Change Point Detection for Poisson Time Series Images with Applications to Astronomy and Astrophysics

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Outline

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Problem Description

- Abrupt changes of intensities, energy spectra and spatial patterns in astronomical sources are of interest to astrophysics.
- E.g., Chandra observations of the Orion Nebula cluster.
- Goal: detect change points in the time direction.

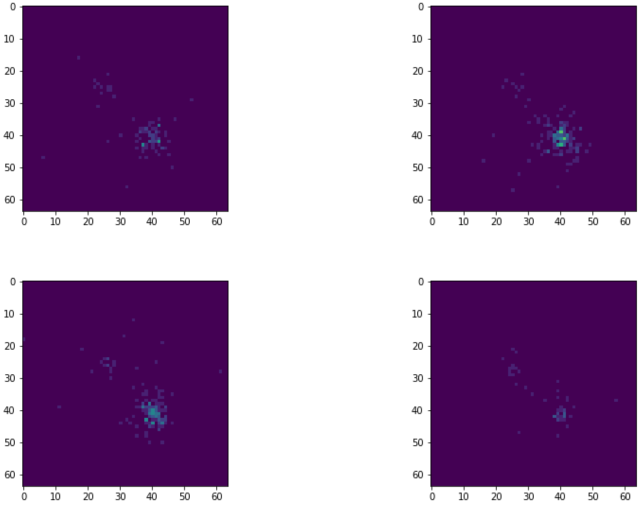
Data

- Data are usually obtained in the form of a list of photons, with four attributes (x, y, t, w):
  - the 2D spatial coordinates (x, y) where the photons were detected
  - the times t they were recorded
  - wavelengths w (i.e., energies)
- We first focus on the spatial and the time attributes of the photons.
- The case for spectral and time has been done - AutoMARK.

Binning

- We first bin the original data into a 3D rectangular grid of equi-volumned boxes (i.e., voxels)
- Thus a 3D table of photon counts indexed by (x, y) and t.
- Can be viewed as a time series of images with photon counts as pixel values.

Binned Data: at four time points



Assumptions: Poisson

- Emission of photons follows a non-homogeneous Poisson process.
- So Poisson counts are independent.
- Observed images $\{y_{i,t}\}, i = 1, ..., n, t = 1, ..., T$ satisfy

$$y_{i,t} \sim \text{Poisson}(g_{i,t}),$$

  where
  - n is the number of pixels in one image,
  - T is number of images,
  - $g_{i,t}$: unknown true value at location i and time t.
  - (i is bivariate: x and y)

Assumptions: Homogeneous Periods

- The T images can be partitioned into $(K + 1)$ homogeneous periods by K change points $\tau_1, \tau_2, ..., \tau_K$.
- (Let $\tau_0 = 0, \tau_{K+1} = T$.)
- Images from the same period are assumed to have the same unknown $g_{i,t}$.

Assumptions: Piecewise Constant for $g_{i,t}$

- Model g with a 2D piecewise constant function:

$$g_{i,t} = \sum_{k=1}^{K+1} I_{\{t \in (\tau_{k-1}, \tau_k]\}} \sum_{\gamma=1}^{m^{(k)}} f_\gamma^{(k)} I_{\{i \in R_\gamma^{(k)}\}}, \quad \forall i, t.$$

- $f_\gamma^{(k)}$: unknown Poisson parameter for the γth region of the kth period.
- $R_\gamma^{(k)}$: index set for the γth region in the kth period.
- Note: $R_\gamma^{(k)} \subseteq \{1, ..., n\}$.
- $m^{(k)}$: number of regions in the image specified by $R_\gamma^{(k)}$

As a Model Selection Problem

- Given data $\{y_{i,t}\}$ such that

$$y_{i,t} \sim \text{Poisson}(g_{i,t}) \quad \forall i, t,$$

  we want to obtain an estimate of $g_{i,t}$.
- In other words, we want to estimate
  - number K and locations of change points τ_k's
  - segmentations (object boundaries) of the images $R_\gamma^{(k)}$
  - Poisson parameters $f_\gamma^{(k)}$.
- The estimation is a model selection problem.
- We will use the minimum description length (MDL) principle.

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What is MDL?

- MDL defines the best model as the one that produces the best compression (minimization of the code length) of the data.
- Code length (or description length): amount of hardware memory to store an object

$$\begin{aligned} \text{CL(data)} &= \text{CL(fitted model)} + \text{CL(data given fitted model)} \\ &= \text{CL(fitted model)} + \text{CL(residuals)} \\ &= \text{CL(explained by model)} + \text{CL(not explained by model)} \end{aligned}$$

  - First term: model complexity
  - Second term: data fidelity (essentially log-likelihood)
- The best fitting model is defined as the minimizer of CL(data).
- Why does MDL work as a model selection method?

MDL for Homogeneous Poisson Image Series

- That is, no change point and all T images share the same true 2D piecewise constant function.
- Our model reduces to, $\forall i$ and t:

$$
\begin{aligned}
y_{i,t} &\sim \text{Poisson}(g_{i,t}) \\
g_{i,t} &= \sum_{\gamma=1}^{m} f_\gamma I_{\{i \in R_\gamma\}}
\end{aligned}
$$

MDL for Homogeneous Poisson Image Series

- m: number of regions
- $R = \{R_\gamma | \gamma = 1, ..., m\}$ as the partition (i.e., object boundaries)

$$\mathrm{MDL}(m, R) = m \log(n) + \frac{\log(3)}{2} \sum_{\gamma=1}^{m} b_\gamma + \frac{1}{2} \sum_{\gamma=1}^{m} \log(T a_\gamma) - \sum_{\gamma=1}^{m} \sum_{t=1}^{T} \sum_{i \in R_\gamma} y_{i,t} \log(\hat{f}_\gamma^{(T)})$$

- a_γ: the "area" (number of pixels) of region R_γ
- b_γ: the "perimeter" (number of pixel edges)
- $\hat{f}_\gamma^{(T)}$: the sample mean of region R_γ when (m, R) are given, which is

$$\hat{f}_\gamma^{(T)} = \frac{1}{T a_\gamma} \sum_{t=1}^{T} \sum_{i \in R_\gamma} y_{i,t}$$

General Case: Change Point Detection

- Unknown K change points $\tau_1, \tau_2, ..., \tau_K$. Let $\tau_0 = 0, \tau_{K+1} = T$.

$$
\begin{aligned}
y_{i,t} &\sim \text{Poisson}(g_{i,t}) \\
g_{i,t} &= \sum_{k=1}^{K+1} I_{\{t \in (\tau_{k-1}, \tau_k]\}} \sum_{\gamma=1}^{m^{(k)}} f_\gamma^{(k)} I_{\{i \in R_\gamma^{(k)}\}}
\end{aligned}
$$

- Let $\lambda_j = \tau_j / T$ for $j = 0, 1, ..., K+1$.
- $\lambda = (\lambda_0, ..., \lambda_{K+1})$.
- Numbers of regions within each period: $m = (m^{(1)}, ..., m^{(K+1)})$.
- Object boundaries for all periods: $R = (R^{(1)}, ..., R^{(K+1)})$, where $R^{(k)} = \{R_\gamma^{(k)} | \gamma = 1, ..., m^{(k)}\}$ for $k = 1, ..., K+1$.

General case: Change Point Detection

The overall MDL is

$$\mathrm{MDL}_{\mathrm{overall}}(K, \lambda, m, R) =$$
$$K \log(T) + \sum_{k=1}^{K+1} \mathrm{MDL}(m^{(k)}, R^{(k)}, \lambda_{k-1}, \lambda_k, \hat{f}(R^{(k)}, \lambda_{k-1}, \lambda_k)),$$

where $\mathrm{MDL}(m^{(k)}, R^{(k)}, \lambda_{k-1}, \lambda_k, \hat{f}(R^{(k)}, \lambda_{k-1}, \lambda_k))$ is the homogeneous MDL for images from $(T\lambda_{k-1} + 1)$ to $T\lambda_k$.

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Consistency in Homogeneous Case

Theorem (Homogeneous case; i.e., no change point)

As $T \to \infty$,

$$\hat{m} \to m^0 \quad and \quad \hat{R} \to R^0,$$

where m^0 *and* R^0 *are the corresponding true values.*

Consistency in General Case

Theorem (General case with change points)

As $T \to \infty$

$$\hat{K} \to K^0, \quad \hat{\lambda}_k \to \lambda_k^0 \quad a.\ s., \quad \hat{m}^{(k)} \to m^{0(k)} \quad and \quad \hat{R}^{(k)} \to R^{0(k)},$$

where K^0, λ_k^0, $m^{0(k)}$ *and* $R^{0(k)}$ *are the corresponding true values.*

To the best of our knowledge, this is the first consistency result for simultaneous change point detection and segmentation for time series of images.

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Optimization: Very Challenging

- Global minimization of $\text{MDL}_{\text{overall}}(K, \lambda, m, R)$ is practically infeasible for not-so-small T and n.
- Two stages:
  1. change points detection
  2. image segmentation
- techniques involved:
  - seeded region growing
  - region merging
  - forward selection/backward elimination
  - some heuristics, and others ...
- still a bit slow, needs improvement (later)

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Simulation

To make the setup more realistic:

- Fit a model to a real dataset with the algorithm described before.
- Generate datasets based on the fitted model.
- Apply the algorithm to the generated datasets

Simulation

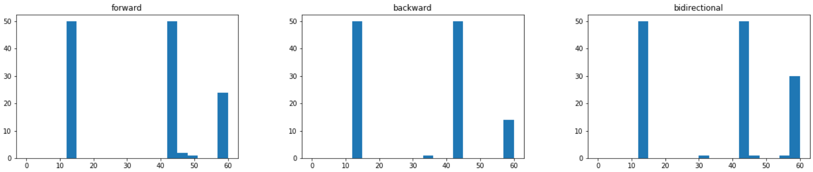
Real data:

- ObsID: 04373_1
- $n = 64 \times 64$
- $T = 60$

Fitted model:

- Change points: 12, 42, 57

Simulation: results from 3 algorithms



Simulation: further results

Method	% correct $\hat{K}$	% exactly the same
Forward	54%	48%
Backward	30%	28%
Bidirectional	58%	56%

Outline

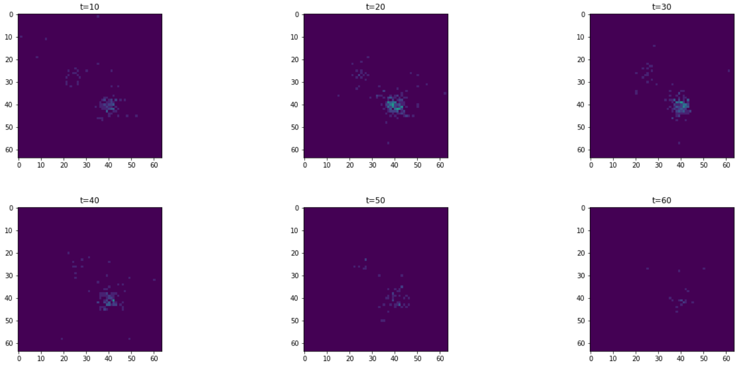
1. Background and Modeling
2. The Proposed Method
3. Theoretical Properties
4. Optimization Algorithm
5. **Practical Performance**
6. Concluding Remarks

Application to real data

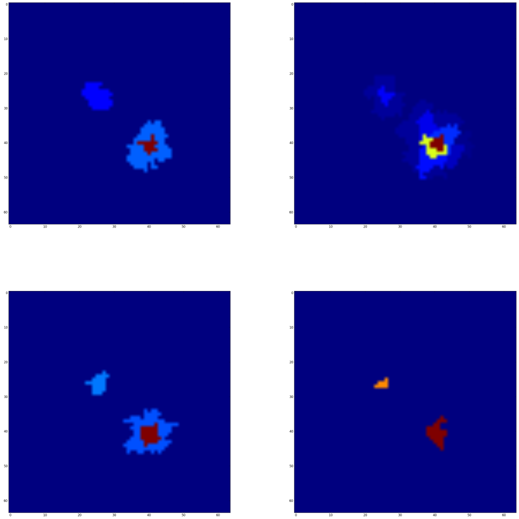
Real data:

- ObsID: 04373_1
- $n = 64 \times 64$
- $T = 60$

Application to real data



- Change points: 12, 42, 57



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Concluding Remarks

Past:

- developed a method for simultaneous change point detection and segmentation for time series of images
- to the best of our knowledge, first time consistency results are established

Present:

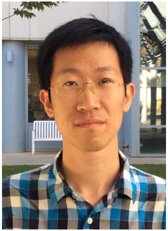
- extension to exponential family (done)
- incorporate spectral information; i.e., include w which leads to a 4D version of the problem (done)
- fused lasso to speed up the algorithm (working on it)

Future:

- relax the piecewise constant assumption; i.e., piecewise polynomial
- successful astro-X applications!

Thank You!

- Thank you!
- collaborators:



Cong Xu

Vinay Kashyap

you?