Inference: A Python Package for Astrostatistics A NASA AISR Project

Tom Loredo, Alanna Connors, Travis Oliphant

Cornell/Eureka Scientific/BYU

Motivation

Many advanced methods are conceptually simple but computationally complex.

Competing methods of very different levels of sophistication are often similar from end-user's perspective.

Principal obstacle to understanding/use is the *art of statistical computing*.

Eliminate this obstacle!

The Inference Project

- The Inference package
 - Library: Deep and broad collection of self-contained tools tailored to astronomers' needs; *multiple methods*
 - Parametric Inference Engine: Framework for parametric modeling; multiple methodologies (χ², likelihood, Bayes) with unified interface
- Use of a modern VHL language: Python
 - Single implementation facilitates depth/breadth
 - VHL features speed development, facilitate testing
 - Easy access for new users and PyRAF users
- Outreach
 - Astrostatistics speakers and sessions at conferences
 - Selected methods from sessions in the package

A Bit About Python

- A general purpose language with a rich standard library
- Sophisticated and fast scientific computing capability
- Simple syntax—resembles "pseudo code," Matlab/IDL
- Use interactively, or via scripts/modules
- Object oriented, with a very simple object model—facilitates high level interfaces, modularity
- Practical rather than "pure"—Selected capabilities of various paradigms (e.g., functional programming, list comprehensions, metaclasses)
- Easily extendible/embeddable with C/C++/Fortran
- Open source, cross-platform, active & growing user community

Scientific Computing With Python

- Array computations
 - Syntax inspired by Matlab/IDL/Fortran90
 - Performance near that of C/Fortran
 - Numeric: Developed by LLNL/MIT scientists
 - numarray: Numeric's successor from STScl
- PyRAF The IRAF command line in Python (STScI)
- SciPy (partly supported by Enthought, NASA)— special functions, linear algebra, FFTs, DSP, quadrature, ODE solvers, optimizers, basic stats
- Plotting: matplotlib, Chaco, various libraries

Simple Example: The Rayleigh Statistic

Search for periodic signals in arrival time series, $\{t_i\}$.

$$R(\omega) = \frac{1}{N} \left[\left(\sum_{i} \sin \omega t_{i} \right)^{2} + \left(\sum_{i} \cos \omega t_{i} \right)^{2} \right]$$

Sample Source Code

```
Python source code
                                   C source code
from Numeric import *
                                   #include <math.h>
def Rayleigh (data, w):
                                  double Rayleigh (int n, double *data,
    wd = w^*data
                                                     double w) {
    return (sum(sin(wd))**2 +
                                    double S, C, wt;
         sum(cos(wd))**2)/len(data) int i;
                                       S = 0.i
                                       C = 0.;
                                       for (i=0; i<n; i++) {</pre>
                                           wt = w*data[i];
                                           S += sin(wt);
                                           C += cos(wt);
                                       }
                                       return (S*S + C*C)/n;
                                   }
```

Library: Tools for Continuous Data

Sampled functions with additive noise, $d_i = f(t_i) + e_i$

- Linear & nonlinear regression: Bershady/Isobe packages, Bayesian EVM, correlated errors
- Detection/measurement of periodic signals
 - Standard approaches: Power spectrum, Schuster periodogram, Lomb-Scargle
 - Bretthorst algorithm (Bayesian periodograms); Kepler periodogram
 - Bayesian piecewise-constant modeling (Gregory method)
- Nonperiodic time series analysis: ARMA, BB, long-mem.
- Robust estimation/outlier detection

Library: Tools for Discrete Data

- Intervals and limits for rates and ratios from counting: Feldman-Cousins likelihood ordering, Bayes, ABC, profile likelihood
- Periodic point processes (period searching in arrival time data): Rayleigh statistic, Z²_N, Bayes log-Fourier models, Gregory-Loredo, adaptive 1-d grid, accelerated (P, P) searching, fractional transforms
- Inhomogeneous point process models for local event detection: Bayes blocks, Poisson "Haar" wavelets
- Survey analyses: Survival analysis (ASURV), Bayes point process + noise
- Nonparametric methods: Adaptive splines, neural nets (interfaces to Max Planck PPI methods), mixture models

Parametric Inference Engine

- Three methodologies: χ^2 , likelihood, Bayes
- Data types: Point samples, binned, folded; on/off; surveys
- Automate standard parameter exploration tasks
 - Exploration on equispaced & logarithmic grids
 - Optimization (unconstrained and with boundary constraints)
 - Exploration of subsets of parameter space (profiling/projection)
 - Hessian/information matrix calculation
- Bayesian computation
 - Marginalization and Bayes factors via adaptive quadrature & Laplace approximation
 - Calculation of 1-d, 2-d, 3-d credible region boundaries
 - Basic Markov chain Monte Carlo (MCMC) support
- Simulate data

Build a model:

class PowerLawModel(ParametricModel):

```
A = RealParameter(1., 'Amplitude')
```

```
alpha = RealParameter(range=(-5,-1), 'Index')
```

```
def signal(self,E):
    return self.A*E**(self.alpha)
```

Associate data with predictor:

- p1 = SampledChisqrPred(data1)
- p2 = BinnedChisqrPred(data2)

Make inferences:

inf = ChisqrInference(PowerLawModel, p1, p2)

```
inf.A.logStep(1., 10., 50)
```

```
inf.alpha.vary()
```

```
grid = inf.opt()
```

Returns a grid object w/ projected $\chi^2(A)$