

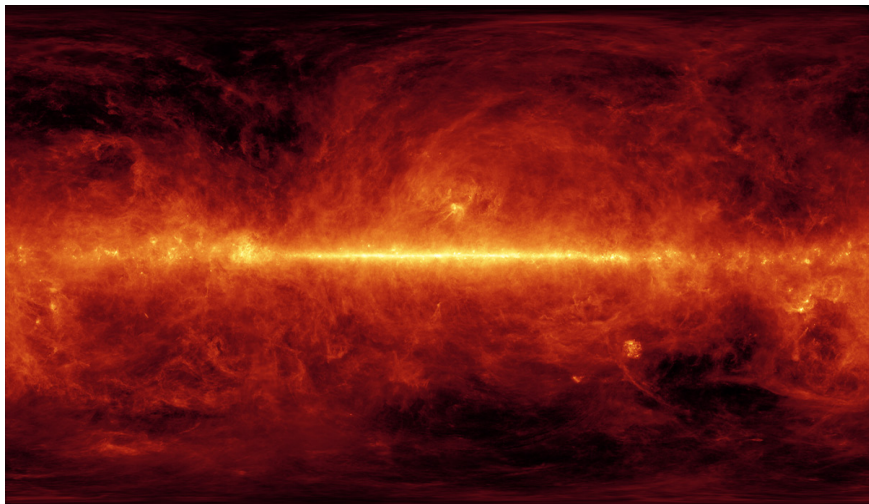
Dust Temperature and Spectral Index Correlation?

Xianchao Xie

Department of Statistics, Harvard University

Joint work with Brandon Kelly

Dust Emission



Spectral Energy Distribution Fitting

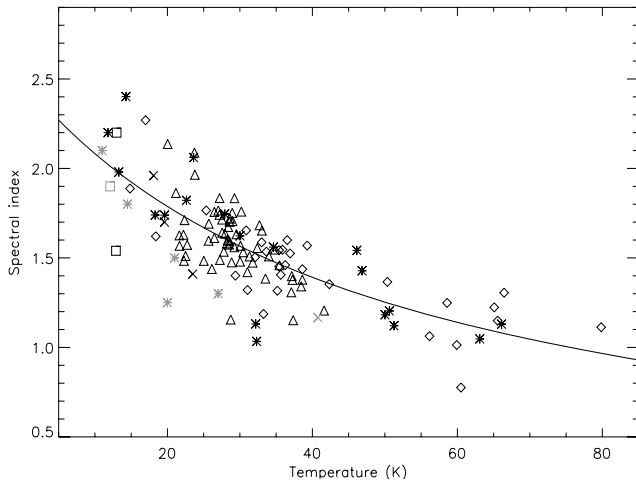
Modified Blackbody Assumption

$$S_\nu \propto C \left(\frac{\nu}{\nu_0} \right)^{\beta+3} \left(\exp \left(\frac{h\nu}{\kappa T} \right) - 1 \right)^{-1}$$

Parameter in the model: (β, T) .

Observations: $S_{\nu_1}, \dots, S_{\nu_J}$.

The Empirical Inverse Correlation



A Physical Law or A Statistical One?

A Scientific Discovery

- Similar patterns on various galactic sources.
- Confirmed in different experiments by independent groups.

A Statistical Fallacy

- There are noises in the measurements.
- Estimates of parameters with noisy data are usually correlated.
- Simulation study has suggested that this might be the reason.

Two Types of Correlation

The Thought Process

$(\beta, T) \rightarrow$ Clean “Data” \rightarrow Dirty Data $\rightarrow (\hat{\beta}(data), \hat{T}(data))$

The Statistical Correlation

$$\text{Corr}(\hat{\beta}, \hat{T})$$

The Scientific Correlation

$$\text{Corr}(\beta, T)$$

Testing the Scientific Hypothesis

A Bayesian Model

Level 1 : $p(\text{Data}|\beta, T)$

Level 2 : $p(\beta, T)$

Scientific Hypothesis

$\beta \perp\!\!\!\perp T$ under $p(\beta, T)$

The Statistical Model I

Likelihood

$$S_{ij} = \delta_j C_i \left(\frac{\nu_j}{\nu_0} \right)^{\beta_i+3} \left(\exp \left(\frac{h\nu_j}{\kappa T_i} \right) - 1 \right)^{-1} e_{ij}$$

$$\delta_j \stackrel{i.i.d}{\sim} N(0, \sigma_\delta^2), C_i \stackrel{i.i.d}{\sim} N(\mu_c, \sigma_c^2), e_{ij} \stackrel{ind}{\sim} N(0, \sigma_{ij}^2).$$

Prior

$$\beta_i | T_i \stackrel{i.i.d}{\sim} N(AT_i^B, \sigma_\beta^2)$$
$$T_i \stackrel{i.i.d}{\sim} 1_{[2,150]}(T_i) dT_i$$

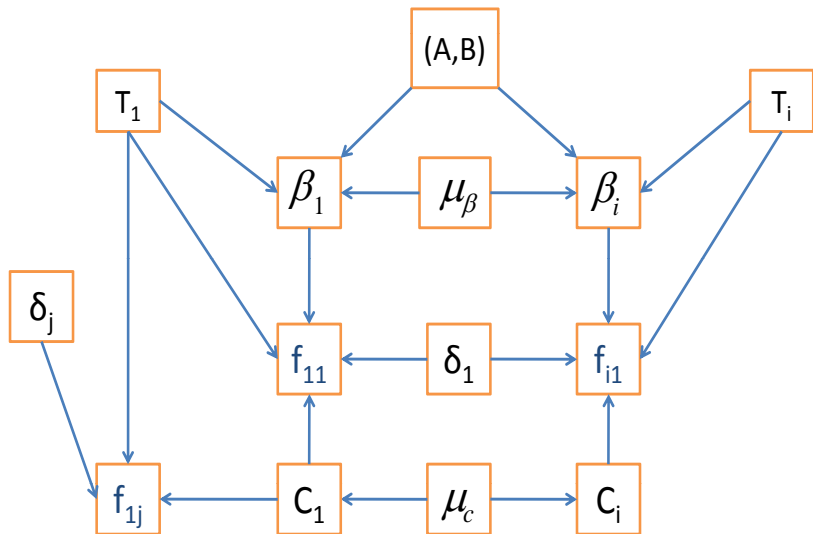
Hyper Prior

$$(\mu_c, \sigma_c^2, \sigma_\beta^2, \sigma_\delta^2) \sim d\mu_c d \ln \sigma_c^2 d \ln \sigma_\beta^2 d \ln \sigma_\delta^2$$

$$A \sim dA$$

$$B \sim 1_{[-2,2]}(B) dB$$

The Graphical Structure of the Model

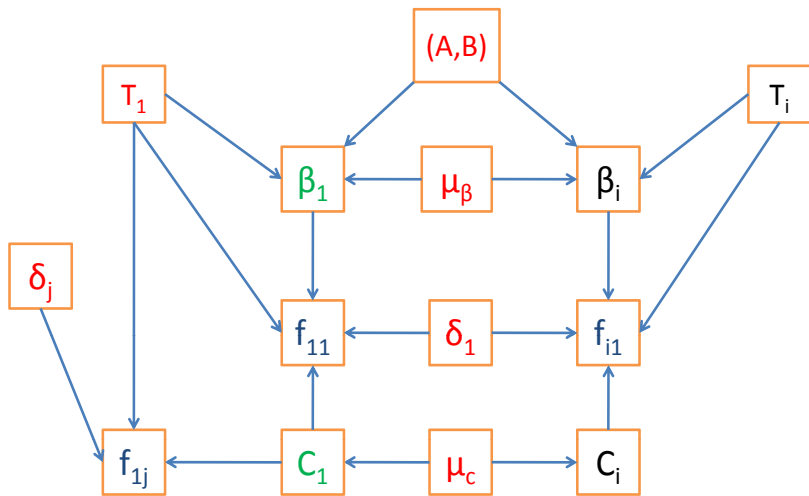


The Plain-Vanilla Gibbs Sampler

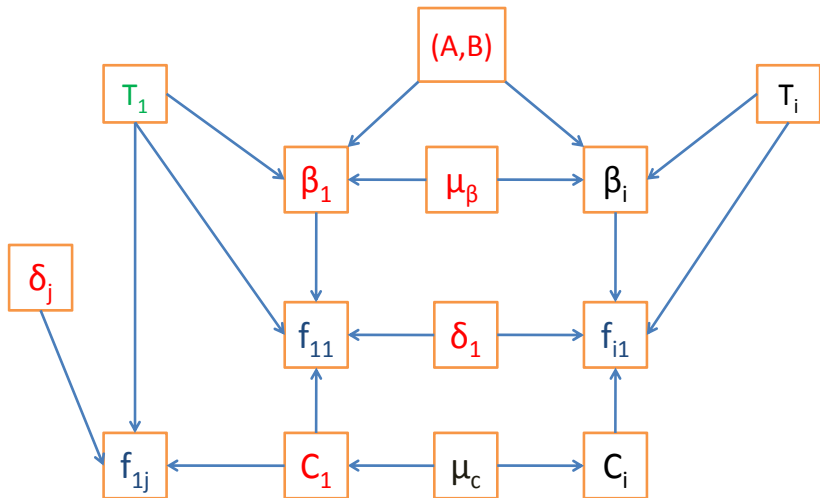
Gibbs Components

- Step I : $(\beta_i, C_i) | (S_{i1}, \delta_1), \dots, (S_{iJ}, \delta_J), T_i, \mu_c, A, B$
- Step II : $T_i | (S_{i1}, \delta_1), \dots, (S_{iJ}, \delta_J), \beta_i, C_i, A, B$
- Step III : $\delta_j | (S_{1j}, T_1, \beta_1, C_1), \dots, (S_{nj}, T_n, \beta_n, C_n)$
- Step IV : $\mu_c, \sigma_c^2 | C_1, \dots, C_n$
- Step V : $\sigma_\delta^2 | \delta_1, \dots, \delta_J$
- Step VI : $A | B, T_1, \dots, T_n, \beta_1, \dots, \beta_n$
- Step VII : $B | A, T_1, \dots, T_n, \beta_1, \dots, \beta_n$

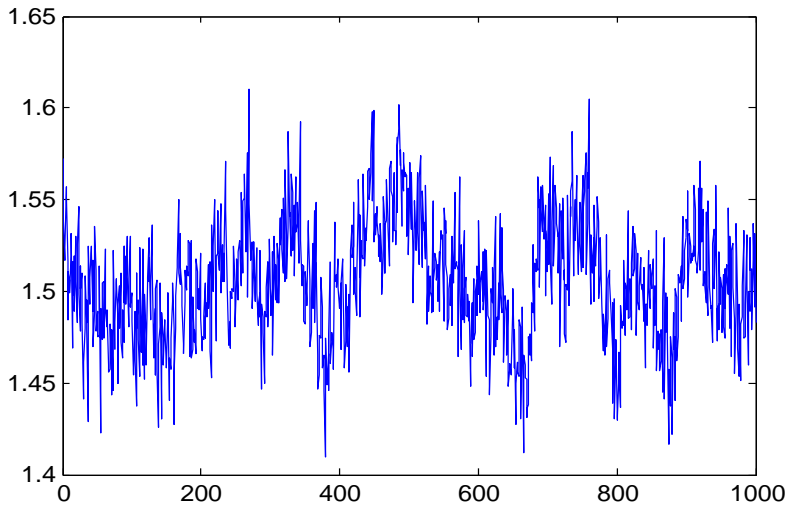
Graphical Illustration of Step 1



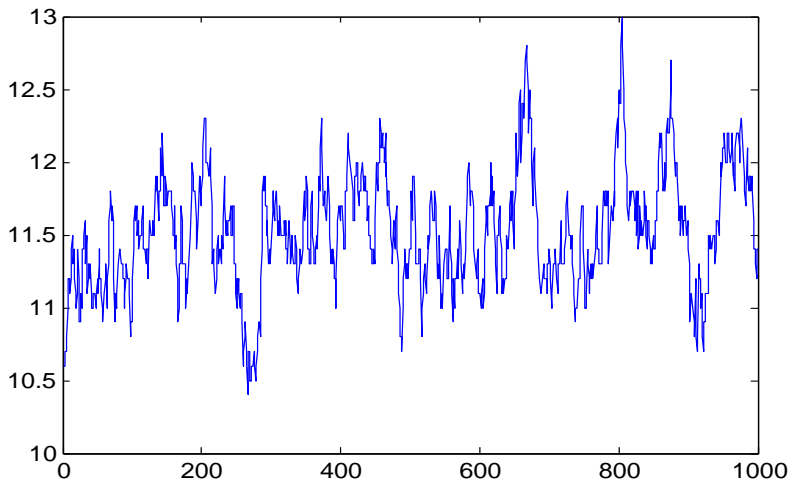
Graphical Illustration of Step II



Trace plot for β_1



Trace plot for T_1



A Better Gibbs Sampler

Gibbs Components

Step I : $(\beta_i, C_i) | (S_{i1}, \delta_1), \dots, (S_{iJ}, \delta_J), T_i, \mu_c, \sigma_c^2, \sigma_\beta^2, A, B$

Step II : $T_i | (S_{i1}, \delta_1), \dots, (S_{iJ}, \delta_J), \beta_i, \mu_c, A, B$

Step III : $\delta_j | (S_{1j}, T_1, \beta_1, C_1), \dots, (S_{nj}, T_n, \beta_n, C_n)$

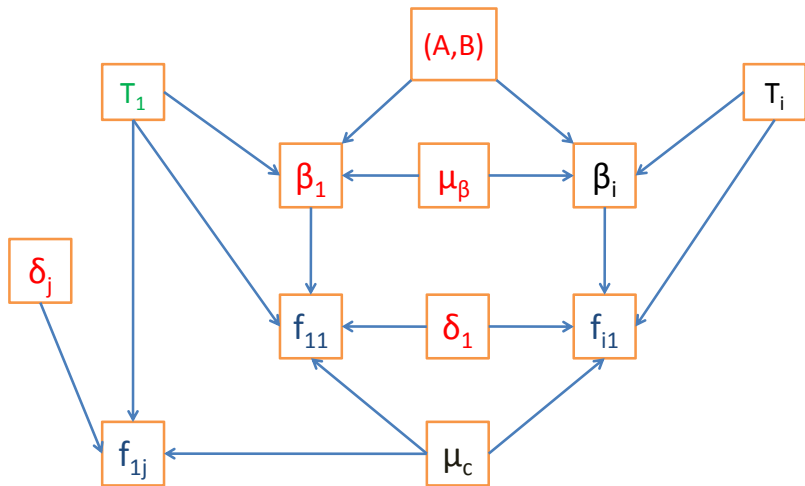
Step IV : $\mu_c, \sigma_c^2 | C_1, \dots, C_n$

Step V : $\sigma_\delta^2 | \delta_1, \dots, \delta_J$

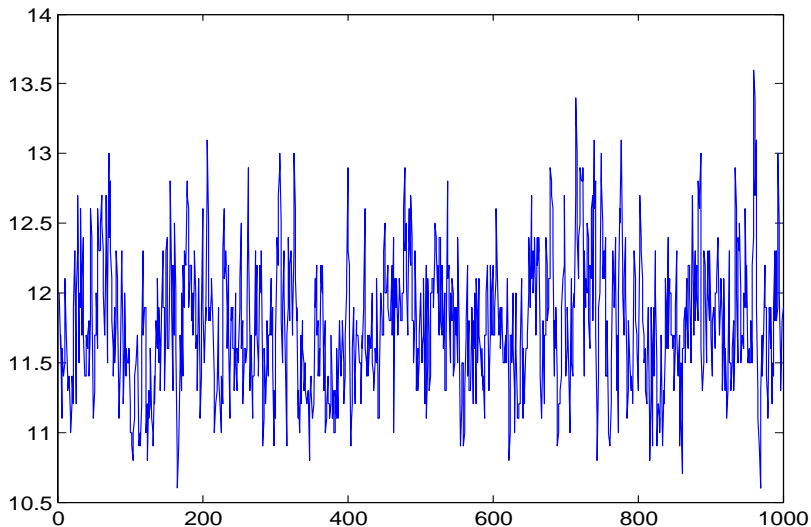
Step VI : $A | B, T_1, \dots, T_n, \beta_1, \dots, \beta_n$

Step VII : $B | A, T_1, \dots, T_n, \beta_1, \dots, \beta_n$

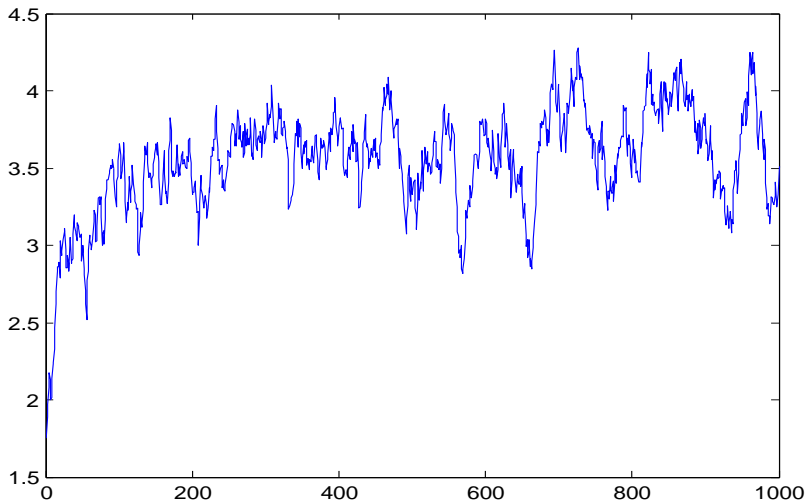
Graphical Illustration of Step II



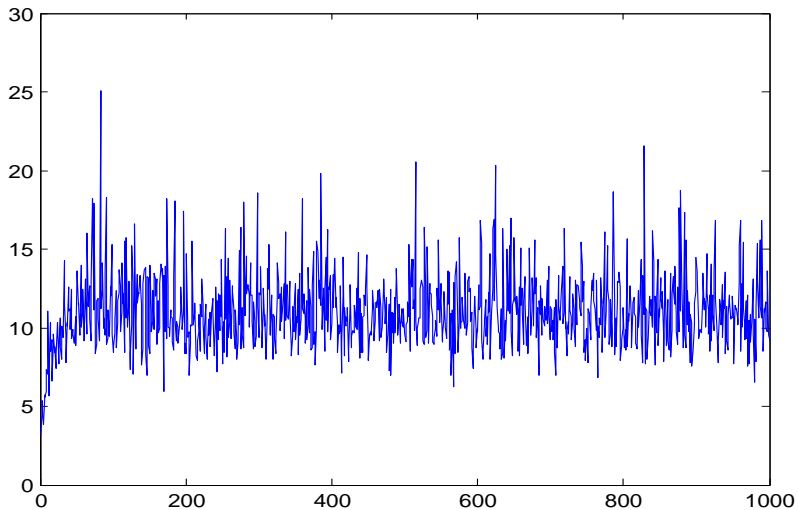
Trace plot for T_1



Application to Real Datasets



What's Wrong: Trace Plot for σ_c^2



How to incorporate the prior?

The Form of the Prior

$$\begin{aligned}(T, A, B) &\sim \pi(T, A, B) \\ \beta | T, A, B &\sim N(AT^B, \sigma_\beta^2)\end{aligned}$$

The Prior Knowledge

$$\beta \sim N(2.0, 0.2^2) .$$