Investigating Nonlinear and Stochastic Variability of Accreting Compact Objects via Recurrence Analysis

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Advisor/PI: Dr. Eric Bellm (UW)

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Outline

● Motivation:
  ○ Long-term monitoring of X-ray Binaries and Active Galaxies
  ○ Traditional time series analysis

● Methods:
  ○ Phase Space and Topology
    ■ Example: 4U 1705-44
  ○ Recurrence Plots
  ○ Quantitative recurrence analysis

● Applications:
  ○ Distinguishing between stochastic and deterministic behavior
  ○ Identifying chaos
  ○ Outstanding challenges
X-ray Binary (XRB)

Active Galactic Nuclei (AGN)
Long-term variability provides a window into the dynamics of accretion.
Time Series Analysis

Guilds

Domain based Methods

Frequency-Domain Methods

Time-Domain Methods

Examples:

Spectral Analysis

Auto-correlation
Time Series Analysis

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Examples:

Uttley+ 2002
Time Series Analysis

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Examples:

Spectral Analysis

Auto-correlation

Statistical methods

Parametric

Non-parametric

Examples:

Moving Averages

Kernel Regression

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Moreno+ 2019
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Spectral Analysis

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Statistical methods

Parametric

Non-parametric

Examples:

Moving Averages

Kernel Regression

Goal:

- Connect power spectrum and statistical features to intrinsic physical properties (black hole mass, spin, etc)

Challenges:

- Assumptions of stationarity, linearity; inconsistencies across bandwidth; influence of noise
Time Series Analysis

Guilds

Domain based Methods
- Frequency-Domain Methods
  - Spectral Analysis
- Time-Domain Methods
  - Auto-correlation

Statistical methods
- Parametric
- Non-parametric
  - Moving Averages
  - Kernel Regression

Phase-Space based methods
- Descriptive Diagrams
- Attractor Invariants
  - Poincaré Plots (return maps)
  - Recurrence Plots

Examples:
- Moving Averages
- Kernel Regression
- Poincaré Plots
- Recurrence Plots
Time Series Analysis

Guilds

Domain based Methods

Frequency-Domain Methods

Time-Domain Methods

Examples: Spectral Analysis, Auto-correlation

Statistical methods

Parametric

Non-parametric

Examples: Moving Averages, Kernel Regression

Phase-Space based methods

Descriptive Diagrams

Attractor Invariants

Examples: Phase-Space based methods

Descriptive Diagrams, Attractor Invariants

1. **Examples:**
   - Moving Averages
   - Kernel Regression

2. **Examples:**
   - Spectral Analysis
   - Auto-correlation
Time Series Analysis

Guilds

Domain based Methods

- Frequency-Domain Methods
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Examples:

Statistical methods

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Examples:

Phase-Space based methods

- Descriptive Diagrams
- Attractor Invariants
  - Poincaré Plots (return maps)
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Advantages:
Analysis for nonlinear & linear systems; more direct probe of dynamics; can be applied to nonstationary, stochastic & deterministic time series
Phase Space

Classically: position versus velocity (or coordinate vs. first derivative)

Simple harmonic oscillator

Damped harmonic oscillator
Phase Space

Damped & Driven Oscillator (Duffing equation):

\[ \ddot{x} + \delta \dot{x} + \alpha x + \beta x^3 = \gamma \cos(\omega t) \]
Phase Space encodes dynamical information
Phase Space encodes dynamical information

**Relative Rotation Rates**: How two trajectories (A and B) in phase space ‘wind’ around each other:

\[
R_{ij}(A, B) = \frac{1}{2\pi p_A p_B} \int \frac{n \cdot (\Delta r \times d\Delta r)}{\Delta r \cdot \Delta r}
\]

where \(\Delta r = [x_B(t) - x_A(t), y_B(t) - y_A(t)]\)

The set of RRRs (a set of integers) are unique to each class of differential equations.

*(Solari & Gilmore 1988)*

If the set of RRRs are the same for two systems -- they likely are produced by the same underlying attractor.

*(Birman-Williams Theorem)*
Phase Space encodes dynamical information

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Phase Space encodes dynamical information

**4U 1705-44**: a low-mass neutron star X-ray binary; **preface**: has evidence for nonlinearity

**Left**: light curve from RXTE All-sky monitor (2-12 keV)
**Right**: 2D phase from the numerical derivative of the flux

Phillipson+2018
Phase Space encodes dynamical information

Phillipson+2018
Phase Space encodes dynamical information

4U1705-44, Close Returns for p2.1 and p2.2

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Differential Phase Space Embedding

Phillipson+2018
Phase Space encodes dynamical information

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Phase Space encodes dynamical information

Q: How to generate phase space of unknown or stochastic systems?

Q: Are there ways to automate the extraction of information encoded in phase space?
Phase Space encodes dynamical information

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The Time Delay Method

The Recurrence Plot
Rossler Attractor (in 3D differential state space)
Time Delay Embedding

Rossler Attractor
(in 3D differential state space)

System dynamics is a black box

Measurable state
Time Delay Embedding

Rossler Attractor (in 3D differential state space)

Takens 1981:
For right choice of time delay and dimension (n) recovers original attractor

System dynamics is a black box

Measurable state

Finding Proper Takens transformation

$x(t) \rightarrow y(t) = (y_1(t), y_2(t), \ldots, y_n(t))$ $y_j(t) = x(t - \kappa_j), \; j = 1, 2, \ldots, n,$
Time Delay Embedding

Rossler Attractor (in 3D differential state space)

System dynamics is a black box

Measurable state

Finding Proper Takens transformation

Kaveh+ 2018
The Recurrence Plot:

Given a dynamical system represented by the trajectory “x” in a d-dimensional phase space, the recurrence matrix is defined as:

\[ R_{i,j}(\epsilon) = \Theta(\epsilon - ||\vec{x}_i - \vec{x}_j||) \text{ for } i, j = 1, ..., N, \]

where \( \epsilon \) is a threshold distance and \( \Theta(\cdot) \) is the Heaviside function.

The following condition holds for two states less than the threshold distance apart:

\[ \vec{x}_i \approx \vec{x}_j \Leftrightarrow R_{i,j} = 1. \]

The result is a binary 2D matrix -- the positions of each entry corresponds to two points in time.

Translation:
Non-zero entries tell us when two points in time are close to each other in phase space. The recurrence plot is the visualization of this binary matrix.
The Recurrence Plot:

- **Time Series**
  - White Noise
  - Sinusoidal
  - Logistic Map + Drift
  - Brownian Motion

- **Recurrence Plots**
  - “Line of Identity (LOI)”
The Recurrence Plot:

<table>
<thead>
<tr>
<th>Signal</th>
<th>RP</th>
<th>Phase Space Plot</th>
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- **Signal**
  - ECG
  - LORENZ
  - RÖSSLER

- **RP**
  - Brownian Motion
  - Sun Spots
  - White Noise

Garciá & Romo 2013
Time Series

Recurrence Plot

Phase Space

Analog: the autocorrelation function
The Recurrence Plot:
Example: X-ray Binaries!
The Recurrence Plot:

Quantify the structure in the RP:
- Recurrence Quantification Analysis (RQA)
- **Examples**: longest diagonal line, average length of diagonal or vertical lines, # lines part of a diagonal feature versus isolated points
- A total of 16 quantities
  - Diagonal features: periodicities, determinism
  - Vertical features: time invariance, state changes

Characteristic Recurrence Plots

- White Noise
- Sinusoidal
- Logistic Map + Drift
- Duffing
Significance of Recurrence Features

The Surrogate Data method (Theiler et al. 2002):

- Data-driven null hypothesis testing
- Generate surrogate light curves that have:
  - the same power spectrum (phase),
    - i.e. take Fourier transform of time series, randomize the phases, and then inverse Fourier transform to obtain the surrogate
  - the same flux distribution (shuffled),
  - or both (IAAFT)
- Apply statistical test to data and ensemble of surrogates:
  - if the data is significantly different, we rule out the hypothesis of the surrogates (e.g. correlated noise)
- Surrogates *do not* retain dynamical information and carry the same noise and systematics as the original light curve
The Surrogate Data Method

Her X-1 (XRB) 
Swift/BAT monitoring

Same PSD + flux distribution

Same PSD only

Same distribution only
Swift/BAT AGN

Hard X-ray (14 - 150 keV) monitoring of 46 AGN from the 70-month catalog, previously observed by power spectra analysis:

- PSD slope of -0.8 for all sources but one;
  Shimizu & Mushotzky 2013

3C 273

(Phillipson et al 2021a - in prep)
Swift/BAT AGN RPs

Variety of behaviors evident in RPs:

- diagonal structures: repeating behavior
- vertical/horizontal lines: trapped states
- large scale inhomogeneities: non-stationarity
- abrupt changes in texture: state changes

(Phillipson et al 2021a - in prep)
Swift/BAT AGN Recurrence Properties

Quantify the structure in the RP to find evidence for:

- Nonlinear behavior
  - Longest diagonal line length (Lmax)
- Determinism
  - Fraction of recurrences that are part of diagonal structures (DET)
- Stochastic behavior
  - Shannon entropy (randomness in the distribution of recurrences; Lentr)

Compare these measures to ensembles of surrogate data.

Are there correlations of significance of recurrence properties with physical characteristics:

- Type 1 vs. Type 2
- Obscured vs. unobscured
- Radio loud vs. radio quiet
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Ongoing Research (& challenges)

**Swift/BAT AGN:**
- Only nominal results comparing to physical characteristics of AGN
- Strong evidence for nonstationary behavior
- **Ongoing:** application to 157-month catalog

**Correlated Timing and Spectral variations for XRBs:**
- Recurrence Plots as a moving window: uncovers changes in the variability as function of time; overlaps with spectral state transitions

**Irregularly Spaced Time Series:**
- The time delay method for embedding in phase space depends on evenly sampled time series
- Other methods for embedding:
  - Legendre polynomials, numerical differentiation
- Developing python package for recurrence analysis, including an alternative recurrence plot that handles irregularly spaced time series (coded for ZTF light curves)

**Generally:** Classification of variable sources using recurrence quantities
Rebecca Phillipson  
(she/her)  
Postdoctoral Scholar | University of Washington  

Slack/Zoom  
raphilli@uw.edu  
@raphillipson  
@beckastrosaurus  

Fun with recurrence plots:  
https://colinmorris.github.io/SongSim/#/rumourhasit
Example: Diverse Variability in Active Galaxies

From distribution of diagonal lines, obtain ‘correlation entropy’, compare to stochastic surrogates — long-term variability distinguishable from stochastic, linear mechanisms

Obtain ‘correlation entropy’ — long-term variability NOT distinguishable from stochastic surrogates

(Phillipson et al 2020)
Diverse Variability in Active Galaxies

“Close Returns”: pseudo-autocorrelation function – quantifies diagonal lines as a function of time delay

Has a QPO (Smith+2018)
Example: Changes in Variability States of XRBs

- Cyg X-1 experienced a series of failed state transitions and soft states (overall MJD 51,000 to MJD 53,900; Grinberg et al. 2013).
- A second, similar transition identified by DET/RR starts to occur at approximately MJD 56,000, where a second pro-longed, very soft X-ray period occurs in 2012 (Grinberg et al. 2013).
Possible Interpretation: Disk-dominated “soft” state corresponds to high determinism and regularity.
**Example: Changes in Variability States of XRBs**

Possible Interpretation: Corona-dominated “hard” state corresponds to high trapping time (laminarity) and low determinism.

(Phillipson et al 2021b - in prep)