Time Delay Lens Modeling Challenge for the Hubble Constant Estimation

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The Hubble constant $H_0$ represents the current expansion rate of the Universe, as well as the age ($= H_0^{-1}$), size, and density of the Universe.
But there have been several different estimates of $H_0$ from various methods.

The most recent estimates from these two methods are

- CMB (Plank collaboration, 2016): $67.8 \pm 0.9 \text{ km s}^{-1}\text{Mpc}^{-1}$.
- CDL (Reiss et al., 2016): $74.3 \pm 2.1 \text{ km s}^{-1}\text{Mpc}^{-1}$.

Is this difference true (new physics) or not (within statistical uncertainty)? Improving statistical accuracy or double-checking by independent methods.
**Time delay cosmography**

*Quasar* is a highly luminous galaxy hosting a supermassive black hole at the center. Since it is extremely bright, it can be seen at a great distance.

*Video Credit: Space.com*
Strong gravitational lensing: The strong gravitational field of the intervening galaxy bends the light rays towards the Earth (like a lens), and thus we see multiple images of the same quasar in the sky.
Time delay cosmography (cont.)

Credit: NASA's Goddard Space Flight Center

Time delay: Light rays take different routes and travel through different gravitational potential, and thus their arrival times can differ $\rightarrow$ time delay!


**TIME DELAY COSMOGRAPHY (CONT.)**

Inference on $H_0$ via an equation for additional travel distance (Refsdal, 1964).

\[
\text{Speed of light (} c \text{) } \times \text{ Time delay (} \Delta t_{ij} \text{)} = \text{ Time delay distance (} D_{\Delta t}(H_0, z, \Omega) \text{)} \times \text{ Fermat potential difference (} \Delta \phi_{ij} \text{)}
\]

Image Credit: Tommaso Treu (UCLA) in “Dark Matter and Strong Lensing (2014)”

- **Time Delay** (Time series data)
- **Fermat potential** (Lens image data)
- **Current expansion rate of the universe,} H_0
Closed-form marginal posterior of $H_o$

Since $\Delta \phi_{ij} = c\frac{\Delta t_{ij}}{D_{\Delta t}(H_o, z, \Omega)}$, Marshall+ (2016) suggest (with fixed $z$ and $\Omega$)

$$
\Delta \hat{\phi}_{ij} \mid \Delta t_{ij}, H_o \sim \mathcal{N} \left[ \frac{c\Delta t_{ij}}{D_{\Delta t}(H_o)}, \sigma^2_{\Delta \hat{\phi}_{ij}} \right],
$$

$$
\Delta t_{ij} \sim \mathcal{N}(\Delta \hat{t}_{ij}, \sigma^2_{\Delta \hat{t}_{ij}}).
$$

Marginally, $\Delta \hat{\phi}_{ij} \mid H_o \sim \mathcal{N} \left[ \frac{c\Delta \hat{t}_{ij}}{D_{\Delta t}(H_o)}, \frac{c^2}{D_{\Delta t}^2(H_o)}\sigma^2_{\Delta \hat{t}_{ij}} + \sigma^2_{\Delta \hat{\phi}_{ij}} \right].$

All but $H_o$ (\sim Unif[50, 90] a priori) are known or (at least) estimable!

- $\Delta \hat{\phi}_{ij}$: Fermat potential difference estimate between images $i$ & $j$.
- $\sigma^2_{\Delta \hat{\phi}_{ij}}$: An uncertainty estimate (variance) of $\Delta \hat{\phi}_{ij}$.
- $\Delta \hat{t}_{ij}$: A time delay estimate between images $i$ and image $j$.
- $\sigma^2_{\Delta \hat{t}_{ij}}$: An uncertainty estimate (variance) of $\Delta \hat{t}_{ij}$.
- $D_{\Delta t}(H_o)$: The time delay distance, a deterministic function of $H_o$. 

Data for a doubly-lensed quasar are two time series (light curves) with known measurement errors.

We can estimate $\Delta$ by the horizontal shift between two time series.
**Time Delay Challenge** (Dobler et al., 2015; Liao et al., 2015)

- A blind competition held by 8 astrophysicists from 2013 to 2014.
- Goal was to improve existing estimation methods.
- 5,000+ simulated data sets with some time delays.
- 13 teams blindly analyzed the simulated data sets.

![Image Credit: HBO website](Image Credit: HBO website)
Time Delay Lens Modeling Challenge

Another blind competition to improve lens-modeling methods (Ding+, 2018+).

Image Credit: https://www.youtube.com/watch?v=iE8x9kDHCFo

Modeling the lens: Lens mass $\rightarrow$ lens potential $\rightarrow$ Fermat potential.
(The mass density is the second derivative of the lens potential.)
Outline of TDLMC

The time delay lens modeling challenge is a three-step blind competition composed of four rungs. Each rung shares the same (simulated) Hubble constant. The difficulty increases as we move up higher rungs.

- **Rung 0**: The true $H_o$ is disclosed for participant’s reference. Two images, one for a doubly-lensed image and the other for a quadruply-lensed image. The point spread function is provided.
- **Rung 1**: 16 images. Due was Sep 8. Real galaxy images for realistic surface brightness are used for simulations.
- **Rung 2**: 16 images. Due is Jan 8. On top of Rung 1’s difficulty, a guess of the point spread function is provided for each image.
- **Rung 3**: 16 images. Due is May 8. In addition to all challenges in Rungs 1 and 2, images are generated by massive early-type galaxies.
Image Data

Image data (from the left): (i) Light intensity (brightness) in $100 \times 100$ pixels, (ii) measurement errors, (iii) point spread function (used in (i)).
**Lens modeling**

We model (i) lens mass, (ii) lens brightness, and (iii) source brightness.

Angular positions (unknown param.)

\[ \beta: \text{Source position in the absence of the lens.} \]
\[ \theta: \text{Lensed image position.} \]
\[ \hat{\alpha}: \text{Deflection angle.} \]
\[ \alpha(\theta): \text{Scaled deflection angle for the image at } \theta. \]
\[ \beta = \theta - \alpha(\theta): \text{The lens equation.} \]

Given the lens mass distribution, we can infer \( \alpha(\theta) \) and \( \beta \).
Lens modeling (cont.)

Outline of lens modeling:

1. Setting (choosing) a lens mass density function, $\Sigma(D_d \theta)$.
2. Deriving a dimensionless surface mass density, $\kappa(\theta) = \frac{\Sigma(D_d \theta)}{\Sigma_{cr}}$, where $\Sigma_{cr}$ is the critical surface mass density. For example, with an elliptical power-law mass density,

$$
\kappa(\theta_{i1}, \theta_{i2}) = \frac{3 - \gamma'}{2} \left( \frac{\sqrt{q \theta_{i1}^2 + \theta_{i2}^2/q}}{\theta_E} \right)^{1-\gamma'},
$$

where $\theta_E$ is the radius of Einstein ring, $q$ is the ellipticity, and $\gamma'$ is the radial power-law slope.
3. Computing $\alpha(\theta) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2 \theta' \kappa(\theta') \frac{\theta - \theta'}{|\theta - \theta'|^2}$.
4. Computing lens potential: $\psi(\theta) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2 \theta' \kappa(\theta') \log |\theta - \theta'|$.
5. Finally, the Fermat potential is computed as $\phi(\theta) = \frac{\alpha(\theta)^2}{2} - \psi(\theta)$. 
The number of unknown parameters is 22 (double) or 28 (quad).

We use a Python package `lenstronomy` (Birrer+, 2015, 2016, 2018) to fit a lens model on the image data. Fitting the model is a two-step procedure; (i) particle swarm optimization to find a global optimum of 20–26 parameters; (ii) MCMC initialized at the global optimum.

Given the observed data (1st), it reconstructs the image (estimate) based on the fitted model (2nd), and shows a residual plot (3rd = 2nd − 1st).
Result of Rung 0

Observed images (1st column), estimated images (2nd column), and residuals (3rd column).
Result of Rung 0 (cont.)

Posterior of one Fermat potential difference from a double-image.

\[ \Delta \phi_{BA} \]

Posterials of three Fermat potential differences from a quad-image.

\[ \Delta \phi_{BA}, \Delta \phi_{CA}, \Delta \phi_{DA} \]
The marginal posterior distribution of $H_o$ is closed-form.

\[
\Delta \hat{\phi}_{ij} \mid H_o \sim N \left[ \frac{c \Delta \hat{t}_{ij}}{D_{\Delta t}(H_o)}, \frac{c^2}{D_{\Delta t}^2(H_o)} \sigma^2_{\Delta \hat{t}_{ij}} + \sigma^2_{\Delta \hat{\phi}_{ij}} \right].
\]

\[H_o \sim \text{Unif}(50, 90).\]

The resulting posterior of $H_o$ based on the four pairs of $\Delta \hat{\phi}_{ij}$ and $\sigma^2_{\Delta \hat{\phi}_{ij}}$ (time delays $\Delta \hat{t}_{ij}$ and their uncertainties $\sigma^2_{\Delta \hat{t}_{ij}}$ are given):

![Graph showing the posterior distribution of $H_o$.]
Result of Rung 1

16 lens image data sets (simulated under the same $H_o$) to be analyzed.
Result of Rung 1 (cont.)

Analytic sequence for one image data set as an example.

1. We fit our model on this image data set with 12 variations each for a combination of four different values of point spread function error inflation (1%, 5%, 10%, 20%) and three different lens light models (1, 2 or 3 lens light models) → 12 Fermat potential difference estimates and their uncertainties.
2. We derive the posterior of $H_o$ using each pair of Fermat potential difference estimate and uncertainty, leading to 12 posteriors of $H_o$: 

![Likelihood plots](image)
3. We collect pairs of Fermat potential estimate and uncertainty that result in the posterior mode of $H_o$ between 50 and 90 (between red vertical dashed lines).
4. We take an average of the collected pairs in three ways:

   (1) Weighted average and variance

   \[ \Delta \hat{\phi}_{AB} = \frac{\sum_{i=6}^{7} \Delta \hat{\phi}_{AB}^{(i)} / \sigma_{\Delta \hat{\phi}_{AB}}^{2}}{\sum_{i=6}^{7} 1 / \sigma_{\Delta \hat{\phi}_{AB}}^{2}} \quad \text{and} \quad \sigma_{\Delta \hat{\phi}_{AB}}^{2} = \frac{1}{\sum_{i=6}^{7} 1 / \sigma_{\Delta \hat{\phi}_{AB}}^{2}}. \]

   (2) Sample mean of estimates, and sample mean of variance

   \[ \Delta \hat{\phi}_{AB} = \frac{1}{2} \sum_{i=6}^{7} \Delta \hat{\phi}_{AB}^{(i)} \quad \text{and} \quad \sigma_{\Delta \hat{\phi}_{AB}}^{2} = \frac{1}{2} \sum_{i=6}^{7} \sigma_{\Delta \hat{\phi}_{AB}}^{2}. \]

   (3) Sample mean of estimates, and sample variance of estimates

   \[ \Delta \hat{\phi}_{AB} = \frac{1}{2} \sum_{i=6}^{7} \Delta \hat{\phi}_{AB}^{(i)} \quad \text{and} \quad \sigma_{\Delta \hat{\phi}_{AB}}^{2} = \sum_{i=6}^{7} (\Delta \hat{\phi}_{AB}^{(i)} - \Delta \hat{\phi}_{AB})^{2}. \]
Result of Rung 1 (cont.)

We applied the estimation routine to 16 images and could successfully analyze 11 images out of 16, leading to 25 Fermat potential difference estimates and their uncertainties.
Result of Rung 1 (cont.)

The following three estimates are reported:

(1) Weighted average
(2) Sample mean of estimates and sample mean of variances
(3) Sample mean and variance of estimates
Result of Rung 1 (cont.)

The two lenses below (marked by red question marks) result in the $H_o$ estimates close to 90. What about removing them?
The following three estimates are additionally reported:

(1) Weighted average

(2) Sample mean of estimates and sample mean of variances

(3) Sample mean and variance of estimates
Our contribution is to provide a way to combine Fermat potential difference estimates obtained from independent image data sets.

- The weighted average method works pretty well.
- The third way to make the representative estimate, i.e., the sample mean and variance of estimates (not using the uncertainty estimates) will not be used for rung 2.
- For rung 2, I will not put my personal curiosity (no additional three submissions), trusting what the data tell us.
- The due for rung 2 is Jan 5.