Joint Spectral-Temporal Analysis of High-Energy Astronomical Sources

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we deal with high-spectral/high-temporal resolution grating data

these data are obtained as lists of photons

for each photon we know

1. the time at which it was recorded \((t)\)
2. its wavelength \((\lambda)\) and hence its energy \((E)\)

one interesting question: does the distribution of energy change over time?
two typical data sets
preprocessing: binning the data

- as a first step, we “bin” the data
- i.e., lay a grid over the data and count how many points in each grid box
- Poisson counts in each grid box (or bin)
- size of grid/bin: needs to be carefully chosen
binned data sets
poisson modeling

- each bin is indexed by two quantities:
  1. $t$: time
  2. $w$: wavelength
- denote the observed counts as $C(t, w)$
- denote the brightness of a source as $\mu(t, w)$ (expected counts per unit area)

$$C(t, w) \sim \text{Poisson} \left\{ \delta t \times \delta w \times \mu(t, w) \sum_{k=1}^{K} A_k(w) \right\}$$

$K$: number of detectors; $A_k(w)$: effective area for the $k$th detector (all known)
about $\mu(t, w)$

- no simple parametric models, so do nonparametric
- which typically requires smoothness assumption
- with *emission lines*, $\mu(t, w)$ is not completely smooth
modeling of $\mu(t,w)$

- for now assume $\mu(t,w)$ is the same for all $t$
- i.e., homogeneous across time
- and model the energy spectrum $\mu(w)$
- split $\mu(w)$ into two parts: smooth part + emission lines
  1. smooth part: radial basis expansion
     (use polynomial of power 3: $1, x, x^2, x^3, |x - "knots"|^3$)
  2. emission lines: delta functions
model for $\mu(w)$

$$g(\mu(w)) = \sum_{j=1}^{P} \beta_j b_j(w) + \sum_{i=1}^{n} \eta_i I_i(w)$$

- $g$: link function as in GLM/GAM, for Poisson data
- $P$: number of basis functions, pre-specifed
- $b_j$: the $j$th basis (radial basis)
- $n$: number of bins in the $w$-direction
- $I_i$: delta function
- $\beta_j$'s and $\eta_i$'s: parameters to be estimated
- note: number of parameters $> \text{number of observations}$
parameter estimation

\[ g(\mu(w)) = \sum_{j=1}^{P} \beta_j b_j(w) + \sum_{i=1}^{n} \eta_i I_i(w) \]

- need to set some \( \beta_j \)'s and \( \eta_i \)'s to zero
- do \( L_1 \) penalty (lasso)
- given tuning parameters \( \gamma \) and \( \rho \), estimate \( \beta \) and \( \eta \) by minimizing

\[ -\log \text{likelihood} + \gamma \{ \rho |\beta|_1 + (1 - \rho) |\eta|_1 \} \]

- fast algorithms exist
selecting the tuning parameters

▶ need to choose $\gamma$ and $\rho$
▶ in classical lasso, they can be chosen say by cross-validation, AIC or BIC
▶ cross-validation: too slow
▶ AIC/BIC: cannot be blindly used here, as "$p > n$"
▶ see Chen and Chen (2008, Biometrika), where an Extended BIC criterion is proposed to handle the "$p > n$" issue
▶ we follow the idea and developed an Extended MDL criterion (Minimum Description Length)
an example (without emission lines)

raw counts

fitted
incorporating time $t$ in modeling

- the energy spectrum typically changes over time
- as a first step, we do piecewise modeling of $\mu(t, w)$
- i.e., $\mu(t, w)$ is the same between any two breakpoints:

\[
\mu(t, w) = \mu_1(w)I_{\{t_0 \leq t < t_1\}} + \mu_2(w)I_{\{t_1 \leq t < t_2\}} + \ldots + \mu_B(w)I_{\{t_{B-1} \leq t < t_B\}}
\]

- the number of breakpoints $B$, and the locations of the breakpoints $t_j$’s, are unknown
selecting $B$ and $t_j$’s

- it is a model selection problem
- MDL has been proven to be very successful in various structural break detection problems
- again, we are in the “$p > n$” scenario
- so direct application of classical MDL won’t work here
- as before, we developed an Extended MDL criterion for choosing the final model
- (essentially a penalized likelihood, with 4 penalty terms)
practical fitting

- involves a non-trivial minimization problem
- a possibility is genetic algorithms
- but slow
- we use a “tree growing” strategy
- i.e., at each time step, choose the best location for adding one breakpoint, repeat until a local minimum is found
- (we could certainly do “tree pruning”, and more)
break detection simulation 1

1 true break, with 200 repetitions

12298

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break detection simulation 2

2 true breaks, with 200 repetitions

13251

17/23
real data set 1
real data set 1

![Graph showing brightness vs wavelength with various curves and shaded regions.](image-url)
real data set 2
real data set 2
concluding remarks

- presented a method for detecting changes of energy spectrum over time
- modern regression techniques and new model selection criteria are used
- future work:
  1. better modeling in $t$
  2. theoretical properties of Extended MDL
The end.

Thank you.