# AXAF Science Center

**Harvard–Smithsonian**
**Center for Astrophysics**

## MEMORANDUM

October 7, 1996

File: /proj/asc/MP/DOCUMENTS/ssa/fom.with.roll.tex

**To:** ASC Aspect Systems, Mission Planning Groups

**From:** M. Garcia, P. Green

**Subject:** Incorporating Roll Error in the SSA FOM

The Figure of Merit (FOM) used in the Star Selection Algorithm (SSA) is meant to approximate the expected error in the measurement of location of the pointing (X) axis; equivalently a measure in the expected error in measurement the spacecraft pitch and yaw. The SSA selects star sets with low FOM values, because minimizing the error in the pointing direction is the goal of solving aspect. The form of the FOM used in the SSA (V1.1, V1.2) was derived by Tom Markert (Memo of 15 July 1994) and is a generalization of the form derived by Murray and VanSpeybroeck (1976) for the case of distinct position errors for individual stars.

Bill Davis of CSC points out that this form leaves out the term which measures the error in the roll about the X-axis. Because of this, the FOM is not sensitive to the spread of stars from the center, *for star sets with mean position near the origin.* For example, the upper three star sets in the figure have identical FOM using the SSA V1.2 FOM.

Bill has independently re-derived the form of the FOM Tom derived, and also generalized the Murray and VanSpeybroeck roll error measurement for the case of distinct position errors. Using the notation of the SSA, the roll error (variance) is:

$$\sigma_{roll}^2 = \frac{1}{\sum_i (1/\sigma_i^2)} \left\{ \frac{1}{((\bar{\xi^2} + \bar{\eta^2}) - ((\bar{\xi})^2 + (\bar{\eta})^2))} \right\}$$

Recall that the X-axis variance is

$$\sigma_x^2 = \frac{1}{\sum_i (1/\sigma_i^2)} \left\{ 2 + \frac{(\bar{\xi})^2 + (\bar{\eta})^2}{((\bar{\xi^2} + \bar{\eta^2}) - ((\bar{\xi})^2 + (\bar{\eta})^2))} \right\}$$

where $(\xi_i, \eta_i)$ is the position of star $i$ in CCD coordinates and the $\bar{\xi}$ and $\bar{\eta}$ are the weighted means (weighted by the uncertainty $\sigma_i^2$)

$$\bar{\xi} = \sum_i \frac{\xi_i}{s_i^2}$$

$$\bar{\eta} = \sum_i \frac{\eta_i}{s_i^2}$$

and where $s_i^2 = \sigma_i^2 / \sum_j (1/\sigma_j^2)$.
where the $\bar{\xi}$ and $\bar{\eta}$ are as defined above and the $\bar{\xi^2}$ and $\bar{\eta^2}$ are defined similarly, *i.e.*,

$$\bar{\xi^2} = \sum_i \frac{\xi_i^2}{s_i^2}$$

$$\bar{\eta^2} = \sum_i \frac{\eta_i^2}{s_i^2}$$

Errors in pitch and yaw translate directly into errors in the ACA (and SI) focal plane, ie, a $5''$ motion in pitch moves a star 1 pixel on the ACA. Errors in roll do not; a $5''$ roll motion about the X-axis moves a star at the edge of the ACA ($1^o$ off axis) only 0.02 pixel.

We propose a new FOM which combines the expected error in the X-axis and the roll. Clearly the roll error must be scaled by some appropriate lever arm, which will be a new parameter in the SSA (ODB), called "ssa.roll.lever.arm", in units of arcmin.

The appropriate value for the level arm depends upon the nature of the science observation, and in theory could be different for each observation. In practice, the HRMA PSF becomes larger than the roll error at some moderate off axis angle, thereby limiting the maximum value for the lever arm. We suggest that a reasonable level arm might be $5'$, based on Figure 6.10 of the SIN; at this radius the HRMA FWHM is $\sim 3.5''$.

Note that the units of the roll variance $\sigma_{roll}^2$ are radians$^2$ (length/length)$^2$, while the units of the X-axis variance is length$^2$. In order to add the two we convert the roll variance to units of length$^2$. Using length units of ACA pixels (at $5''$/pix) we define a new FOM, equal to the total variance in pitch,yaw, and roll

$$FOM = \sigma_t^2 = \sigma_x^2 + \sigma_{roll.x}^2$$

with

$$\sigma_{roll.x} = units.scale * (ssa.roll.lever.arm/5') * \sigma_{roll}$$

Note that the units for $\sigma_x$ and $\sigma_{roll.x}$ MUST match, therefore we have added the term "units.scale". If $\sigma_x$ is in units of pixels, then $units.scale = 60\ pix$ for the $5'$ lever arm. If the X-axis error $\sigma_x$ and the roll error $\sigma_{roll}$ are both measured in radians, then the units.scale factor would be 0.00145 for the $5'$ lever arm, from

$$\begin{aligned}\sigma_{roll.x}(rad) &= ssa.roll.lever.arm(') \times [(1^o/60') * (\pi\ rad/180^o)] * sin(\sigma_{roll}(rad)) \\ &\sim 5'(1\ deg/60')(\pi\ rad/180^o) \times \sigma_{roll} \\ &= 0.00145\sigma_{roll}\end{aligned}$$

or more generally,

$$\sigma_{roll.x}(rad) = 0.00145(ssa.roll.lever.arm(arcmin)/5') \times \sigma_{roll}$$

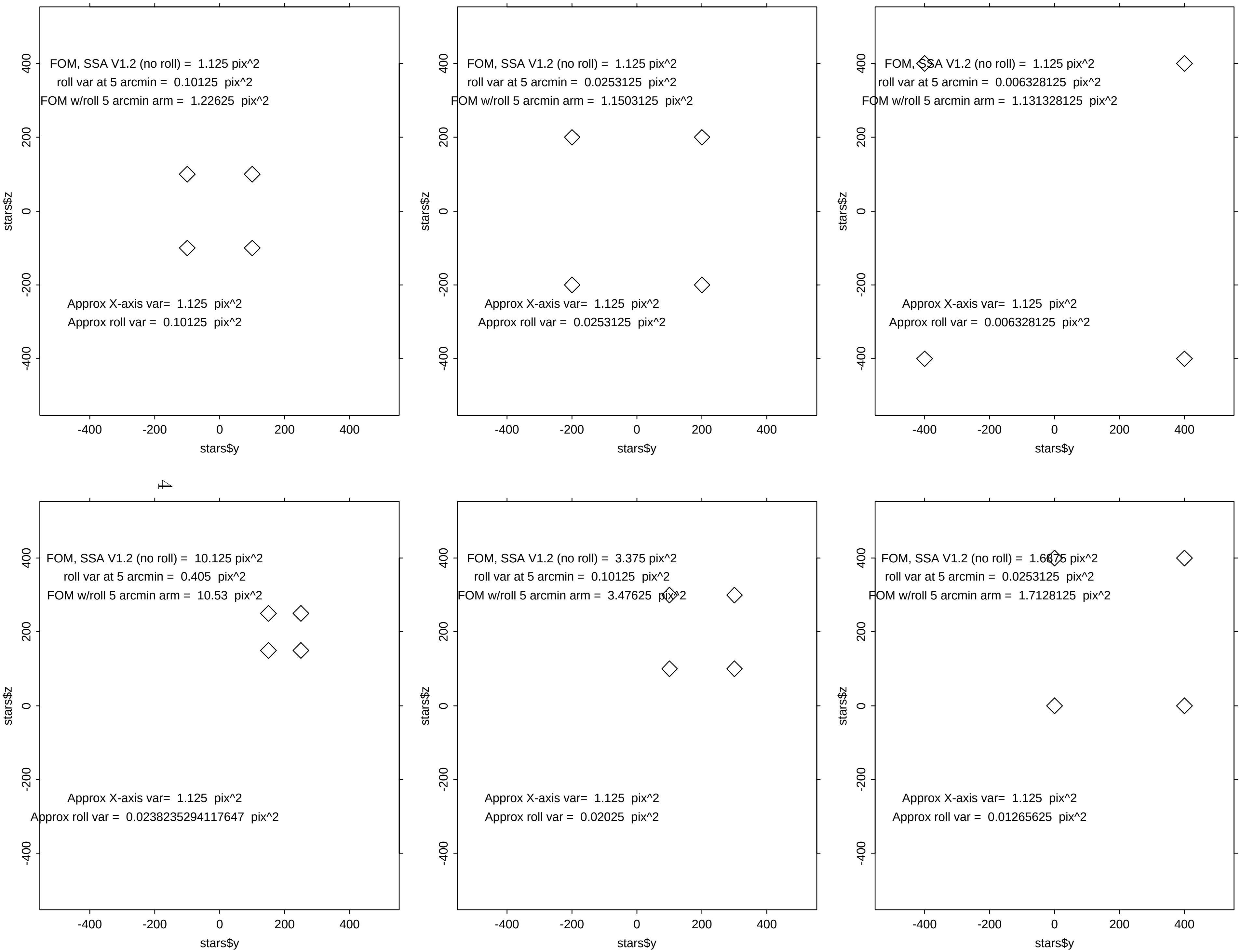
The values of this FOM are listed on Figure 1 for a few selected star sets. The position error for each star $\sigma_i = 1.5pix$. For the purposes of comparison, we also list a the quadrature sums of the individual star position and roll errors. For the X-axis variance this is

$$\frac{2}{\sum(1/\sigma_i)^2}$$

For the roll, this is

$$\sigma_{roll}^2 = \frac{1}{\sum(1/\sigma_{roll,i})^2}$$

where $\sigma_{roll,i} = \sigma_i/r_i$, $r_i$ =radial distance from center of ACA to star $i$.


FOM, SSA V1.2 (no roll) = 1.125 pix^2
roll var at 5 arcmin = 0.10125 pix^2
FOM w/roll 5 arcmin arm = 1.22625 pix^2
Approx X-axis var= 1.125 pix^2
Approx roll var = 0.10125 pix^2
FOM, SSA V1.2 (no roll) = 1.125 pix^2
roll var at 5 arcmin = 0.0253125 pix^2
FOM w/roll 5 arcmin arm = 1.1503125 pix^2
Approx X-axis var= 1.125 pix^2
Approx roll var = 0.0253125 pix^2
FOM, SSA V1.2 (no roll) = 1.125 pix^2
roll var at 5 arcmin = 0.006328125 pix^2
FOM w/roll 5 arcmin arm = 1.131328125 pix^2
Approx X-axis var= 1.125 pix^2
Approx roll var = 0.006328125 pix^2
FOM, SSA V1.2 (no roll) = 10.125 pix^2
roll var at 5 arcmin = 0.405 pix^2
FOM w/roll 5 arcmin arm = 10.53 pix^2
Approx X-axis var= 1.125 pix^2
Approx roll var = 0.0238235294117647 pix^2
FOM, SSA V1.2 (no roll) = 3.375 pix^2
roll var at 5 arcmin = 0.10125 pix^2
FOM w/roll 5 arcmin arm = 3.47625 pix^2
Approx X-axis var= 1.125 pix^2
Approx roll var = 0.02025 pix^2
FOM, SSA V1.2 (no roll) = 1.6875 pix^2
roll var at 5 arcmin = 0.0253125 pix^2
FOM w/roll 5 arcmin arm = 1.7128125 pix^2
Approx X-axis var= 1.125 pix^2
Approx roll var = 0.01265625 pix^2
stars$y
stars$z
-400 -200 0 200 400
4