The Equivalent Width for spectral lines in astronomy

Edwin Kellogg
Harvard/Smithsonian Center for Astrophysics

Historically, spectral lines were first observed in absorption. Astronomers were confronted with a spectrum of a typical star, like the Sun that had a continuous distribution in wavelength , usually measured in angstroms. An absorption line attenuates light exponentially as a function of obscuration from a continuous spectrum of emitted radiation. Therefore the observed spectrum consists of a continuous function of wavelength (or energy), in units of power per unit wavelength (or energy) with localized regions of depleted power due to the absorption lines. These absorption lines can have various functional forms, such as a lorentzian, typical of atomic emission lines, or a gaussian, typical of random doppler shifts. Here we will use lorentzian and gaussian lines as examples. To express this visually in a graph, we will assume a flat continuum, i.e. the power per unit E (rather than wavelength, since we currently deal with energy spectra in Chandra) is constant $I(E)=I_c$, and a lorentzian or gaussian absorption line at E_0 with a full width at half maximum characterized by Γ, and a σ for the gaussian. The relative amount of absorption is represented by a normalized thickness of absorber t, so t = relative optical depth at line core. Below we express these in equation form and graph them. We also illustrate the Equivalent Width. The EW is defined as the width of continuum with the same power (= integral of intensity over spectral range) as the line (see Ridpath, Ian, "Oxford Dictionary of Astronomy" 2003, Oxford U. Press and Mitton, Jacqueline, "Cambridge Dictionary of Astronomy" 2001, Cambr. U. Press. See the end of this treatise for an explanation of the unrelated definition of EW by Lang.

Absorption line case

Lorentzian example

The apparent width increases quickly with saturation. If we imagine a photographic plate showing contrast at 0.6, the apparent width at 0.6 ranges from < 50 eV for a weak line to ~1200 eV for a saturated one in the plots below. Thus, the apparent width of different lines on a plate of the same Γ was a measure of their strength. This has been canonized as Equivalent Width. Below, we calculate and plot the line shape for six values of t and plot them semilog and linear. The Lorentzian function of energy with FWHM Γ is L(E) (σ is indeterminate for a Lorentzian):

$$L(E) = \frac{1}{\pi}\cdot\frac{\frac{1}{2}\Gamma}{\left(E-E_0\right)^2+\left(\frac{1}{2}\Gamma\right)^2} \qquad \int_{-\infty}^{\infty} L(E)\,dE = 1 \qquad L\left(E_0\right) = \frac{2}{\pi\,\Gamma}$$

See **Weissenstein** "Lorentzian Function" www.mathworld.wolfram.com/LorentzianFunction.html

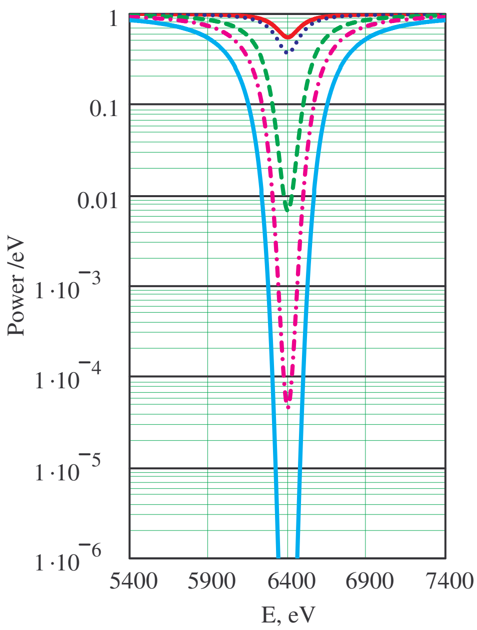
We use:

$$\frac{\pi\cdot\Gamma}{2}\cdot L\left(E_0\right) = 1 = \frac{\Gamma^2}{4\left[\left(E-E_0\right)^2+\left(\frac{\Gamma}{2}\right)^2\right]}$$

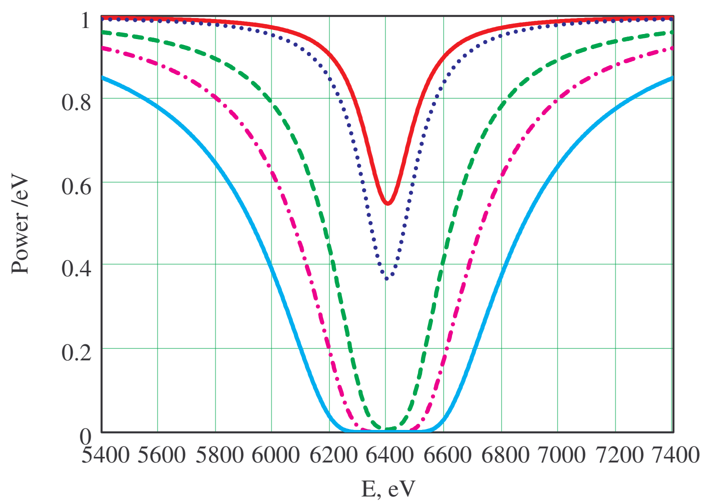
so the bottom of the absorption line has a depth of e^{-t}.

$I_c = \frac{1}{eV}$ $E_0 = 6400$ eV $\Gamma = 180$ eV

Line strength $I2(E,t,E_0,\Gamma,I_c) = I_c \cdot \exp\left[\frac{-t\cdot\Gamma^2}{4\left[(E-E_0)^2 + \left(\frac{\Gamma}{2}\right)^2\right]}\right]$



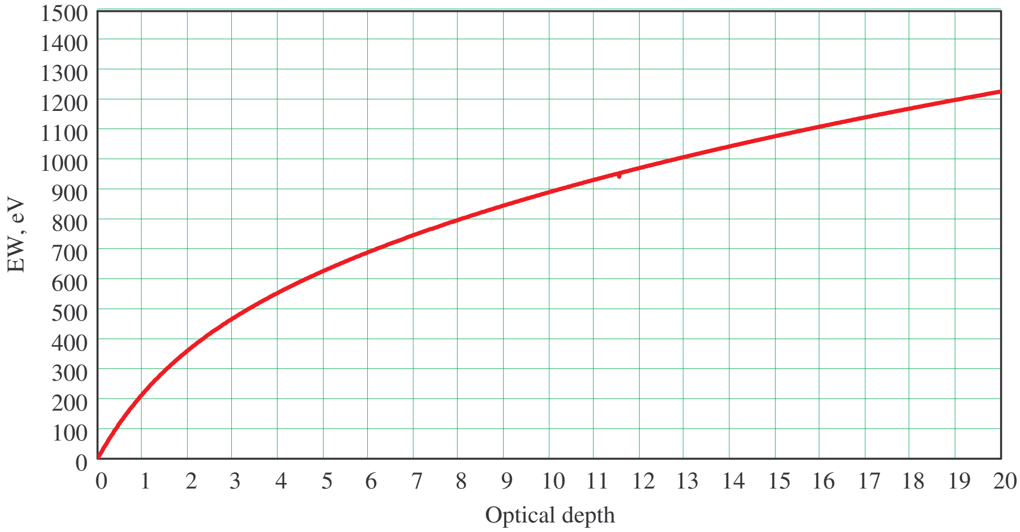
Optical depths are 0.6, 1, 5, 10, 20.



Equivalent Width: we define the EW as the difference between the area in a chunk of continuum 20Γ wide and the area in the function plotted above integrated over the same 20Γ. We choose a range of 20Γ to capture the wings of the Lorentzian. This gives the width of continuum equivalent to the area of the absorption line.

$$EW(t) = \left[20\cdot\Gamma\cdot I_c - \left(\int_{E_0-10\cdot\Gamma}^{E_0+10\cdot\Gamma} I2\left(E,t,E_0,\Gamma,\frac{1}{eV}\right) dE\right)\right]$$

The plot below gives EW as a function of t, the optical depth (a function of the strength of the line).



The EW increases with optical depth, i.e. strength of the absorber, at constant Γ !

Gaussian example

Here, the apparent width at 0.6 varies from ~80 eV for a weak line to ~ 400 eV for a saturated line, with little variation in Equivalent Width at high saturation. We use the form of a gaussian that has a height of unity at E_0.

$$I2(E,t) = I_c \cdot \exp\left[(-t)\cdot \exp\left[\frac{-(E-E_0)^2}{2\cdot\sigma^2}\right]\right] \qquad \sigma = \frac{\Gamma}{2\cdot\sqrt{2\ln(2)}}$$

See **Weissenstein**
www.mathworld.wolfram.com/GaussianFunctio-
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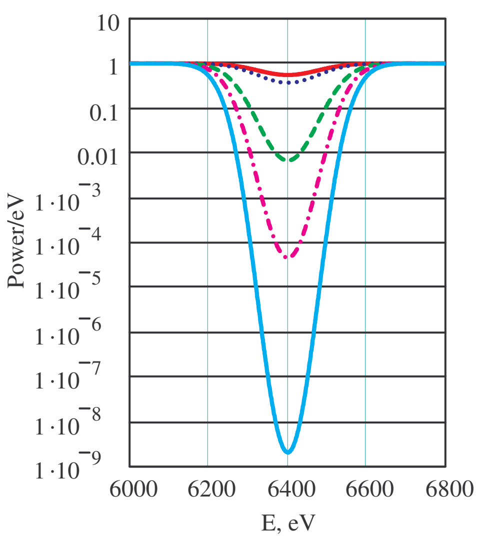
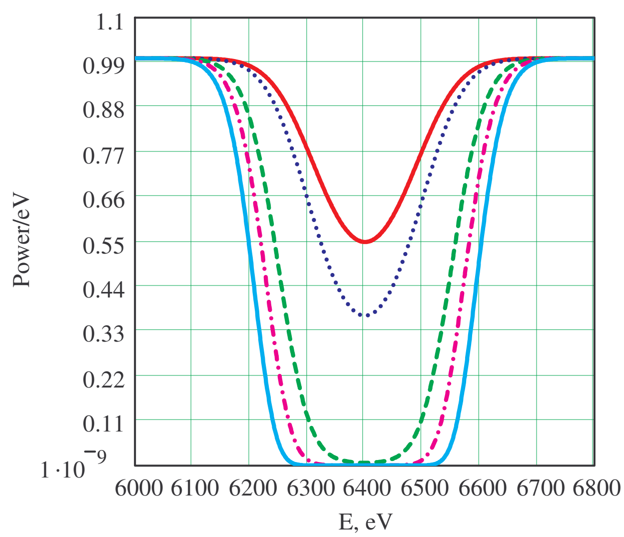
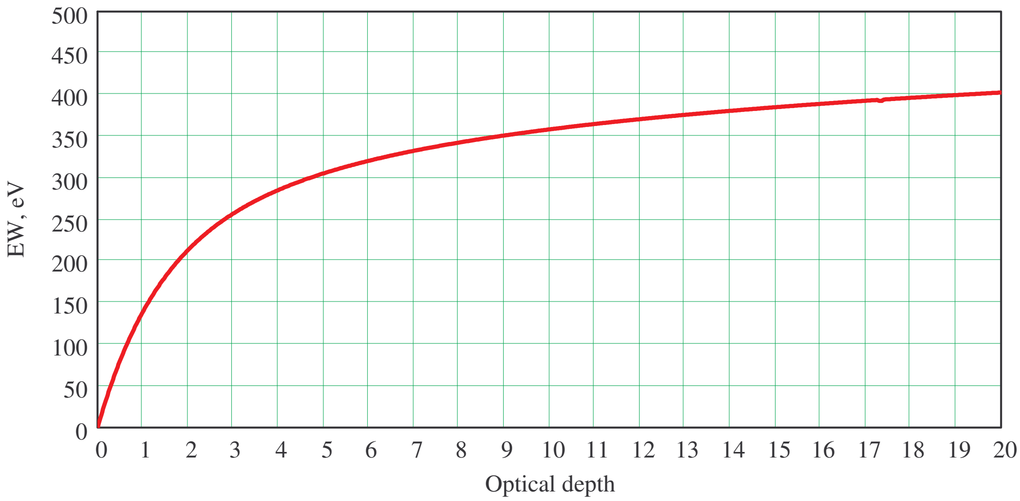




Equivalent Width: $$EW(t) = \left[20\cdot\Gamma\cdot I_c - \left(\int_{E_0-10\cdot\Gamma}^{E_0+10\cdot\Gamma} I2(E,t)\,dE\right)\right]$$



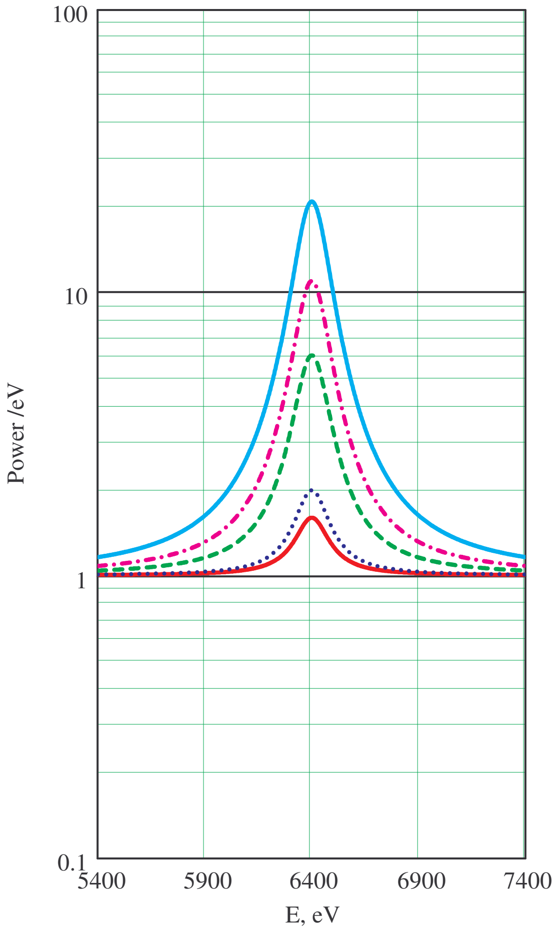
The EW again increases with optical depth, i.e. strength of the absorber, at constant Γ (or σ, if you prefer for a gaussian, since they are related). However, the EW does not increase as rapidly at large optical depth as the Lorentzian does.

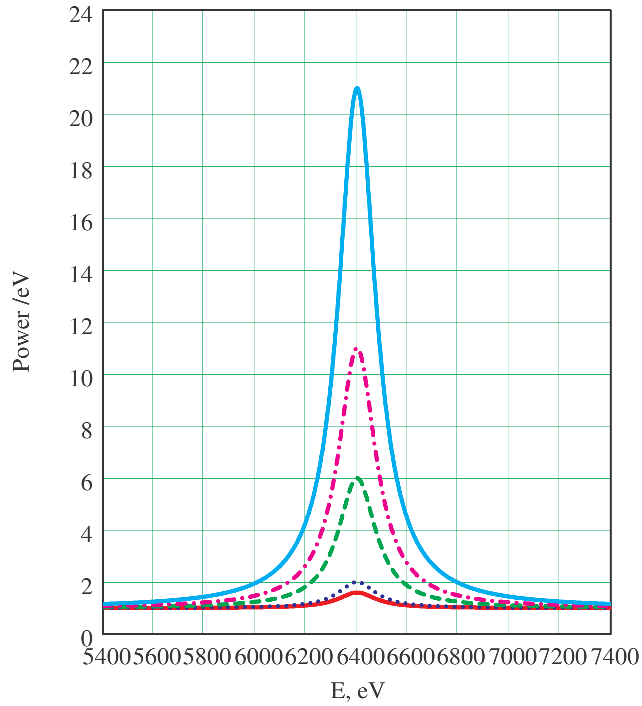
Emission line case

Lorentzian example:

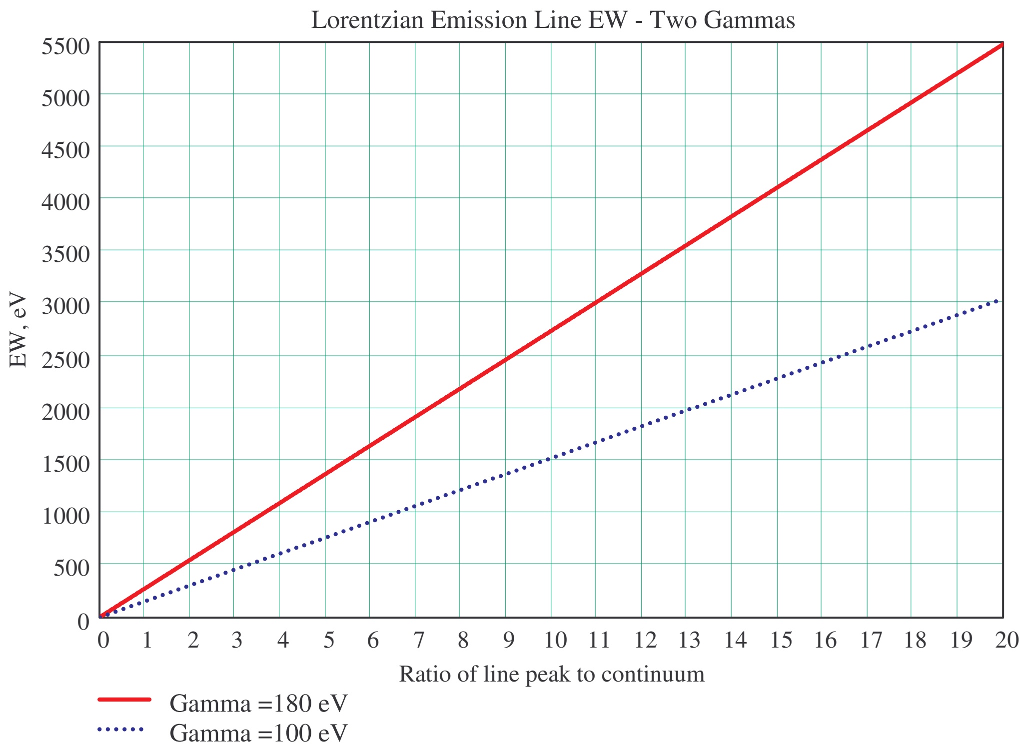
$$I2(E, I_l, E_0, \Gamma, I_c) = I_c + \frac{I_l}{eV}\cdot\frac{\Gamma^2}{4\left[(E-E_0)^2 + \left(\frac{\Gamma}{2}\right)^2\right]}$$

I_l/I_c = 0.6, 1, 5, 10, 20 in ratio of powers.



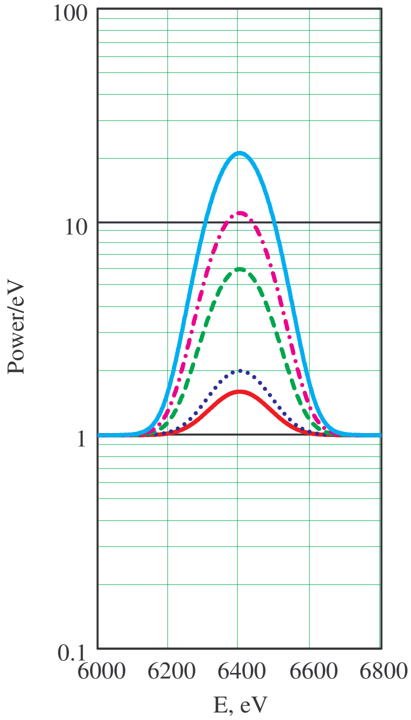


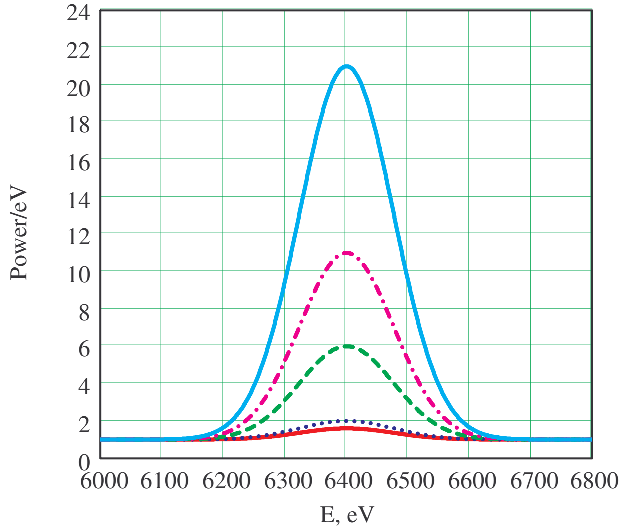
$$EW\left(I_l, \Gamma\right) = \left[\left(\int_{E_0-10\cdot\Gamma}^{E_0+10\cdot\Gamma} I2\left(E, I_l, E_0, \Gamma, \frac{1}{eV}\right) dE\right) - 20\cdot\Gamma\cdot I_c\right]$$



Gaussian example:

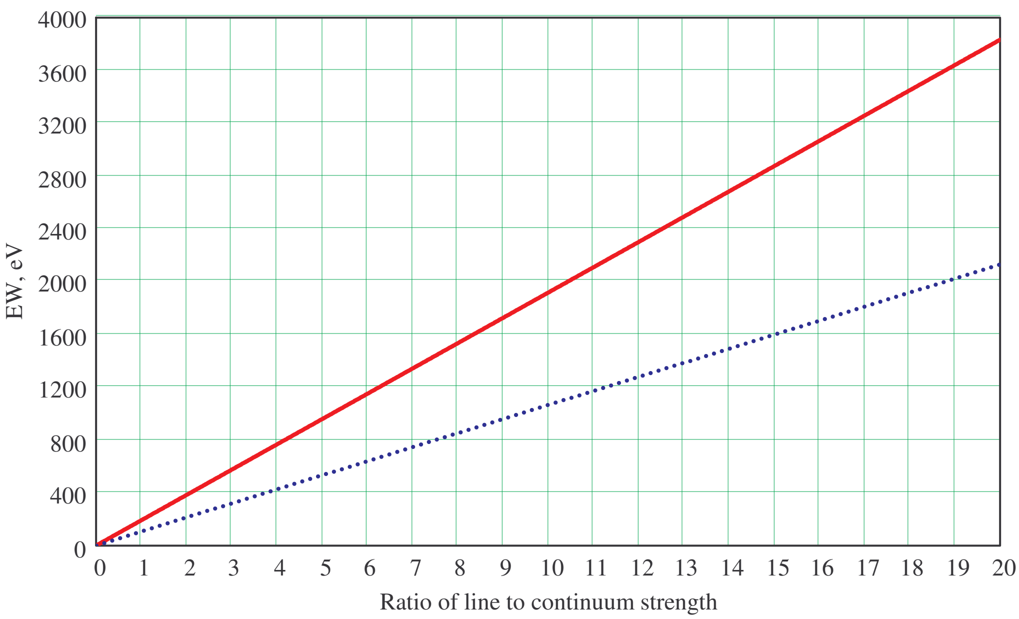
$$I2\left(E,\Gamma,I_l\right) = I_c + \frac{I_l}{eV}\cdot\exp\left[\frac{-\left(E-E_0\right)^2}{2\cdot\left(\frac{\Gamma}{2.3548}\right)^2}\right]$$





Equivalent Width:

$$EW(\Gamma, I_l) = \left(\left(\int_{E_0-10\cdot\Gamma}^{E_0+10\cdot\Gamma} I2(E,\Gamma,I_l)\,dE\right)\right) - 20\cdot\Gamma\cdot I_c$$



Lang's unrelated definition of EW

To clear up some confusion, Kenneth Lang, in "Astrophysical Formulae", 2 ed. p. 180 (and 3 ed. vol. I of II, p.174), gives a definition of the "equivalent width of a *function*" (Italics are mine). This by itself is not relevant to the analysis of astrophysical spectra. His EW definition is "the area of a function divided by its central ordinate." This definition cannot compare a line shape function with continuum. Lang goes on to derive that EW = 2.5066 σ. This is easy to show as follows. First, the normalized gaussian, whose integrated area = 1 is

$$g(E) = \frac{1}{\sigma\cdot\sqrt{2\pi}}\cdot\exp\left[\frac{-(E-E_0)^2}{2\cdot\sigma^2}\right] .$$

See **Weissenstein**
www.mathworld.wolfram.com/GaussianFunction.html

Therefore, the central (maximum) ordinate, or height of g(x) is $g(E_0)$:

$$g(E_0) = \frac{1}{\sigma\cdot\sqrt{2\pi}} .$$

Since the area of this gaussian is 1, his EW is

$$EW = \frac{1}{g(E_0)} \qquad EW = \sigma\cdot\sqrt{2\pi} \quad \text{and} \quad \sqrt{2\pi} = 2.5066 \quad \text{so} \quad EW = 2.5066\cdot\sigma$$

However, this is completely unrelated to our problem. It makes no reference to another function, the continuum, with which we are trying to compare.

Reference:C:\Documents and Settings\emk\My Documents\mathcad\UNITS.MCD