Constraining $\sigma_8$ and $\Omega_m$ with the Velocity Distribution Function

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*Scheinbare Geschwindigkeiten im Comahaufen.*

\[
\begin{align*}
v & = 8500 \text{ km/sek} & 6900 \text{ km/sek} \\
7900 & & 6700 \\
7600 & & 6600 \\
7000 & & 5100 (\text{?})
\end{align*}
\]
In order to obtain the observed value of an average Doppler effect of 1000 km/s or more, the average density in the Coma system would have to be at least 400 times larger than that derived on the grounds of observations of luminous matter. If this would be confirmed we would get the surprising result that dark matter is present in much greater amount than luminous matter.
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In order to obtain the observed value of an average Doppler effect of 1000 km/s or more,
Virial Theorem

\[ 2K + U = 0 \]
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\[ K = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 \]

\[ U = -\sum_{i=1}^{N} \sum_{j<i} \frac{G m_i m_j}{r_{i,j}} \]
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\[ \sigma_v^2 = \alpha_R \frac{GM}{R} \]

\[ \sigma_v \propto M^{1/3} \]
$M(\sigma)$ POWER LAW
M(σ) Power Law

Reasons for scatter:
- triaxiality
- infalling matter or mergers
- galaxy selection
- bias between dark matter particle dispersion & galaxy dispersion
Signatures in the Velocity PDF

- Infalling matter & mergers lead to flatter velocity distributions
THE CLUSTER CATALOG
Ideal Cluster Catalog
Impure Cluster Catalog

Line of sight velocity

Aperture
THE HALO MASS FUNCTION
THE HALO MASS FUNCTION

It’s impolite to ask a galaxy cluster its mass.
Halo Mass Function

\[ \log[dn/d\log(M)] = \log(M(M_{\odot} h^{-1})) \]

\[ M = M_{\text{true}} \]

\[ \sigma_8 = 0.860, \Omega_m = 0.320 \]
\[ \sigma_8 = 0.860, \Omega_m = 0.300 \]
\[ \sigma_8 = 0.823, \Omega_m = 0.307 \]
\[ \sigma_8 = 0.780, \Omega_m = 0.320 \]
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Eddington Bias in Dynamical Masses

Scatter in the $M(\sigma)$ relationship, coupled with the steeply-declining HMF, alters the observed HMF.
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Halo Mass Function

\[ M = M_{\text{dyn}} \]

\[
\log[dn/d\log(M)] = \log\left(\frac{M}{M_{\odot} h^{-1}}\right)
\]

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**χ^2** analysis for constraining $\sigma_8$ & $\Omega_m$

$$\chi^2(y|\sigma_8, \Omega_m) = (\bar{y} - y^*)^T \hat{\Psi}^{-1} (\bar{y} - y^*)$$

- Compare the mock observed HMF to that predicted by an analytic HMF.
Constraining Cosmological Models

- Measurement error biases to low $\Omega_m$ and high $\sigma_8$.
- Fiducial model lies outside of the 99% likelihood contour.
A FORWARD MODELING APPROACH:
The Velocity Distribution Function (Mocks)
Velocity PDF

\[ \log [PDF(\mathbf{v})] \]

\[ 14.50 \leq \log(M) < 14.75 \]
\[ 14.75 \leq \log(M) < 15.00 \]
\[ 15.00 \leq \log(M) < 15.25 \]
\[ 15.25 \leq \log(M) < 15.50 \]
Velocity Distribution Function

\[
\frac{dn}{dv}(v) \equiv \frac{1}{V} \sum_{i=1}^{N} [\text{PDF}(\mid v \mid)]_i
\]

- Sum the most massive or the richest - or simply the observed - clusters in a volume
Velocity Distribution Function

\[ \log[dn/dv] = v_{\text{true}} \]

\[ v = v_{\text{true}} \]

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\end{align*}
\]
Velocity Distribution Function with velocity error

\[ \log[dn/dv] = v = v_{\text{err}} \]
$\chi^2$ analysis for constraining $\sigma_8$ & $\Omega_m$

$$\chi^2(y|\sigma_8, \Omega_m) = (\bar{y} - y^*)^T \hat{\Psi}^{-1} (\bar{y} - y^*)$$
$\chi^2$ analysis for constraining $\sigma_8$ & $\Omega_m$

$$\chi^2(y | \sigma_8, \Omega_m) = (\bar{y} - y^*)^T \hat{\Psi}^{-1} (\bar{y} - y^*)$$
Constraining Cosmological Models with the VDF

- Constraints can be approximated as a band in the $\Omega_m$-$\sigma_8$ plane.
Constraining Cosmological Models with the VDF

- Measurement error introduces a nearly-negligible bias.
Constraining Cosmological Models with the VDF

- HMF and VDF give similar constraints when true cluster properties are known.
- VDF is less sensitive to measurement error than the HMF.
PRELIMINARY COSMOLOGICAL CONSTRAINTS WITH THE VDF APPLIED TO HECS-SZ CLUSTERS
HeCS-SZ Clusters

• Spectroscopic follow up of an SZ-complete survey of 83 clusters
• Selected from SDSS DR6 and DR10
• $z<0.3$ with a footprint of 20%-28% of the full sky

HeCS-SZ: the Hectospec Survey of Sunyaev-Zeldovich-Selected Clusters.
Kenneth J. Rines, Margaret J. Geller, Antonaldo Diaferio, and Ho Seong Hwang
Interlopers

Line of sight velocity

Aperture
HeCS-SZ Clusters

The figure shows a distribution of clusters with red markers indicating the HeCS data and blue markers indicating the mock data. The $z$ axis represents redshift, and the $N$ axis represents the number of clusters. The data points are compared at different redshifts, with error bars indicating the uncertainty. The title indicates the study of HeCS-SZ Clusters.
HeCS-SZ Clusters

![Graph showing the distribution of HeCS-SZ clusters with Mocks.](image-url)
HeCS-SZ Clusters

![Graph showing distribution of M500/(10^{14} M_\odot h^{-1}) vs. redshift (z) for different populations, including selection, HeCS-SZ, and mock.](image)
Radial Distribution of Galaxies

![Graph showing RDF with different lines and shading for HeCS-SZ and mocks. The x-axis represents $R_{sep}$ (Mpc $h^{-1}$) ranging from 0.3 to 1.1, and the y-axis represents RDF ranging from 0.9 to 1.4. The graph includes shaded regions for different models.]
\[ \log\left[\frac{dn}{dv}\right] = 0 \]

- \( \sigma_8 = 0.900, \Omega_m = 0.270 \)
- \( \sigma_8 = 0.900, \Omega_m = 0.340 \)
- \( \sigma_8 = 0.823, \Omega_m = 0.307 \)
- \( \sigma_8 = 0.730, \Omega_m = 0.270 \)
- \( \sigma_8 = 0.730, \Omega_m = 0.340 \)
Planck SZ (2014)

CMB TT
Planck 2016 CMB & SZ Cluster Constraints
Highlights

• Forward modeling with the Velocity Distribution Function reduces bias in cosmological constraints caused by measurement error.

• Preliminary analysis of the HeCS-SZ clusters shows a tension with the CMB TT constraints (but in agreement with other LSS probes).