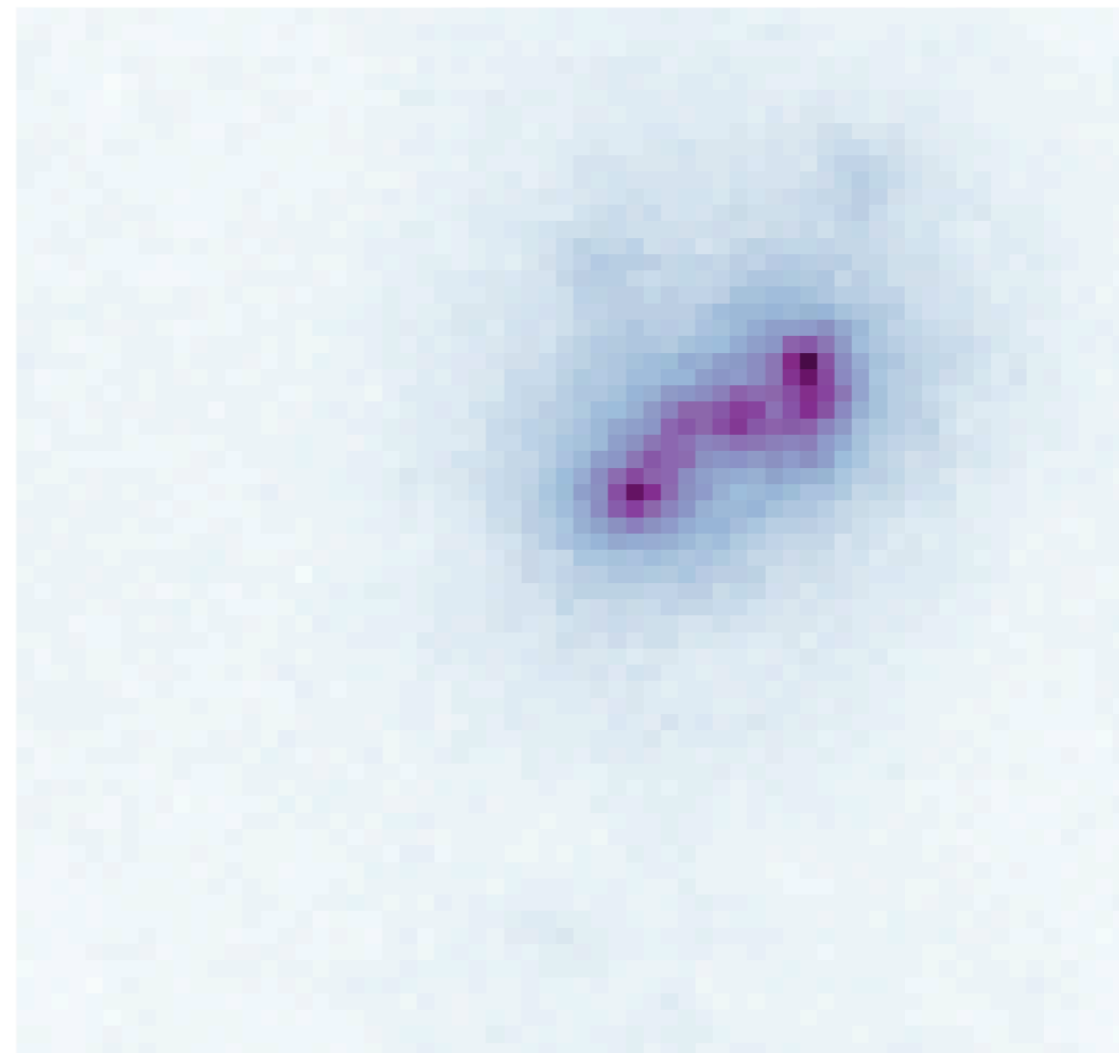
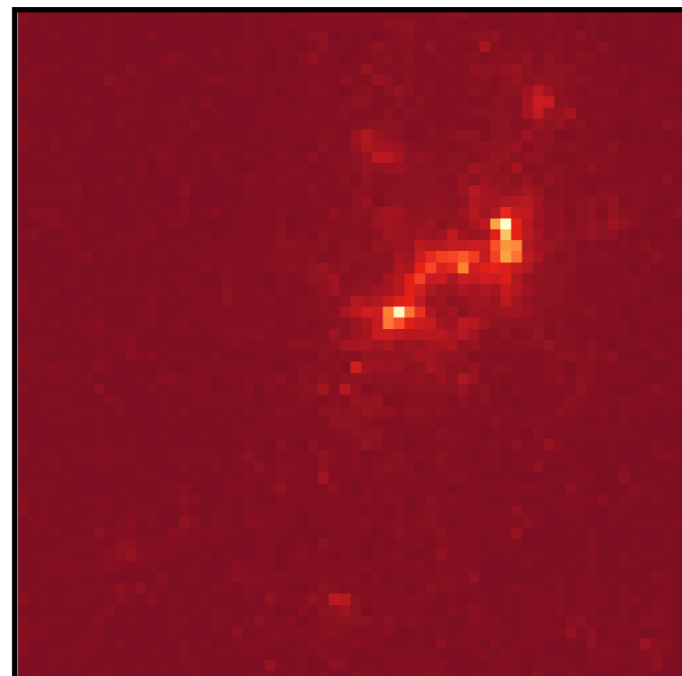
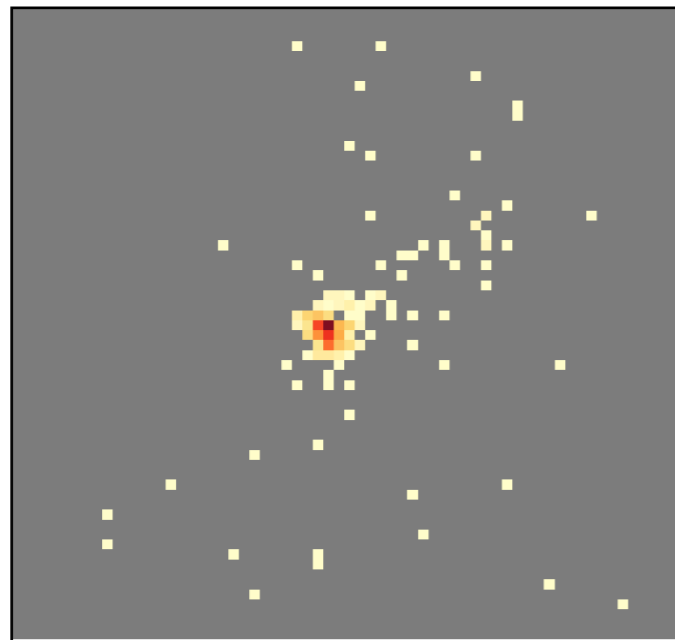
ISING-LIRA ITERATIONS

1. Get many posterior draws from LIRA
2. Apply Ising step to each LIRA draw
3. Average across LIRA-Ising iterations to get probability map.



PROBABILITY MAP

- ▶ Probability each pixel is a member of the ROI:



LOCATION PROBABILITY

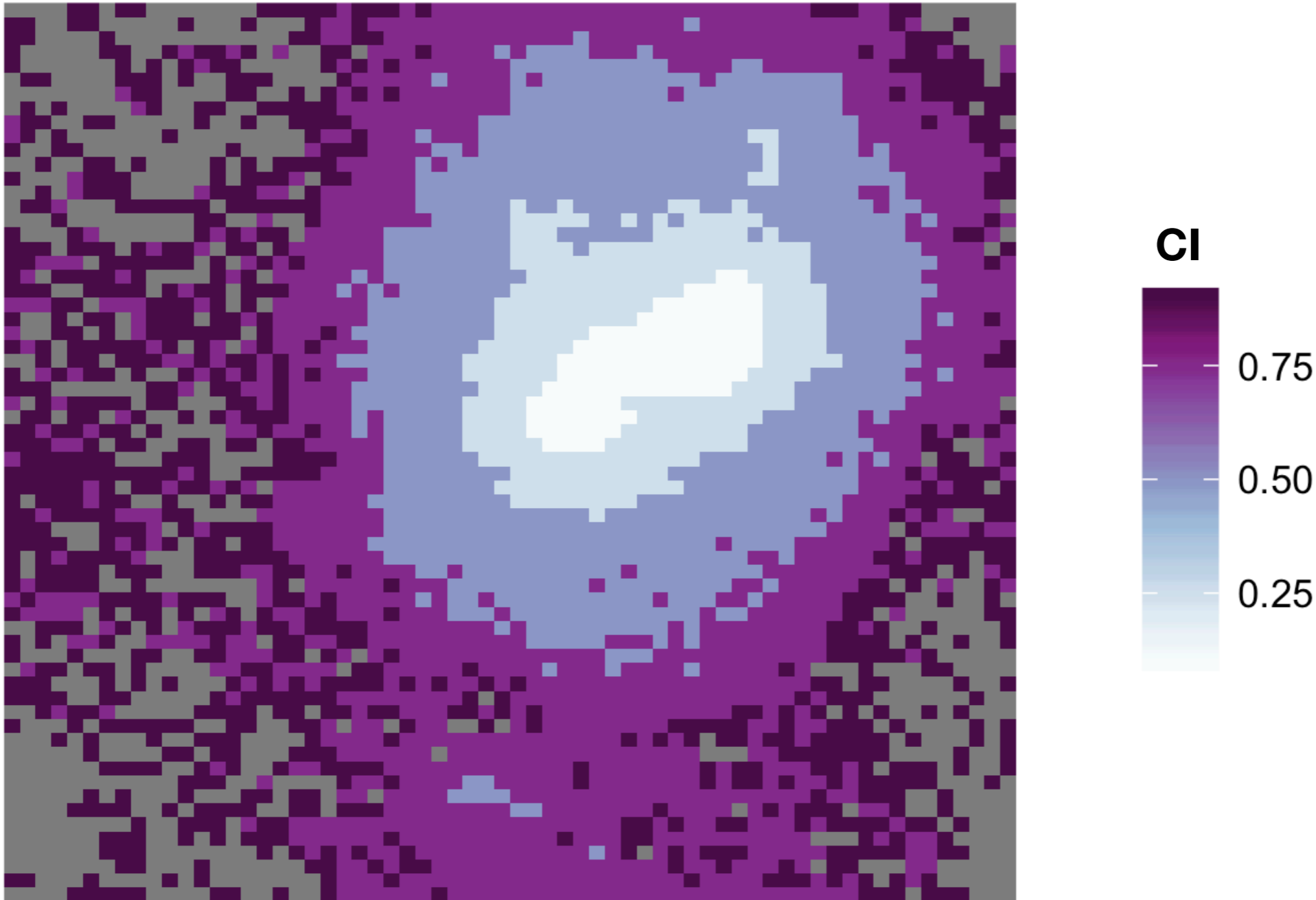
- ▶ Think about location as a random variable

$$P(z_{ij} = 1 | -) \rightarrow P(z = 1 | l = (i, j), -)$$

- ▶ Transform to probability ROI is in location (i, j)

$$P(l = (i, j) | z = 1) = \frac{P(z = 1 | l = (i, j))}{\sum_{ij} P(z = 1 | l = (i, j))}$$

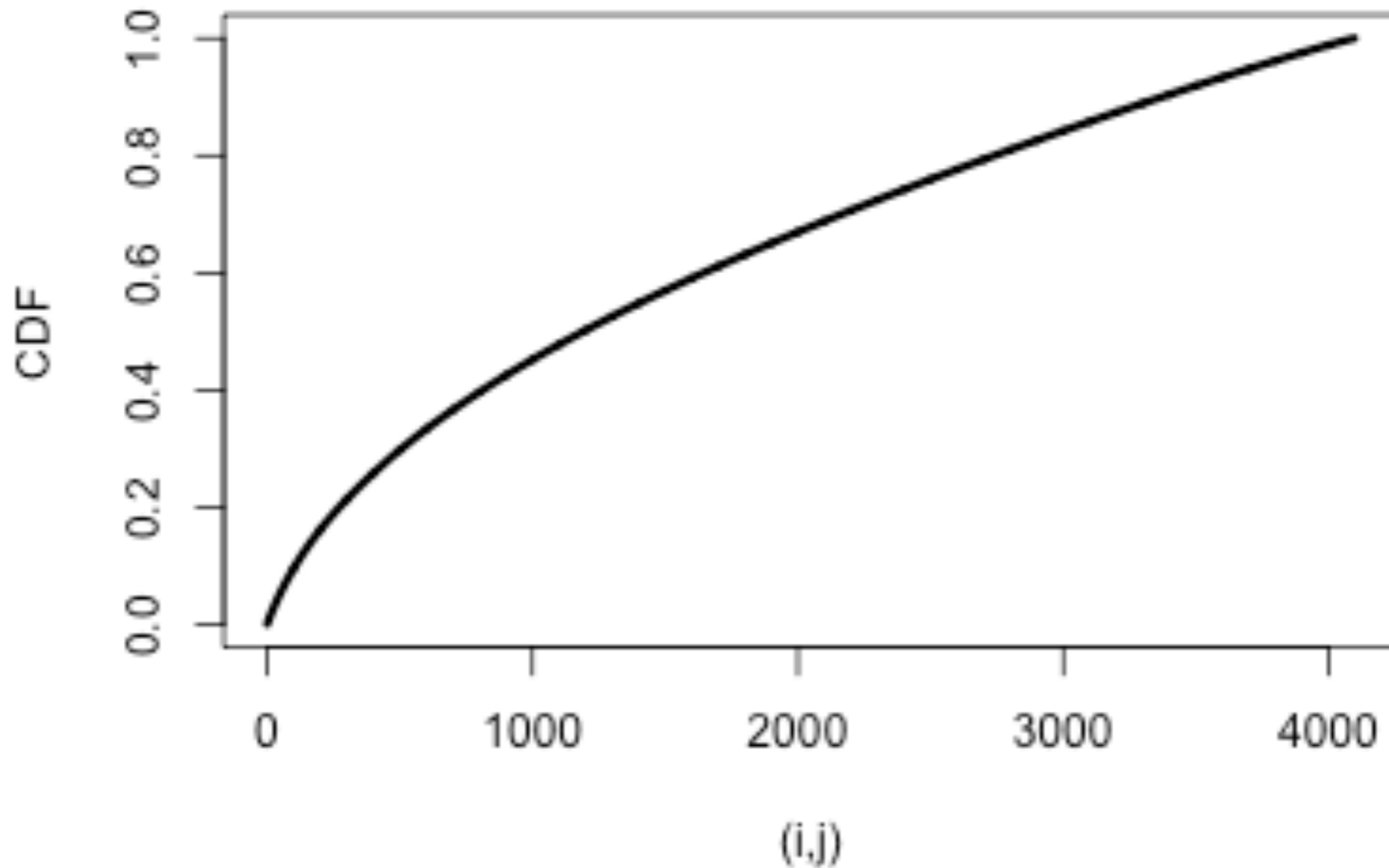
LOCATION PROBABILITY – CREDIBLE INTERVALS



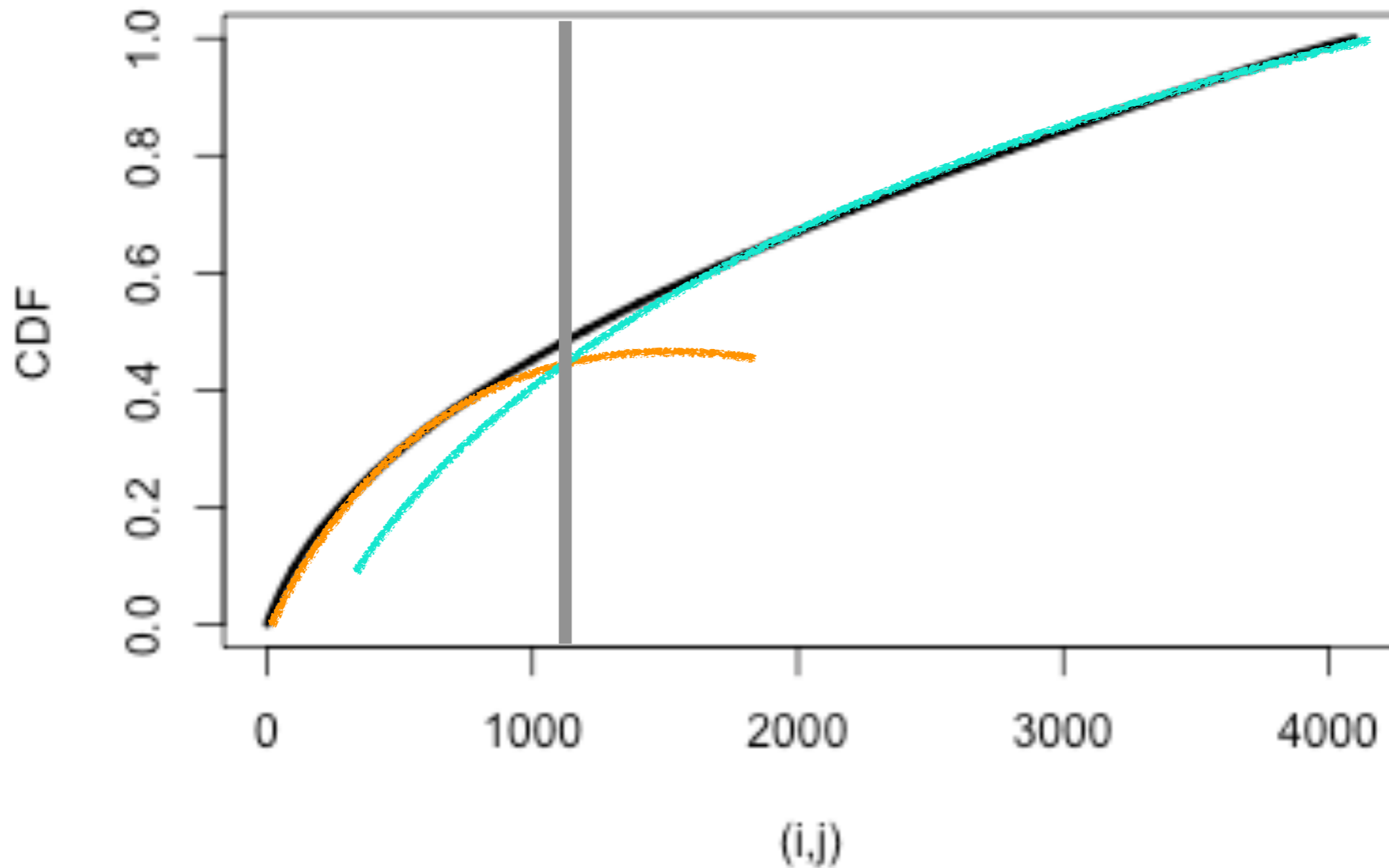
DROPPING POINT

- ▶ **Dropping Point** - The point where the ROI becomes indistinguishable from the background
- ▶ Can be defined as a change point in the ordered CDF of the location

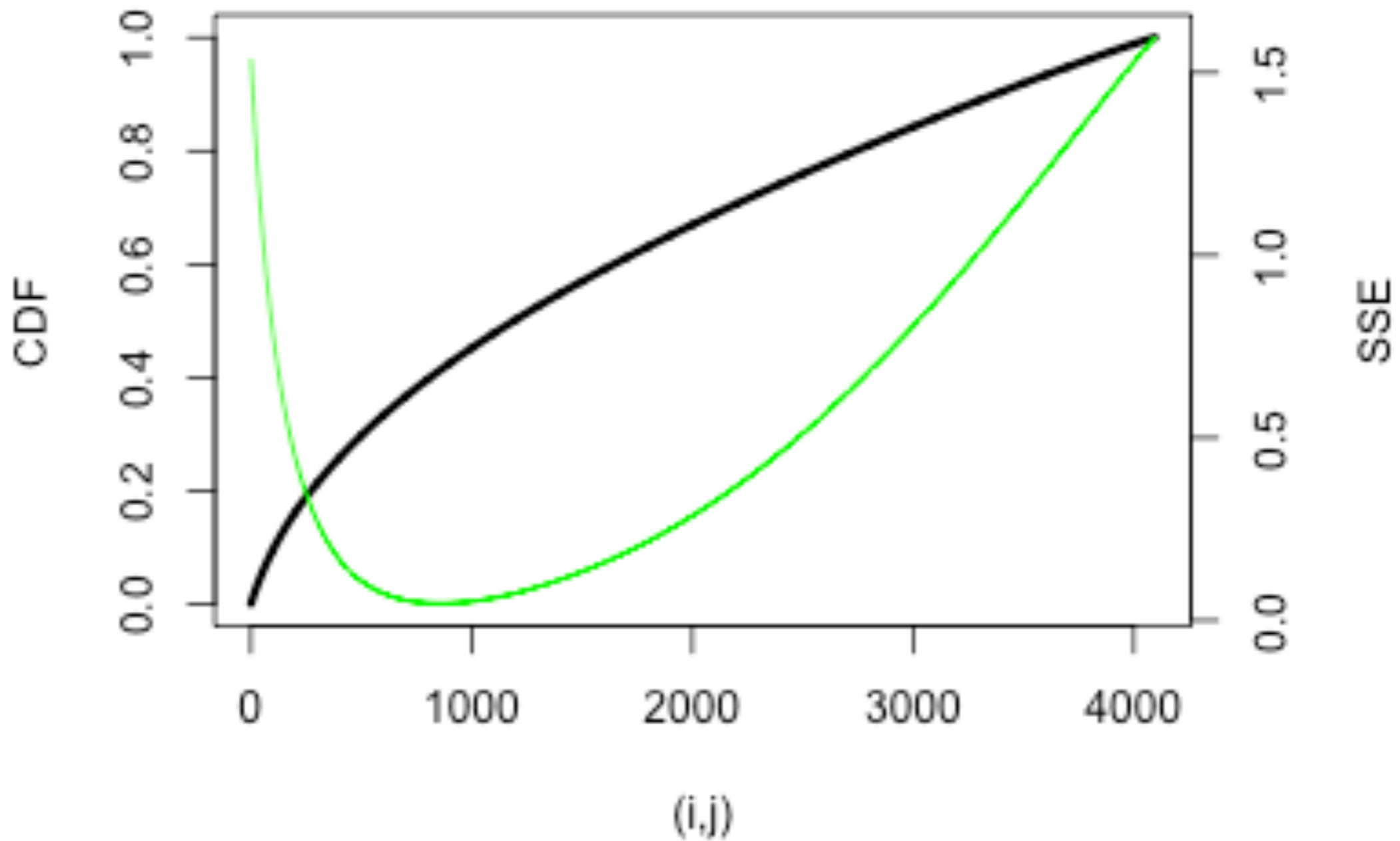
ORDERED LOCATION CDF



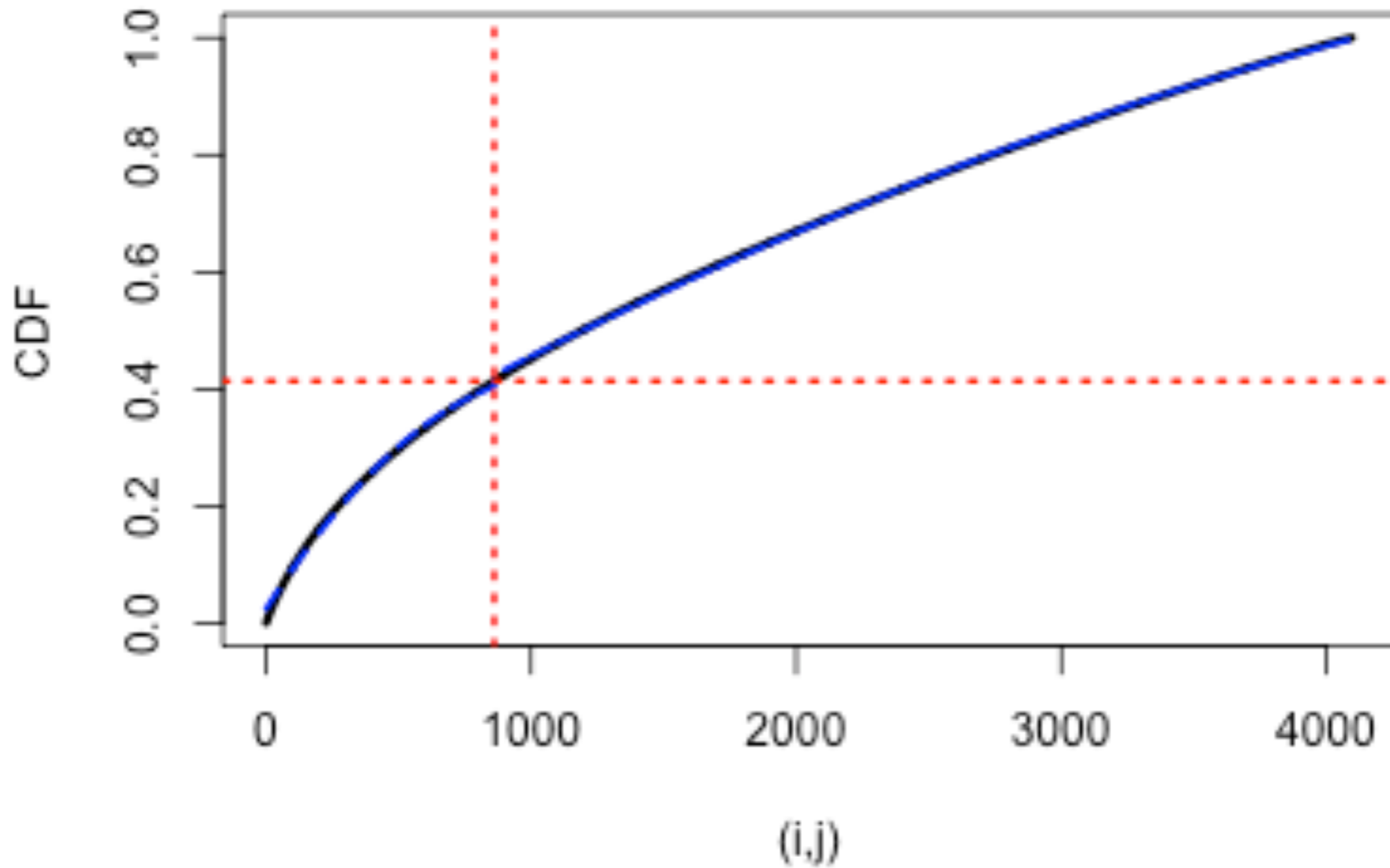
FINDING DROPPING POINT



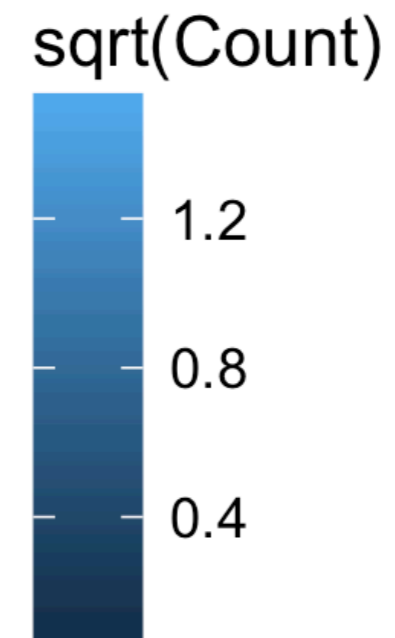
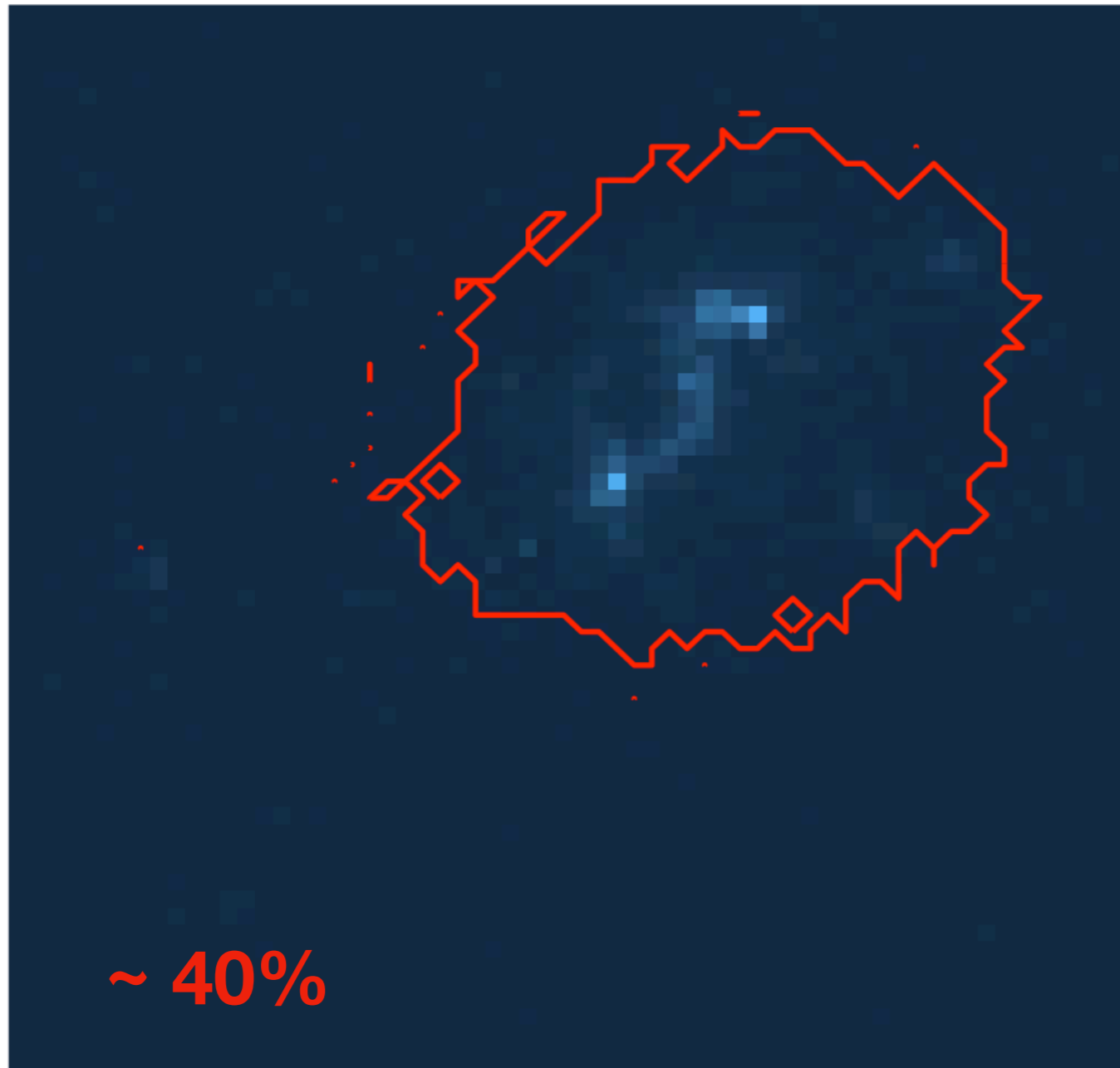
FINDING DROPPING POINT



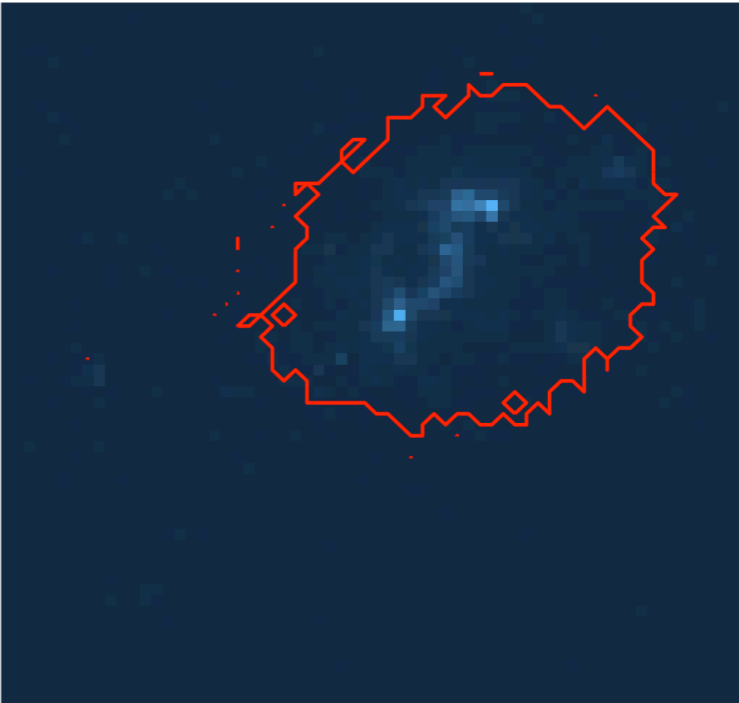
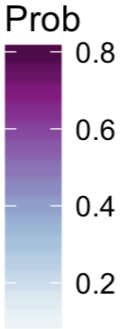
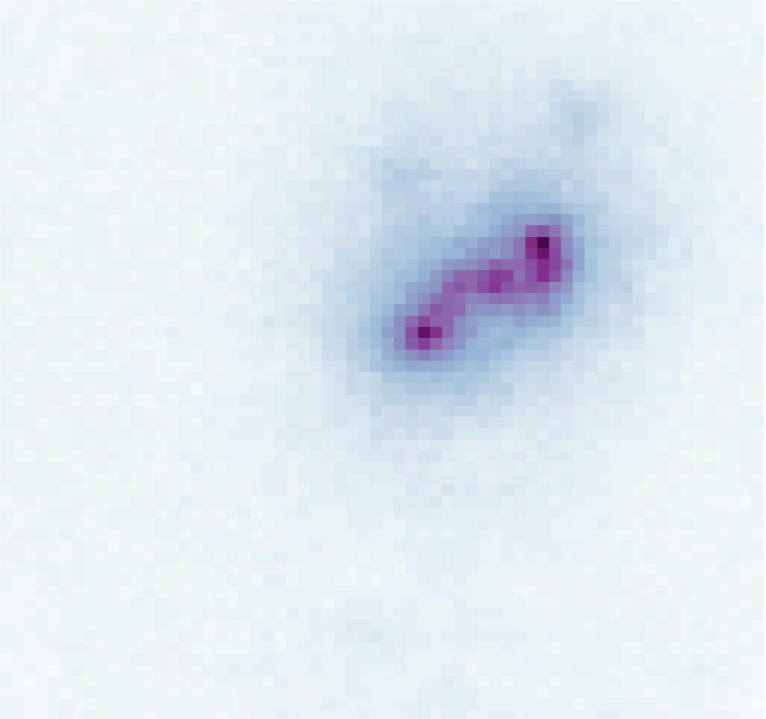
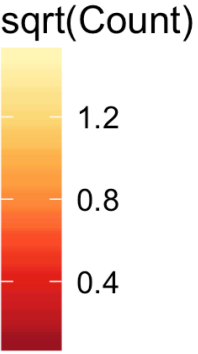
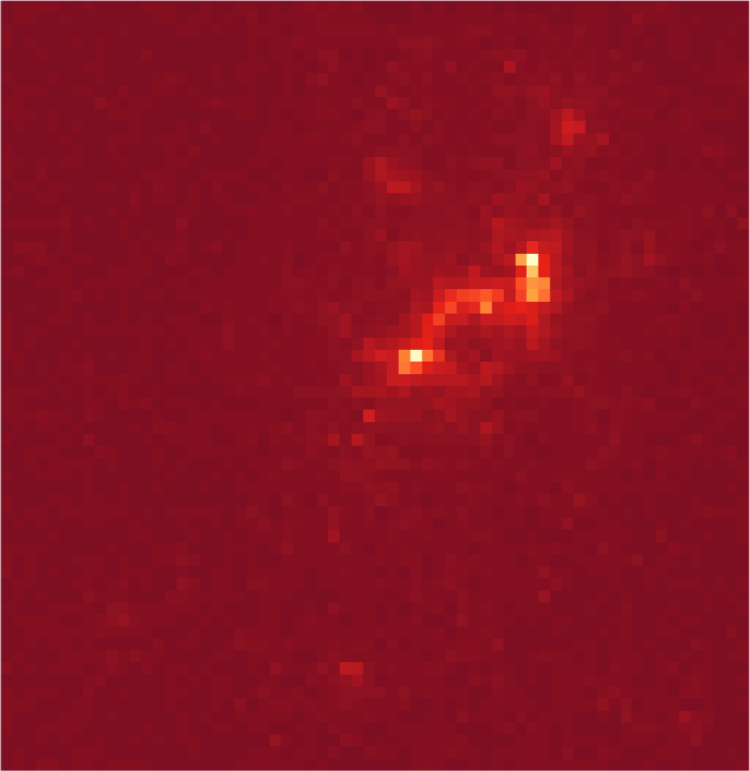
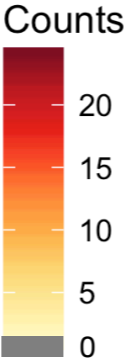
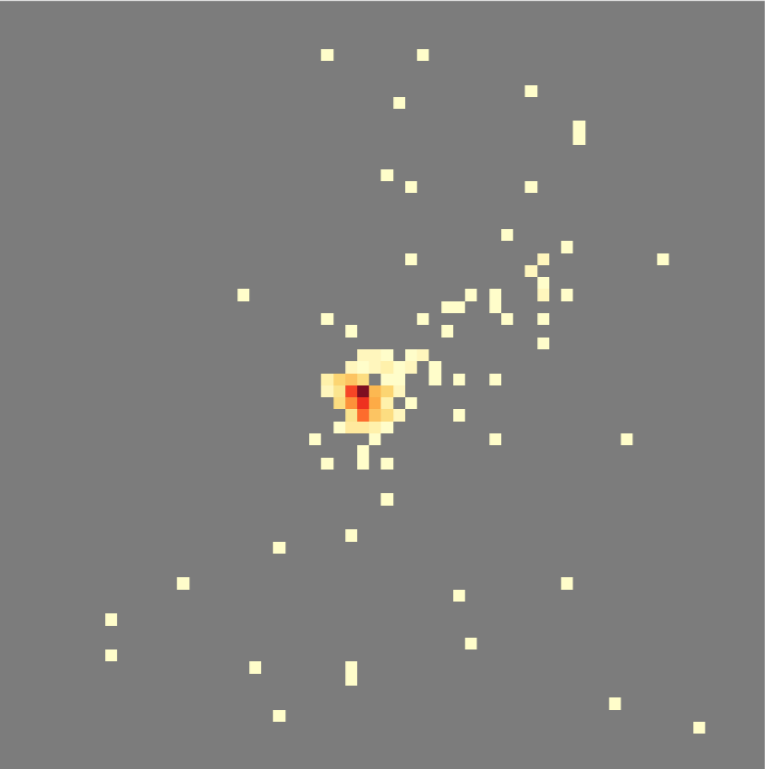
DROPPING POINT



DROPPING POINT – BOUNDARY ESTIMATE



RESULTS



Joint Model

$$P_{\mathcal{J}}(\tilde{\lambda}, z|Y) \propto f(Y|\tilde{\lambda}, z)\pi_{\mathcal{J}}(\tilde{\lambda}, z)$$

Multi-Step Process

$$\begin{aligned} P_{\mathcal{S}}(\tilde{\lambda}, z|Y) &= P_{\mathcal{S}_1}(\tilde{\lambda}|Y)P_{\mathcal{S}_2}(z|\tilde{\lambda}) \\ &\propto f(Y|\tilde{\lambda})\pi_{\mathcal{S}_1}(\tilde{\lambda})\frac{P_{\mathcal{S}_2}(\tilde{\lambda}|z)\pi_{\mathcal{S}_2}(z)}{P_{\mathcal{S}_2}(\tilde{\lambda})} \end{aligned}$$

REFERENCES

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